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THE TUTORIAL PHYSICS.

VOLUME IV.

A TEXT BOOK OF MAGNETISM AND ELECTRICITY.

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NOTE TO THE FOURTH EDITION.

IN this edition much new matter appears in the additional chapter on the practical application of the science and in the appendix. For most of this Dr. R. H. Jude is responsible. These additions, it is hoped, will bring the work quite up to date.

As in the first edition, the scope of the work is still of a strictly elementary character, but the author hopes that readers will find in it a suitable foundation for a more detailed and advanced study of the science.

October, 1899.

PREFACE TO FIRST EDITION.

THIS Text-Book forms a companion volume to my *Text-Books of Heat and Light*, and, like them, is primarily written for the use of candidates for the Matriculation, Intermediate Science, and Preliminary Scientific Examinations of the University of London. The articles dealing with subjects outside the requirements of the Matriculation Examination are marked with an asterisk, and may be omitted by students preparing for that examination, or by readers desirous of taking an elementary course. The treatment of the subject throughout is, however, of a strictly elementary character, and the book will, I hope, be found suitable for general use as an elementary introduction to the study of electrical science.

I have to acknowledge the help of Mr. W. P. Winter, B.Sc., in the preparation of diagrams and in the revision of the proofs.

I shall feel indebted to all readers who may be kind enough to communicate to me any errata which they meet with, or any suggestions which occur to them.

R. W. S.

September 27th, 1892.

CONTENTS.

PART I.

ELECTROSTATICS.

			PAGE
CHAPTER	I.	ELECTRIFICATION	1
		CALCULATIONS	25
,,	II.	ELECTROSTATIC INDUCTION	29
,,	III.	DISTRIBUTION OF ELECTRICITY ON CONDUCTORS	52
,,	IV.	FRICTIONAL ELECTRICAL MACHINES	61
,,	V.	ELECTRICAL POTENTIAL AND CAPACITY	66
		CALCULATIONS	94
,,	VI.	INDUCTION ELECTRICAL MACHINES	108

PART II.

MAGNETISM.

,,	VII.	FUNDAMENTAL PHENOMENA	117
,,	VIII.	TERRESTRIAL MAGNETISM	154
,,	IX.	SIMPLE MAGNETIC MEASUREMENTS	173
		CALCULATIONS	181

PART III.

CURRENT ELECTRICITY.

		PAGE
CHAPTER X.	INTRODUCTORY	185
,, XI.	GENERAL EFFECTS OF CURRENTS	204
,, XII.	MAGNETIC EFFECTS OF CURRENTS	223
,, XIII.	OHM'S LAW	246
	TABLE OF ELECTRICAL UNITS	275
,, XIV.	CHEMICAL EFFECTS OF CURRENTS	276
,, XV.	HEATING EFFECTS OF CURRENTS	282
,, XVI.	MAGNETO-ELECTRIC INDUCTION	290
	CALCULATIONS	310
,, XVII.	THERMO-ELECTRICITY: ELEMENTARY FACTS	323
,, XVIII.	PRACTICAL APPLICATIONS	331

APPENDIX FOR SCIENCE AND ART STUDENTS:

	A. SPARK AND BRUSH DISCHARGE	347
	B. FORCE EXERTED ON A CONDUCTOR CARRYING A CURRENT	349
	C. MUTUAL ACTION OF CURRENTS	350
	D. RESISTANCE OF LIQUIDS	352
	E. SECONDARY BATTERIES	355
	F. CALCULATION OF THE E. M. F.	358
	G. COEFFICIENT OF SELF INDUCTION	359
APPENDIX ON TRIGONOMETRY		361
RESULTS IN MENSURATION		369
ANSWERS		370
INDEX		373

ELECTRICITY AND MAGNETISM

PART I.

ELECTROSTATICS.

CHAPTER I.

ELECTRIFICATION.

1. Introductory. If a piece of sealing wax be lightly rubbed with a piece of flannel it acquires the peculiar property of being able to attract light bodies, such as cork dust, fine shreds of paper, pieces of pith, and other similar objects. Many other substances beside sealing wax exhibit this property, and Thales of Miletus, who lived about 600 B.C., mentions amber as one of these. It is probable that amber and jet were the only substances known by the ancients to possess this property, and it was not until the year 1600 A.D. that Dr. Gilbert, physician to Queen Elizabeth, showed by experiment that many other substances are similarly affected by friction. Such substances he called *electrics*, from the Greek word for amber (*ēlektron*), and since his time the name "electricity" has been applied to the agent to which these and other electrical phenomena are due. An electric, when excited so as to be capable of exercising this property of attraction, is said to be *electrified* or *charged with electricity*. The question as to the real nature of this agent, electricity, need not here be discussed, for we shall be chiefly concerned with simple experimental facts; but the

student cannot too soon learn that, unlike heat, light, and sound, electricity is not in itself a form of energy; nor is it a form of matter in the ordinary meaning of the term. In the past, men of science were led to consider electricity as a subtle material fluid, and the nomenclature of the subject is based on the fluid theories then framed. This nomenclature is still, in some measure, retained, but the theories have been abandoned in their original terms to reappear in a new bearing but in a very similar form. Remembering this, the student will be prepared to read of the "flow" of electricity without assuming that any material fluid theory is thereby endorsed, and he will ultimately find that the mode of thought and language acquired is readily adapted to the more modern views of the science.

Fig. 1.

2. Attraction of Neutral Objects by Electrified Bodies. This simple phenomenon derives importance from the fact, already mentioned, of its being the starting-point of electrical science. It supplies the simplest method of testing whether a body is electrified or not; and until we are able to employ more satisfactory methods we shall consider this a sufficient test of electrification.

If a rod of sealing wax be lightly rubbed with warm, dry flannel or fur, and then held near a few pith balls or other light objects, it will be found to attract them strongly. A rod of ebonite rubbed with fur, or a glass rod or tube rubbed with silk, will be found to exercise a similar attraction. These substances are therefore said to be electrified and, by experimenting in a similar way, it will be found that many substances can be electrified by friction with a suitable rubber. In practice, instead of employing a number of light objects to indicate the attraction of an electrified body, it is more convenient to suspend a single pith ball by a thread from a suitable stand (Fig. 1), and to test for the electrification of any body by bringing it near the suspended ball.

This arrangement is very sensitive, and can be employed to detect very feeble electrification. It is, in fact, a simple form of *electroscope* or instrument for indicating whether a body is electrified or not. Other convenient forms of this instrument may be readily devised by the student; the earliest form was that employed by Dr. Gilbert, and consisted (Fig. 2) of a stift straw freely balanced on a needle point.

Let us now consider whether this property of attraction exhibited by *electrified* bodies is exercised generally and on all substances, or whether it is confined to very light substances. On consideration, we at once conclude that in all probability the property is general, but that light substances are chosen to exhibit it, because, being light, they are easily put in motion by the attracting force.

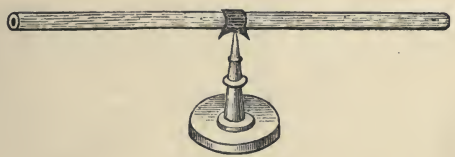


Fig. 2.

To verify this then, it is evident that if the body to be attracted be not light it must be suspended or pivoted in some way so as to be free to move For example, take a lath or rod of wood and suspend it horizontally by a thread, or balance it on the bottom of a small Florence flask. On holding an excited glass tube near one end of the rod it will be seen that the latter is attracted, and rotates so as to approach the tube.

We must further determine whether this attraction is mutual—that is, whether the electrified body, in addition to attracting other bodies, is itself attracted by these bodies. Newton's third law of motion at once tells us that this must be the case, and we can verify the fact by suspending the electrified body (Fig. 3) in a wire stirrup attached to a *silk* thread. When thus suspended it will

be found that it is attracted by any body brought near it, just as the rod of wood mentioned in a previous experiment was attracted by the electrified body.

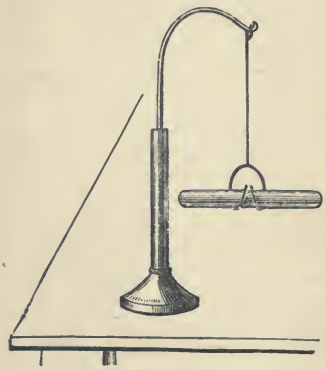


Fig. 3.

So far, then, we see that some substances on being rubbed by others acquire the property of attracting *neutral* bodies or bodies in their normal condition, and that this attraction is general * and mutual.

3. Electrification by Friction. If *any* two bodies be rubbed together it will be found that, in the majority of cases, neither of the bodies will, under ordinary conditions, exhibit the property referred to above. In a few cases, however, it may be found that one of the bodies is distinctly electrified while the other is not; and in one or two cases, it is possible that both bodies may exhibit feeble electrification. If we choose one of the substances showing distinct electrification, and test it by rubbing it with others, we shall find that its power of attracting light substances varies with the substance used as a rubber—that is, the electrification produced on a given substance varies with the rubber producing it, and it is found that if the rubber be exactly similar physically to the substance rubbed, then no electrification whatever is produced. It can also be shown that the electrification is confined to the external surface, for if the body be made in the form of a tube it will be found that the electrification produced on it, under given conditions, is the same as on a solid rod of the same diameter, and is quite independent of the thickness of the walls of the tube.

On experiment it will also be noticed that only that portion of the surface of the body which is subjected to

* See Art. 11.

friction becomes electrified; for example, if one end of a rod of sealing wax be rubbed with flannel, only that end, and no other portion of the surface of the rod, will be excited. This shows that, in the cases here considered, the electrification produced on one portion of the surface does not spread over the whole surface, but appears to be confined to the area on which it is produced.

Again, if the substance be excited so as to exhibit distinct electrification, and then tested from time to time, it will be found that it gradually loses its electrification, and in a few minutes returns to its original *neutral* condition. This loss of electrification will be found to depend on external circumstances; for example, if the air is moist the rate of loss is very rapid, but if dry, as on a bright frosty day, the body may remain electrified for hours. The same fact is emphasised by holding a freshly excited body in a current of steam or dipping it into water; it at once loses its electrification, and returns to the neutral state. These experiments show that when *dry* air is in contact with an electrified surface it tends to prevent loss of electrification, but that the presence of moisture * tends to accelerate the loss. Similar experiments would show that all dry gases tend to prevent loss of electrification, but that liquids behave in various ways; some, such as water, mercury, dilute acids, and solutions of metallic salts, etc., admit of instant loss of electrification, while others, including oils and various organic compounds, more or less completely prevent this loss. Similarly, if the excited body be drawn through the hand it will be found to have almost entirely lost its electrification, thus proving that contact with the hand or person of the operator results in loss of electrification.

The action of solids in this relation is less easily determined by direct experiment. There is evidently no difficulty in securing intimate contact between an electrified surface and a gas or liquid, but with a solid good contact

* Dry aqueous vapour is *itself* an insulator; the loss of electrification resulting from the presence of moisture is due to the *conducting* film of liquid deposited on the surface of the conductor.

can be obtained at only a few points, and the loss of electrification produced by such contact would depend more upon the action of the intervening air-film than upon that of the solid. With one solid, however, the electrified body itself, there is no difficulty, for there is already intimate contact between the electrified portion of the surface and the adjacent portions of the same surface. Hence, it is evident that if the substance of a body be such as to admit of loss of electrification from an electrified surface with which it is in contact, then it will be impossible, without special precautions, to electrify that substance. For if any portion of the surface be electrified by friction it immediately loses its electrification through adjacent portions and the hand of the experimenter, and thus electrification is lost as quickly as it is produced. Similarly, if a body can be electrified, it must evidently be composed of a substance which does not admit of loss of electrification. We are thus able to divide all substances, solid and fluid, into two classes—one class composed of substances which, when placed in contact with an electrified surface, admit of loss of electrification, the other composed of substances which, under similar circumstances, prevent this loss. The substances in the former class are called *conductors*, those in the latter *insulators*.

We are now able to understand why electrification is not always evident when any two substances are rubbed together. It is not because none is produced, but because its exhibition depends upon whether the substances employed are insulators or conductors. If both are conductors, then neither appears to be electrified; if one is an insulator and the other a conductor, then the former becomes electrified, the latter remaining neutral; if both are insulators, then both may exhibit electrification.

That electrification is produced on a conductor may be made evident by *insulating* it—that is, mounting it on a handle or stand of insulating substance, and then rubbing it with some suitable substance. The escape of the electrification is prevented by the insulator, and the conductor becomes electrified, but it will be found that the electri-

fication appears over its whole surface, although only a small portion of that surface may have been directly excited by friction. This is evidently a result to be expected from the nature of the substance, and corresponds to the fact mentioned above, that the electrification on the surface of an insulator is confined to the area directly subjected to friction.

Thus if *any* two dissimilar bodies be *first insulated* and then rubbed together, it may be shown that both are electrified. Further, if two insulated *conductors* be put in contact it can be shown that, without any friction taking place, both become electrified, but the electrification developed is extremely feeble. This indicates that the true cause of electrification in the cases considered above is *contact of dissimilar substances* and not friction; but in the case of insulators, intimate contact at nearly every point on the surface is necessary, for the electrification cannot, as in the case of a conductor, spread over the surface from one or two points of contact. Friction secures this intimate contact, and helps the transfer of electrification along the surface. It is thus evident that the degree of electrification produced in any given case cannot depend on the mechanical work done in friction but rather on the nature of the substances considered.

We may now summarise by stating that the contact of any two dissimilar substances tends to produce electrification of both, and that the degree of electrification produced depends only on the nature of the substances and the conditions of contact.

4. Conductors and Insulators. The general meaning of the terms insulator and conductor has been given in the preceding article. From the explanation there given we know that the contact of a conductor with an electrified body causes loss of electrification, while contact with an insulator results in no such loss. It must, however, be remembered that this is only generally true. The terms conductor and insulator are comparative terms, there being no perfect conducting or insulating substance. An insulator may be considered as a very bad conductor, or a conductor

as an extremely bad insulator; and, as might be expected, there are many substances, such as wood, ivory, stone, paper, etc., which occupy an intermediate position, being neither good conductors nor good insulators. Hence, in practice, all substances admit of some loss of electrification —that is, all substances are conductors,—but if the loss is inappreciable then, for all practical purposes, the substance is an insulator or *non-conductor*; if, however, the loss is readily appreciable by experiment, the substance is classed as a conductor in the usual meaning of the term. Further, the conducting power of any given substance often varies with its physical condition; for example, glass at ordinary temperatures is a typical insulator, but if its temperature be raised its conducting power increases, and at temperatures just below a red heat it is a fairly good conductor. Similarly the conducting power of the different forms of carbon is very different; the diamond acts as an insulator, while the dense carbon obtained from gas retorts is a good conductor. The properties of insulators and conductors may be illustrated by a great many simple experiments. In 1729, Stephen Gray, who first directed attention to the phenomena of conduction and insulation, found that a cork inserted into the end of an electrified glass tube exhibited attraction for light bodies. Similarly a rod of wood stuck into the cork was found to be electrified, and even a pith-ball pendulum attached to this rod by a long hemp-rope exhibited slight electrification. If, however, the ball be suspended by a *dry* silk thread the ball does not become electrified, showing dry silk to be an insulator, but if the thread be moistened it at once acts as a conductor, and the ball becomes charged.

These experiments are readily repeated, any simple form of electroscope being employed to detect the electrification; and it may be easily shown that electrification is readily conducted from one conductor to another through considerable lengths of conducting substance. For example, if a moist cotton thread or a metal wire several yards long be supported on silk loops and connected at each end with an insulated conductor, it will be found that if one of the con-

ductors be directly electrified the other also becomes electrified, and can be made to affect any electroscopic * arrangement near it.

If an insulated electrified conductor be placed in contact with another insulated but non-electrified conductor, it will be found that the former at once transfers a portion of its electrification to the latter; that is, an electrified conductor readily communicates its electrification to any other conductor brought into contact with it. If, however, an electrified insulator be brought into contact with a conductor, little or no transfer of electrification takes place. At the point or points of contact direct transfer may take place, but the portion of the surface from which this transfer takes place is confined to the actual points of contact, and the electrification of the rest of the surface is unaffected.

Similarly, if an electrified conductor be brought into contact with an insulator, transfer of electrification takes place only at the points of contact. These facts emphasise the distinctive properties of insulators and conductors, and correspond to those given in Art. 3 in connection with the electrification of conductors and insulators. This transfer of electrification from conductor to conductor supplies the most convenient test of the conducting power of any given substance. For example, let a conductor be insulated and electrified; if not further interfered with, it will retain its charge until it is gradually dissipated by leakage through the insulator and the surrounding air, but if put in direct contact with the earth it at once loses its charge, thus showing that the earth acts as a conductor, and that after sharing its charge with so large a conductor as the earth, any charged body is practically discharged.

Similarly, if the electrified body be touched with the finger, it instantly loses its electrification by transfer to the conducting body of the experimenter and the other conductors with which he is connected. Now, under ordinary circumstances, the experimenter communicates with the earth by conducting substances, and therefore in the case considered

* In these experiments it is best to employ the gold-leaf electroscope described in Art. 12.

the electrified body shares its charge with the earth, and all conducting bodies on its surface, and thus has practically no charge left. If before touching the electrified body the experimenter insulates himself from the earth by standing on a stool with glass (insulating) legs, it will be found that although the body loses the greater portion of its electrification by transfer to the experimenter, both still exhibit feeble electrification.

Hence, if an insulated electrified body be touched with any conducting substance, in direct or indirect contact with the earth, it at once loses its electrification. On the other hand, if touched with an insulator it retains its charge or loses it very gradually, according as the substance tested is a good or bad insulator.

We can now understand why it is necessary to insulate any body which we wish to electrify. If the body is a conductor this is effected by attaching it to a stem or handle of some insulating substance, such as glass or ebonite; if an insulator, any portion of it may be considered as already insulated from the other portions, and may therefore be electrified. The phenomenon of transfer of electrification from conductor to conductor is applied in the use of the *proof plane*. This simple instrument consists of a small disc of metal, or cardboard covered with tinfoil, attached to an insulating handle, such as an ebonite penholder. It is used for testing the electrification of any body. When placed in contact with the body it receives by transfer a small charge, which may be removed with it, and used as an indicator of the nature and magnitude of the original charge.

It is somewhat difficult to give a correct list of conductors and insulators in the order of their conducting powers, for different specimens of the same substance vary greatly in this respect. The following table, however, gives a general idea of the relative conducting or insulating powers of the substance referred to. Taken as a table of conductors in the order of their conducting power, the list should be read downwards; as a table of insulators arranged in the order of their insulating power, the list should be read upwards.

Bad Insulators.	Silver. Copper. Other metals Gas Coke. Charcoal. Graphite. Acids. Metallic Salts. Water. The Body.	Good Conductors.
Partial Insulators.	Linen. Cotton. Wood. Stone. Marble. Paper. Ivory.	Partial Conductors.
Good Insulators.	Oils. Porcelain. Wool. Silk. Resin. Sulphur. Guttapercha. Shellac. Sealing Wax. Ebonite. Paraffin. Glass. Dry Air.	Bad Conductors.

5. The Two Electrical States. If an ebonite rod be electrified by means of a fur rubber and held near a few pith balls the latter will, as we know, be attracted by the rod. If, however, we watch the behaviour of any one of the attracted balls we shall find that there is something further to explain. The ball is attracted up to the rod; but directly it touches the latter it seems to cling to it for an instant, and is then cast off to be again attracted as before. If this action is allowed to go on for some time without again exciting the rod it will be found that each successive time the ball touches the rod its time of contact is lengthened, and that it moves in an erratic way over the surface of the rod before being cast off. This action

becomes more and more marked as the electrification of the rod is gradually lost, until at last the ball ceases to be attracted at all. Further, it will be found that under the circumstances of this experiment the rod loses its electrification much more quickly than if it were merely exposed to the air. Some of the phenomena here noticed we can readily explain. The pith ball acts as a conductor, and directly it comes in contact with the electrified rod it becomes slightly electrified by transfer of electrification at the point of contact. When it is cast off by the rod it carries this electrification with it, and finally gives it up to the earth through the table or other conducting stand to which it returns. It thus becomes neutral, and is again attracted by the rod, and the process described is repeated. We thus see that the pith balls act the part of carriers, and convey the electrification of the rod to the earth, thus causing it to lose its electrification more rapidly than it otherwise would. Further, if the rod is strongly electrified the ball in contact with it quickly becomes charged, but if feebly electrified then the time of receiving the charge is extended, and the rod, being an insulator, the electrification cannot pass over its surface to the ball; so the latter rolls over that surface, gradually collecting a charge, and, finally, when sufficiently electrified, leaves it as before. We have now explained some of the minor points of the experiment, but the most important fact—viz., that the ball after becoming electrified ceases to be attracted, is still unexplained. If, instead of experimenting with loose pith balls lying on the table, we suspend one by means of a dry silk thread, we shall find, on holding the electrified rod near it, that the ball is at once attracted, but after contact it is quickly repelled; and if the rod be again brought near, the ball, instead of being attracted, is strongly repelled. Similarly, if two pith balls be suspended from the same stand, they will be seen to repel one another after both have become electrified by contact with the ebonite rod. From these experiments we come to the conclusion that bodies charged with the *same* electrification repel one another. If the same experiment be

repeated with an excited glass rod exactly similar results will be obtained. If, however, the suspended pith ball be charged by contact with an electrified ebonite rod, and an excited glass rod be then brought near, it will be found that the charged ball is strongly attracted instead of being repelled, as it is by the ebonite rod. Similarly, if the pith ball be charged by contact with an electrified glass rod, and an excited ebonite rod be then brought near it, the ball will be seen to be attracted, although repelled by the glass rod. Further, if an ebonite rod be excited and suspended on a wire stirrup, as shown in Fig. 3, it will be found to be distinctly repelled by another similarly excited ebonite rod. Also, if the experiment be repeated with glass rods excited with silk, sticks of sealing wax excited with flannel, or with any two *similar* substances similarly excited, the same result will be obtained; that is, the two similarly excited rods will exert mutual repulsion. If, however, we take a glass rod excited by silk, and an ebonite rod excited by fur, and suspend either in the stirrup, it will be found to be *attracted* by the other. It is not, however, necessary to employ rods of different substances to exhibit attraction instead of repulsion, neither is it necessary to employ rods of the same substance to exhibit repulsion. Two pieces of the *same* substance may, by the use of appropriate rubbers, be electrified to exhibit mutual attraction, and rods of different substances may be so excited as to exhibit mutual repulsion; for example, glass rubbed with silk is attracted by glass rubbed with fur, and sealing wax rubbed with flannel repels glass rubbed with fur. So far, all this is empirical; but experiment shows that if any two electrified substances attract or repel the same electrified body, then they invariably repel one another, but if one attracts while the other repels the given body, then they invariably attract one another. From these observations it has been inferred that *there are two opposite kinds of electrification, and that bodies similarly electrified repel one another, while bodies oppositely electrified attract one another.* Further, if a given substance be electrified by a given rubber it will be found that, if proper precautions be taken to ensure

insulation, the rubber and the substance rubbed *always* exhibit mutual attraction—that is, *they are oppositely electrified*. For example, if a rod of ebonite be rubbed with a piece of fur mounted on a glass or ebonite handle, the rod, when suspended in the wire stirrup, will be found to be attracted by the fur. Hence we see that if any two substances are rubbed together they become oppositely electrified; and the nature of the electrification developed on either depends not only on the nature of its own material, but also on that of the other. Symmer and Du Fay independently discovered and investigated these phenomena, and, in explanation of them, the latter suggested that there might be two kinds of electricity which exercised mutual attraction but self-repulsion. The electricity which appears on glass when rubbed with silk he called *vitreous* electricity, and that which appears on sealing wax and other resinous substances when rubbed with wool or fur he called *resinous* electricity.

These terms are misleading, for they are founded on the erroneous assumption that only one kind of electrification can be produced on a given substance; they have, therefore, been discarded in favour of the terms *positive* and *negative*, introduced by Franklin. A rod of glass excited by a silk rubber is said to be *positively* electrified, and any resinous substance excited by a fur or woollen rubber is said to be *negatively* electrified.* This is also expressed by saying that a positively electrified body is *charged* with *positive electricity*, and a negatively electrified body with *negative electricity*.

Similarly, a neutral body may be said to possess equal quantities of positive and negative electricity, each neutralising the other, and thus accounting for the neutral state of the body, and indicating how, by a separation of these neutralising constituents, two neutral bodies may exhibit opposite electrifications.

The student must, however, remember that the use of

* That is, it is agreed to call one kind of electrification positive, and the other negative; but there is nothing but general convention to decide which shall be considered positive, and which negative.

these terms does not necessarily imply that there are two different and opposite kinds of electricity. They arose out of the various attempts to frame a theory of electrification in accordance with observed facts, and they are still necessary for the purposes of speaking and writing.

From a number of experiments it is possible to draw up a list such that when any substance in the list is rubbed with another lower down on the list, the former becomes positively, and the latter negatively electrified. The list given below is arranged on this principle, but it must be remembered that the order of the list might be different for *different specimens* of the *same* substances.

Fur.	Silk.
Wool.	The Hand
Ivory.	Metals.
Wood.	Caoutchouc.
Shellac.	Amber.
Resin.	Sulphur.
Sealing Wax.	Ebonite.
Glass.	Guttapercha.
Cotton	Collodion.
Paper.	

The relative positions of substances standing close together on the list is somewhat uncertain. When substances far apart on the list (for example, fur and ebonite) are rubbed together, each becomes strongly electrified, but when substances close together (for example, sulphur and ebonite) are dealt with, the electrification produced is very feeble, and it is therefore difficult to decide as to its nature for either of the substances.

6. Electrical Attraction and Repulsion. The force of attraction between two oppositely electrified bodies, or the force of repulsion between two similarly electrified bodies, is found to depend on three things—the degree of electrification of each of the bodies, their distance apart, and the nature of the intervening medium. The more strongly electrified the bodies are, the greater the force of attraction or repulsion exerted by them. This condition may be more definitely stated by considering an electrified body as charged with electricity, and saying that the magnitude

of the force depends upon the quantity of electricity with which each of the electrified bodies is charged—that is, upon what are called their *charges*. This way of dealing with the question evidently involves the necessity for some method of measuring quantity of electricity; but, assuming for the present that this is possible, let us consider *how* the force between two charged bodies varies with the magnitude of their charges. Imagine two small positively charged bodies separated by any definite distance, and suppose the charge on *one* to be doubled, then evidently the force of repulsion between them will also be doubled. Similarly, if the charge on the other body be doubled, then the magnitude of the force is evidently again doubled—that is, by doubling the charge on *each* of the bodies the force is quadrupled. Similarly, if one charge be doubled while the other is trebled, the magnitude of the repelling force will be increased to six times its original value. This result can be expressed generally by saying that *the force exerted between two charged bodies varies as the product of their charges.*

Considering the second condition on which the magnitude of the force depends, it is found *by experiment* (Art. 8) that if the distance between two charged bodies be doubled then the force is reduced to one-fourth of its original value, that is, *the force exerted between two charged bodies varies inversely as the square of the distance between them.* This law is rigorously true only when the charges are supposed to be concentrated at points. This involves a physical impossibility, but the law may be taken to be very approximately true when the dimensions of the charged body are small compared with the distance between them.

With regard to the influence of the medium surrounding the two bodies, it is at once evident that it must act as an insulator, or the conditions of the question are entirely altered. The phenomena of attraction and repulsion depend upon the action of the medium. If it is perfectly insulating then the charges are confined to the conductors considered; but we know by experience that a body always tends to lose its charge, that is, the charge always tends to pass into the surrounding medium. This tendency

of electricity to pass into the adjacent insulating medium is the result of a stress existing in the medium, and the phenomena of attraction and repulsion are direct consequences of the existence of this stress. From this we infer that the magnitude of the force of attraction or repulsion between any two charged bodies varies with the nature of the surrounding medium. Experiment shows that this is the case, but that the laws stated above hold for any given medium. That is, in any medium the force exerted between two charged bodies varies directly as the product of their charges, and inversely as the square of the distance between them; but the magnitude of the force, under given conditions, varies with the nature of the medium. These laws may be concisely expressed by means of a formula. Let two small spheres, A and B

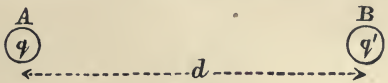


Fig. 4.

(Fig. 4), be charged respectively with quantities of electricity denoted by q and q' and let the distance between their centres be denoted by d, then, as explained above, we have—

$$f \propto \frac{qq'}{d^2}.$$

where f denotes the force exerted between the charged spheres. That is,

$$f = \frac{1}{\kappa} \frac{qq'}{d^2},$$

where κ is a constant depending in magnitude upon the nature of the medium surrounding A and B. For any given medium κ has a definite constant value for all values of q, q' and d, but this value is different for different media. For example, if in one medium the force exerted between two charged bodies is twice as great as it is under the same conditions in another medium, then the value of κ for the

first medium would be half as great as its value for the second. It is evident that this constant, κ, for any medium is a specific constant for that medium, depending in magnitude upon that property of the medium which determines the facility with which electric force acts through it. To determine the absolute value of κ for any given medium it would evidently be necessary to know the absolute value of f corresponding to known absolute values of q, q' and d. The values of f and d can be determined by experiment, but those of q and q' cannot be so determined. Hence, instead of dealing with absolute values, it is more convenient to decide on some convenient standard substance, for which the value of κ shall be taken as unity. The substance chosen as a standard is dry air, hence in this medium we have

$$f = \frac{qq'}{d^2},$$

and in any other medium we have

$$f = \frac{1}{\mathrm{K}} \frac{qq'}{d^2},$$

where K is a constant expressing the ratio of the magnitude of f in air to its magnitude under the same conditions in the given medium. For example, if in one medium the force exerted between two charged bodies is twice as great as it is under the same conditions in air, then the value of K for the first medium would be $\frac{1}{2}$.

From the relation $f = qq'/d^2$ we are now able to deal with the measurement of quantity of electricity, for if f and d each equal unity, and $q = q'$, then we must have $1 = (1 \times 1)/1 = 1$; that is, *unit quantity of electricity is that quantity which, when placed in air at unit distance from another equal and similar quantity, repels it with unit force.* Hence, if unit quantity of electricity be placed at unit distance from a given charge, the magnitude of the force which it experiences is a measure of the quantity of electricity constituting the given charge. A charge of q units of positive electricity is denoted by q, or $+q$, and an equal charge of negative electricity is denoted by $-q$. With this notation, the force of repulsion exerted between two *similar* and equal charges is evidently given by $f = q^2/d^2$ whether the charges be positive or negative, but for two dissimilar

equal charges $f = -q^2/d^2$, that is, when f is positive the force is one of repulsion, but when negative it is one of attraction.

It is found by experiment that if any number of conductors whose charges are all of the same sign are made to touch and then separated, the sum of their charges is unaltered, though the individual charges are usually altered. But if some of the conductors are positively and some negatively charged before contact, they are all similarly charged after contact, and the sum of their charges after contact is the *algebraic* sum of their charges before contact, positive and negative charges being considered of opposite signs. If two equal conducting spheres are made to touch, their charges after contact are equal.

7. Field of Electrical Force. Consider a charged body surrounded by an unlimited insulating medium. If another similarly charged body be brought near the first it will experience a force of repulsion varying in magnitude with its position, the amounts of the charges, and the nature of the insulating medium. The whole space surrounding the charged body in which this electrical force is exerted is called the *field of force*, due to the charge on that body. The *intensity of the field* at any point, or the magnitude of *the electric force at any point in the field*, is determined by the force which a *unit positive charge* would experience if placed at that point. In the case of a point-charge the direction of this force is along the line joining the point considered to the position of the charge.*

If the field is due to a single charge then the direction of the force at any point is readily obtained by drawing a line passing through that point and the centre of the charge, and in this way the field can be mapped out by a

* The term *point-charge* implies that the charge considered is supposed to be concentrated at a point. In practice, however, an electrical charge is always spread over the surface of the body which is said to be charged. The centre of the charged body, even if it could always be indicated, is not however necessarily the centre of the charge, and only in a limited number of cases does a point exist at which the charge may be supposed to be concentrated so as to have the same external field as it has when distributed over the surface of the body. This point, when it exists, may be called the centre of the charge.

series of lines each indicating the direction of the force at the points through which it passes. These lines are called *lines of force*, a line of force being a line such that its direction at any point through which it passes gives the direction of the electric force at that point. Thus, in the field due to a single charged sphere, the lines of force are radial straight lines, diverging from the centre of the sphere, and the direction of the force for all points lying on any given line is therefore the same, but the magnitude of the force diminishes as the square of the distance from the centre of the sphere increases. If, however, the field is due to two or more charged bodies, then to determine the direction of the electric force at any point in the field it is necessary to determine the magnitude and direction of the force at that point due to each charge individually, and then to take the resultant of the several forces thus determined. The direction and magnitude of this resultant determines the electric force at the given point due to the several charges considered. In this case it is also possible to map out the field by lines of force, such that the resultant direction of the electric force at any point is determined by the direction of the line of force passing through that point. In general, however, the lines will not be straight but curved, showing that the direction of the force varies from point to point along any given line, the direction at any point being given by the tangent to the line at that point. Hence, a line of force may be defined generally as a curve such that the direction of the electric force at any point through which it passes is given by the tangent to the curve at that point. (Compare Figs. 64-67). By a conventional arrangement the lines of force may be made to indicate, not only the direction, but also the magnitude of the electric force at any point in the field. Theoretically, the number of lines that can be drawn in any field is infinite, for at all points in the field the electric force has a definite direction. It is, however, agreed to indicate the intensity of the field by the number of lines drawn according to the following convention. Let the intensity at any point of a given field be

denoted by F, then the number of lines of force per unit area at that point crossing a plane at right angles to the line of force passing through the point shall also be denoted by F—that is, the number of lines of force per unit of area at the point is a measure of the intensity of the field at the point considered. Hence, when we speak of the number of lines of force passing through a given area, we refer to the *flow of force* across that area.

8. The Torsion Balance. One of the earliest attempts at quantitative experiment was that made by Coulomb, by means of the Torsion Balance, which he invented for the purpose of comparing the forces of repulsion exerted between two similarly charged bodies under different conditions of distance and charge. He also employed it to make corresponding magnetic measurements, as described in Chapter VIII. The instrument (Fig. 5) consists essentially of a fine silver wire, *w*, suspended within a glass case from a brass head, B, and carrying at its lower extremity a light lever, *ab*, made of shellac, or some light insulating substance. This lever moves in a horizontal plane, and is furnished at one extremity, *b*, with a gilt pith ball. The brass suspension head, B, fits into a brass tube, in which it can be easily rotated, and carries a pointer which moves over a brass disc, D, on which is cut a circular scale divided into degrees. This arrangement for the suspension and control of the wire is called the *Torsion head.* On that portion of the glass case which encloses the lever a scale, *ss*, is etched round its circumference at the level of the horizontal axis of the lever, and through an aperture in the cover a second gilt

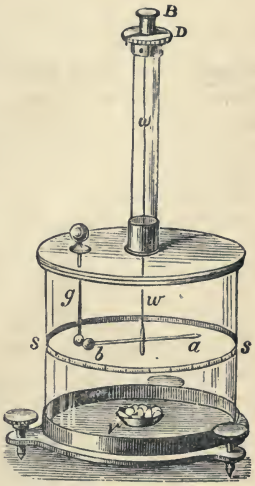


Fig. 5.

pith-ball, *c*, attached to the end of a glass rod, *g*, can be introduced in the position shown in the figure. To absorb moisture and thus improve the insulation a small vessel, V, containing calcium chloride, or pumice stone soaked in sulphuric acid, is placed in the bottom of the case. The principle of the action of the instrument consists in balancing the force of repulsion between the two charged balls, *b* and *c*, against the torsion of the wire. The torsion head is adjusted until the balls, *b* and *c*, are lightly in contact. The pointer of the torsion head should then be at the zero of its scale, and the ball *b* should be opposite the zero of the scale *ss*. The ball *c* is now removed and charged; on being replaced it shares its charge with *b*, and mutual repulsion ensues. As a result, *b* is repelled until the moment of the force due to the torsion on the wire balances that due to the force of repulsion between the balls. As the instrument is used to make comparative measurements of electrical force, it is evidently essential that the law connecting the angle of torsion with the force producing it should be known. The law is known from experiment, and it is found that the angle of torsion is directly proportional to the couple producing it; that is, for example, the couple necessary to twist a given wire through 20° is twice as great as that necessary to twist it through 10°. Hence, if in one experiment the force of repulsion between the charged balls is sufficient to repel *b* through 10°, and in a second experiment it is sufficient to repel it through 20°, then the force due to torsion is twice as great in the second case as in the first. But before we can compare the charges on the balls in this way, we must eliminate the effect due to the change of distance between the balls. This can be done by adjusting the torsion head so that the distance between the balls shall be the same in each experiment. Thus in the example given above the distance between the balls in the second experiment is twice as great as in the first experiment, but by turning the torsion head so as to force *b* nearer to *c*, the ball *b* may be brought back to its position in the first experiment at 10° from *c*. Let it be necessary in order to effect this adjustment to turn

the torsion head through 70°. The twist on the *wire* is now 70° due to the torsion head above, and 10° due to the deflection of the lever below, that is, 80° in all. In the first experiment the torsion head was untouched, and the twist on the wire was due only to the 10° deflection of the lever. Hence at the *same* distance apart the force of repulsion between the balls is eight times as great in the second case as in the first—that is, the charge on each ball is $\sqrt{8}$, or $2\sqrt{2}$ times * as great in the second case. Similarly from the data of the second experiment the law of inverse squares can be verified. Assuming what is approximately true only for small angles, that the linear distance between the balls is proportional to their angular distance, we see that by halving the distance between the balls, the force of repulsion is increased to four times its original value. For when the deflection is 20° the twist on the wire is also 20°, but when the deflection is reduced to 10°, the twist on the wire has to be increased to 80°. Similarly in one of Coulomb's experiments the angle of deflection after charging the balls was found to be 36°, the corresponding twist on the wire being also 36°. To reduce this deflection to 18° it was found necessary to turn the torsion head through 126°, thus producing a twist of 144° (126°+18°) on the wire. That is, by halving the distance between the charges, the force exerted between them was increased fourfold, as shown by the increase of the balancing twist on the wire from 36° to 144°. Coulomb also investigated the force of attraction between two oppositely charged balls, by comparing the relative amounts of torsion required to keep them at different distances apart.

The method of calculation adopted above gives approximately correct results only when the angle of *deflection* † is small—say less than 10°. For large angles, a more correct method (see Ex. I. 5) should be employed; but owing to the

* From the formula, $f = \frac{q^2}{d^2}$, f is evidently proportional to q^2, that is, q is proportional to $\sqrt{f}$, d being constant. The balls b and c are here supposed to possess equal charges, each denoted by q.

† The law of torsion is true for all angles, large or small.

difficulty of ensuring good insulation, the disturbing effect of the size of the balls, and various other defects, the experimental results obtained with the Torsion Balance are very rough, and it is therefore of no practical value to adopt rigorously exact mathematical methods. Further, owing to the advance of electrical science, the methods of measurement have greatly changed, and the Torsion Balance is now but seldom used and is of little more than historical interest.

9. Energy of Electrification. An electrified body is capable of attracting and repelling other electrified bodies, and is thus capable of doing work. This implies that the electrification of any body is associated with a gain of energy, and hence before a body can be electrified work must be done. The mechanical work done during friction does not furnish the energy of electrification, for we have already seen that it has no direct bearing on the quantity of electricity produced. Electricity itself, whatever be its nature, is not energy. Hence we have to consider how this energy is produced and in what it consists. It has already been stated that when electrification is produced by the contact or friction of two dissimilar substances, these substances become oppositely electrified. Hence, during contact, the substances attract one another, and the work that has to be done against this force of attraction before the substances can be separated is the equivalent of their energy of electrification.

10. CALCULATIONS.

THE force exerted in air between two small bodies having charges q and q', and separated by a distance d, is given by $f = \frac{qq'}{d^2}$. 1. Art. 6.

If q and q' are expressed in terms of the unit defined in Art. 6, and d in centimetres, then f is given in *dynes*. The direction of the force is along the line joining the two bodies.

The intensity at any point of the field due to any charge or system of charges is determined in magnitude and direction by the force which a unit positive charge would experience if placed at that point. Thus at a point at a distance r from a charge q the intensity of field due to that charge is given by—

$$\frac{q \times 1}{r^2} \text{ or } \frac{q}{r^2}.$$

EXAMPLES I.

1. Two small spheres are charged respectively with 6 units of positive and 4 units of negative electricity. Find the force between them when they are placed 12 cms. apart.

Here—

$$\begin{aligned} q &= \quad 6 \text{ units of quantity} \\ q' &= -4 \quad \text{" " "} \\ d &= \quad 12 \text{ cms.} \end{aligned}$$

$$\therefore \ f = \frac{-24}{(12)^2} = -\frac{1}{6} \textit{ dyne.}$$

That is, either sphere *attracts* the other along the line joining them with a force of $\frac{1}{6}$ *dyne*.

2. A small brass ball is charged with 12 units of positive electricity, and is then placed in contact with an equal and similar brass ball. Find the force exerted between the balls when they have been separated by 9 centimetres.

On contact the two balls, *being equal*, will share the charge of 12 units equally between them, and will therefore each have a charge of 6 units.

Hence, we have—

$$f = \frac{36}{(9)^2} = \frac{4}{9} \textit{ dyne.}$$

That is, either ball repels the other along the line joining them with a force of $\frac{4}{9}$ *dyne*.

3. A brass ball is charged with 100 units of positive electricity. Find the intensity of the field due to this charge at a distance of 50 cms. from the ball.

The intensity of the field at the point considered is measured by the force which a positive unit of electricity would experience if placed at that point. Here, therefore, we have—

$$f = \frac{100 \times 1}{50^2} = \frac{1}{25} \textit{ dyne.}$$

That is, the intensity of the field at a point 50 cms. from the charge is $\frac{1}{25}$ *dyne*, in a direction away from the charge.

4. In an experiment with the torsion balance the lever was deflected through 10°. Find the torsion necessary to reduce this deflection to 5°, supposing the charges on the balls to remain constant.

If the deflection be reduced from 10° to 5°, that is, halved, the force exerted between the charged balls will be quadrupled, and therefore the torsion on the wire must be quadrupled. Hence if the *torsion head* be turned through T° before the deflection is reduced to 5°, the twist on the wire will be (T + 5), and we must have—

$$\begin{aligned} T + 5 &= 4 \times 10, \\ \text{or } T &= 40 - 5 = 35° \end{aligned}$$

5. In an experiment with the torsion balance the lever is deflected through 90°. Find the torsion necessary to reduce this deflection to 60°.

[In this example the deflections are too large to admit of the application of the method of the preceding example, and the following more exact method must be adopted.]

Let the balls each be supposed to carry a charge q. Then from Fig. 6 we have—

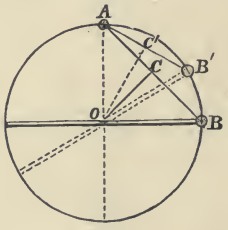


Fig. 6.

$$f_1 = \frac{q^2}{(A\,B)^2}$$

where f_1 denotes the force of repulsion between the balls when the

deflection is 90°. The moment of this force round $O = f_1 \times OC = q^2 \frac{OC}{(AB)^2}$.

Similarly, if f_2 denote the force exerted between the balls when the deflection is 60°, we have the moment of f_2 round $O = q^2 \frac{OC'}{(AB')^2}$.

Now these moments are in each case balanced by the moment due to the torsion on the wire, and as the moment of torsion is proportional to the angle of torsion we have—

$$90 : T :: q^2 \frac{OC}{(AB)^2} : q^2 \frac{OC'}{(AB')^2}$$

where T denotes the twist *on the wire* when the deflection is 60°.

$$\therefore 90 : T :: \frac{OC}{(AB)^2} : \frac{OC'}{(AB')^2}$$

or

$$\frac{90}{T} = \frac{OC}{OC'} \times \left(\frac{AB'}{AB}\right)^2,$$

And from the geometry of the figure—

$$\frac{OC}{OC'} = \frac{\sqrt{2}}{\sqrt{3}} \text{ and } \frac{AB'}{AB} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

$$\therefore \frac{OC}{OC'} \cdot \left(\frac{AB'}{AB}\right)^2 = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{1}{2} = \frac{1}{\sqrt{6}}.$$

$$\therefore \frac{90}{T} = \frac{1}{\sqrt{6}} \text{ or } T = 90\sqrt{6}.$$

That is, T = 220°.45 nearly.

To produce this twist *on the wire*, the torsion head must be turned through 220.45 − 60 = 160°.45.

[If this question be worked by the method of Ex. 4 we have—

$$\frac{90}{T} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

$$\therefore T = \frac{810}{4} = 202°.5.$$

That is, the torsion head must be turned through 202.5 − 60 = 142°.5].

6. A small brass sphere is charged with 30 units of positive electricity, and is then made to touch another equal sphere having a charge of 10 units of negative electricity. Find the force exerted between the charged spheres when they are separated by a distance of 10 cms.

7. Two charges of + 10 and − 10 units are placed at two corners of an equilateral triangle of 10 cm. side. Find the magnitude and direction of the resultant force acting on a charge of + 10 units placed at the third corner of the triangle.

8. Three charges of + 10, + 20, and − 10 units are placed at three corners of a square of 1 metre side. Find the intensity of the field at the fourth corner due to these three charges.

9. Two equal positive charges of 10 units each are placed at the extremities of the diameter of a circle of 5 cms. radius. Find the intensity of the field due to these charges at the middle point of the semicircular arc joining the charges.

10. Assuming the balls on the lever and rod of a torsion balance to be equal, compare the charges given to the latter when the deflections produced are respectively 10° and 15°.

11. In an experiment with the torsion balance a deflection of 5° is produced. Explain in what direction and how far the torsion head must be turned to increase this deflection to 15°.

12. Six equal charges are placed at the corners of the base of a hexagonal pyramid. If the slant height of the pyramid is equal to the diameter of its base, find the intensity of the field at the apex due to the charges at the base.

CHAPTER II.

ELECTROSTATIC INDUCTION.

11. General Phenomena. Let us consider what takes place when a neutral insulated conductor, AB (Fig. 7), is placed in the neighbourhood of a positively charged conductor, C. The medium surrounding AB is in a state of strain, and the electric force at a point near A being greater than at a point near B, positive electricity is urged from A to B.

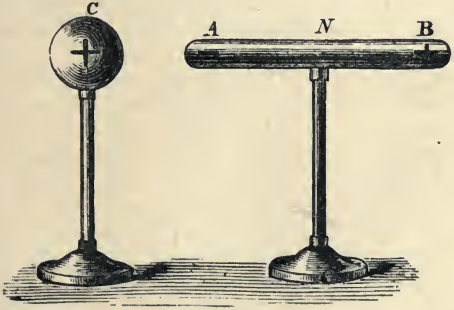


Fig. 7.

Since AB is a conductor, it is unable, like the surrounding medium, to set up any opposing stress, and, as a result, positive electricity passes from A to B, the end B thus becoming positively electrified and the end A negatively electrified. This action, to which every neutral conductor placed in an electric field is subjected, is called *electrostatic induction.* In simple terms it may be said that, in the case

considered, the positive electricity on C acts *inductively* on AB, attracting negative electricity to the end A, and repelling positive to the end B. As before (Art. 5), however, the student must be warned that this is merely a convenient way of expressing the facts; it is a somewhat misleading way, for the function of the medium, the most important factor of the phenomenon, is liable to be lost sight of, but for the purposes of speaking and writing it is concise and convenient.

As a result of this inductive action it is evident that AB is attracted by the charged conductor C, for, the negative charge at A being nearer C than the positive charge at B, the force attracting the former is greater than that repelling the latter, and therefore the conductor AB is, on the whole, attracted by C. This, then, explains why neutral bodies are attracted by electrified bodies—the latter act inductively on the former, and mutual attraction is the result.

From what has been said it is evident that only *conducting* bodies can be acted on inductively. For example, in Fig. 7, imagine the conductor AB to be removed, and the space it occupied to be filled with air. The stress in the medium external to this space urges positive electricity from A to B, but the air, being an insulating substance, is able to set up an opposing stress, and thus no separation of electricity is produced. Similarly, if the space AB be supposed to be occupied by any insulating substance, little or no electrical separation can result from the inductive action to which it is subjected on account of its position in the electric field. As a direct consequence of this it follows that insulating substances are but slightly attracted by an electrified body. Experiment indicates that this is the case, for if a rod of sealing-wax, ebonite, or other imperfect insulator, be suspended as in Fig. 3, it will be found that it is but feebly attracted by electrified bodies which strongly attract a similar rod of wood or other light conducting substance.

Let us again consider the inductive action of a charged conductor C on two small conductors A, B (Fig. 8), connected by a wire or thread of conducting substance. A and

B, together with the connecting wire, form one conductor, and therefore, as explained above, under the inductive

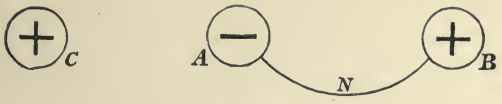


Fig. 8.

action of the charge on C, negative electricity is attracted to A and positive repelled to B. While this distribution of electricity obtains, the connection between A and B may be cut by an insulated pair of scissors, and the conductors A and B will retain their *induced charges.*

Similarly, if B be removed and A connected by a wire to the earth, negative electricity is attracted to A, and positive repelled to earth, and, when the wire is removed, A is left negatively charged. The same result may be obtained by touching A with the finger instead of connecting it to earth by means of a wire—the positive electricity is then repelled through the body to the earth, and, on removing the finger, A is left negatively charged as before. Hence, when a body is charged in this way—*by induction*—the sign of its charge is opposite to that of the inducing charge. If, in the case illustrated by Fig. 8, the conductor B be touched with the finger for an instant, the positive charge repelled into it by C is further repelled to the earth. Hence, on removing the finger, B is left neutral, and A negatively charged as at first. If, however, C be now removed, this negative charge on A is freed from the attraction of the charge on C; that is, it ceases to be "bound," and is free to distribute itself over the whole conductor formed by A, B, and the connecting wire. Further, if instead of touching B the finger be placed upon A, the positive charge originally repelled to B will now be repelled through the body of the experimenter to the earth as the most distant conductor, and the negative charge in A, being bound by the attraction of the charge in C, will not be disturbed, so that on removing the finger the result is similar to that obtained

on touching B. Hence, we may say, generally, that if an insulated conductor, or system of connected conductors, be subjected to induction, electrical separation results, and a charge of sign opposite to that of the inducing charge is attracted by the latter to the nearer portions of the system, while a charge of the same sign is repelled to the more distant portions. Also, if *any* point of the system be connected to earth, the repelled charge passes to earth, leaving the portions originally occupied by it neutral, and producing no change in the distribution of the attracted charge. If, however, the inducing charge be removed, this "bound" charge is free to distribute itself over the whole system.

If an insulated conductor be placed in an electric field due to several charges, each of the latter acts inductively on it, and the total effect produced is the sum of these individual actions. For example, if an insulated conductor be placed midway between two similar and equally charged conductors, the two inductive actions balance one another, and the electrical condition of the conductor is not affected. Similarly, if several insulated conductors be placed in an electrical field, each is acted on inductively, not only by the charges to which the field is due, but also by the induced charges on the other conductors; that is, the distribution on any one conductor is determined by the distribution—initial and induced—on all other conductors in the field.

From what has been said in Art. 6 it will be evident that induction is modified by the medium through which it takes place. For example, in the case illustrated by Fig. 7, the electrical displacement resulting from the inductive action of C on AB will vary with the nature of the surrounding medium, and the variation depends on the same property of the medium as that which determines the variation of the electric force between two charged bodies. Here, however, we have to consider the *induced charge due to a given intensity of field* at the point where the conductor subjected to induction is placed, whereas in Art. 6 we are dealing with the *intensity* at any point *of the field*

due to a given charge. Hence induction will vary *directly* with that property of the medium on which force *inversely* depends, that is, the value of the constant K for any medium determines the facility with which electrostatic induction takes place through it, and gives this inductive power or *inductive capacity* of the medium relative to dry air taken as a standard. For example, if for any medium the value of K is 2, then induction takes place twice as well through that medium as through air. That is, the *specific inductive capacity* of the medium relative to air as a standard is denoted by 2.

12. The Gold-leaf Electroscope. A simple form of this instrument is shown in Fig. 9. It consists essentially of two rectangular strips of thin gold leaf (*g*, *g*) attached by their upper edges to a short T-piece of brass, which forms the lower extremity of a thin brass rod, *b*. This rod passes through the stopper S of the glass case G, and carries a small circular brass disc, D, at its upper extremity. This disc is sometimes called the *cap* of the instrument. The glass case which is fixed on the base board of the instrument serves to protect the leaves and also to insulate the brass rod *b*. In order to improve this insulation the stopper *S* is stuffed with shellac or some good insulating substance, and the outer surface of the glass is lightly varnished with good shellac varnish. This prevents the formation of a film of moisture on the outer surface of the case, and the interior is kept dry by means of calcium chloride, or pumice stone soaked in sulphuric acid, contained in a small vessel, *V*. The principle of action of the instrument depends upon the fact that two similarly charged bodies repel one another. If a charge is given to the cap of the electroscope it com-

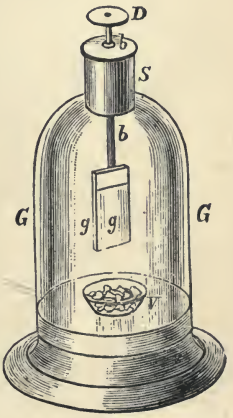


Fig. 9.

municates a portion to the gold leaves, which thus become similarly charged and repel each other. Being very light this repulsion is at once indicated by a divergence of the leaves. For example, if the brass cap be lightly beaten with a piece of fur, the electrification of the brass is at once indicated by a slight but gradually increasing divergence of the leaves.

By means of this convenient instrument, the phenomena described in the preceding article are readily illustrated. Thus, if a charged ebonite rod be brought near the cap of an electroscope, the leaves at once diverge, showing that induction is taking place, the negative charge on the rod being supposed to attract positive electricity to the cap, and to repel negative to the leaves. If, while the leaves still diverge, the cap be touched with the finger, they at once collapse, showing that the negative charge originally repelled to the leaves now passes to earth, the attracted positive remaining "bound" on the cap. If the charged rod be now removed, this bound charge spreads to the leaves, and they will be seen to again diverge, but this time with positive instead of negative electricity. The electroscope is now *charged inductively* with a charge of opposite sign to that on the ebonite rod. If it be required to charge the leaves with a charge of the same sign, it cannot be readily done by bringing the charged rod into direct contact with the cap; for the rod, being an insulator, will not readily communicate its charge to the cap, and the only effect of the contact is to intensify the induction by the close proximity of the inducing charge.* One method of obtaining the desired result is to charge any insulated conductor by the inductive action of the rod, and then to charge the electroscope by the induction of the charge on this conductor.

The nature of the charge on any body can be readily determined by means of the electroscope. Suppose the instrument to be positively charged, say by the induction

* If this experiment be tried, it will be found that the electroscope becomes slightly charged by induction, owing to the escape of some of the repelled charge from the thin edges of the gold leaves. The rod must not be too strongly excited, or the leaves may be torn off.

of a charged ebonite rod. If a negatively charged body—for example, the ebonite rod just used—be now brought near the electroscope, negative electricity is repelled to the leaves, and partially neutralises their positive charge. As a result, the leaves partially collapse, and if the charged body be brought nearer the cap, they may be made to totally collapse, and finally again diverge, with excess of the repelled negative charge. Similarly, if a positively charged body, such as an excited glass rod, be brought near, a positive charge is repelled to the leaves and increases the positive charge they already possess, that is, the divergence of the leaves is increased. Hence, if an electroscope be charged with electricity of a given sign, the approach of a similarly charged body causes increased divergence of the leaves, but the approach of an oppositely charged body causes first a diminished divergence, then total collapse, and finally a re-divergence of the leaves. Unless this second divergence takes place the latter test is not conclusive, for if a *neutral conducting* body be brought near a charged electroscope, the leaves show a very sensible decrease of divergence; for example, if the hand be held over the cap of a charged electroscope, the divergent leaves at once partially collapse. This is due to the fact that the neutral body is acted on inductively by the charge in the electroscope, and the attraction of the nearer induced charge withdraws some of the inducing charge from the leaves to the cap of the instrument, and thus causes a partial collapse. This effect of a neutral body will evidently be more or less marked, according as the body is a good or bad conductor; for example, with a perfect insulator no such effect would be observed, for no electrical displacement would result from the inductive action of the charge in the electroscope, but the greater the conductivity of the body the more complete is this displacement, and the more marked is the effect on the electroscope. Hence, this effect indicates a rough method of testing the relative conductivity of different substances, but it is essential that the test for each substance be made under exactly similar conditions. The relative conductivity of different substances may also be

exhibited by means of the following experiment, illustrative of induction. A metal can, C (Fig. 10), is insulated from the earth by means of a block of paraffin, I, and connected by a wire, *ab*, with the cap of an electroscope, E. On bringing a charged body near C, the leaves of the electroscope at once diverge, showing that the repelled charge passes from C, through *ab*, to E. By varying the material of the connection *ab*, it will be found that the divergence of the leaves varies with the conductivity of the material; for example, with metal wire the divergence is immediate and distinct, with *dry* silk thread no divergence is obtained, with dry cotton thread the effect is not quite so marked as with wire, with a wet thread (silk or cotton) the divergence is quite distinct, and with no connection other than the intervening air no divergence is obtained. These results show that air and dry silk are insulators, metals and water good conductors, and cotton a partial conductor. Perhaps the readiest

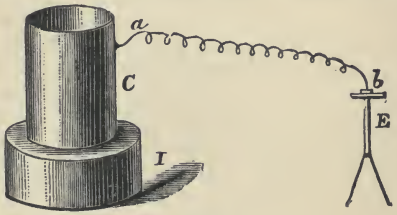


Fig. 10.

test of the conductivity of any given substance is afforded by the result of touching the cap of a charged electroscope with the substance in question. If that substance be a conductor, the instrument is at once discharged and the leaves collapse; if a perfect insulator, then no effect is produced; but if a partial conductor (or insulator), then the electroscope is gradually discharged, and the leaves slowly collapse, the relative rapidity of collapse giving a rough indication of the relative conductivity of the substances.

Being now able by means of the electroscope to determine the nature of any given charge, we can proceed to verify

the statements made in the preceding article with regard to the distribution of electricity on a conductor subjected to the inductive action of a neighbouring charge. For example, in the case illustrated by Fig. 7 a negative charge is supposed to be collected at the end A of the conductor AB and a positive charge repelled to the end B. If a *proof-plane* (Art. 4) be placed in contact with AB at A it should on removal possess a negative charge; on testing it at the electroscope it will be found that this is so. Similarly, if the charge on the end B be tested it will be found to be positive. Further, if different points on the surface of AB be tested, it will be found that about one half of the conductor, from A to N, is negatively charged, while the other half, from N to B, is positively charged, but that the degree of electrification very rapidly decreases from the ends towards the middle of the conductor. At the boundary line between the two charges the electrification is *nil*, and this line is therefore sometimes called the *neutral line*. Again, if AB be touched at *any* point, C still remaining in position, testing as before will show that the distribution of the negative charge is much the same as before, but that the surface originally charged with positive electricity is now neutral. If, however, C be removed, then the whole surface of AB will be found to be negatively electrified. The same results are more conveniently illustrated by the

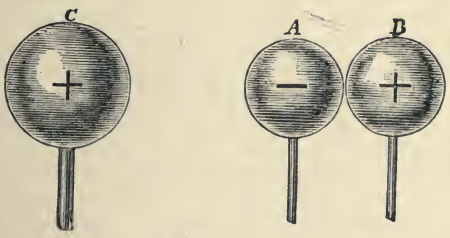


Fig. 11.

following experiments. Two insulated brass spheres, A and B (Fig. 11), are placed in contact, and a positively charged

conductor, C, is brought near them. Under the influence of this charge a negative charge is induced in A and a positive charge in B. If the spheres be now separated, each retains its induced charge, and, on testing by the electroscope, the nature of the charges will be found to be as first stated. If either conductor is touched before separation is effected, A will still be found to be negatively charged, but B will be neutral. If, after touching either conductor, C is removed before separation, then both A and B will be found to be negatively charged.

13. Field of Induction. The electrical field (Art. 7) surrounding any charged body is sometimes called the field of induction due to the charge on the body. The lines of force in the field indicate at any point the direction in which the electric force due to the charge tends to effect electrical displacement; that is, they indicate the direction of the inductive action, and for this reason are sometimes called *lines of induction.* Let us imagine a charged insulated conductor to be placed on a table in the middle of a room. Each surrounding object is subjected to the inductive action of this charge; if it be a conductor, then electrical separation takes place without producing any strain in the substance, and therefore without calling up any opposing stress; if an insulator, then slight electrical displacement takes place, until the stress produced by the strain thus induced balances the displacing electric force, and equilibrium obtains. As results, therefore, we find induced *charges* on the conductors and a *strain* in the insulators. If a conductor be insulated, then the repelled charge on the side remote from the inducing charge transmits the inductive action into the space beyond; if earth-connected, then the repelled charge goes to earth, *and the inductive action does not travel further.* Thus, in the case considered, the field of induction is bounded partly by the earth-connected walls of the room, and partly by the earth-connected objects which intercept the inductive action before it reaches the walls.*

* The portions of those walls, which are, as it were, in the shadow of these objects, possess no induced charge.

If there be any gaps in these walls, or if any portion of them be made of insulating material, or of insulated conducting material, then these areas do not act as portions of the bounding surface, and the space beyond them may become part of the field of induction. Thus, all conducting objects within the room possess induced charges, and the air and all other insulating substances in the field are in a state of strain. In fact the field may be briefly defined as all portions of space in which a state of strain associated with the inducing charge is found. This definition evidently excludes the interior of all conducting bodies in the field, for a strain cannot be produced in such bodies; and this will be the case whether the body be solid or hollow, for in the latter case, whatever the cavity may contain, the strain cannot be transmitted to it through the substance of the surrounding conductor.

Further, if a charged body be surrounded by an earth-connected conductor made of wire gauze, this conductor will limit the field of induction as effectually as a completely continuous conductor, for the lines of force will all tend to terminate on the oppositely charged conducting surface and not on the gaps; that is, the inductive action will be concentrated on this surface, and no strain will be transmitted through the gaps to the external medium. This result is evident if we remember that a line of force may be defined as the direction along which a positive unit of electricity will travel, for, if we imagine a positive unit to be repelled by the inducing charge along any one of the lines of force in the field, we can understand that, as it approaches the negative charge on the inner side of the conductor, it will experience attraction tending to draw it up to this surface and preventing it from passing out through any of the neighbouring gaps. This, of course, is true only when the wires of the gauze are sufficiently closely set. Similarly, the space within a conducting surface, perforated with a number of small gaps, is effectually protected from inductive action from without.

Some of these facts are readily illustrated by simple experiments. For example, if a plate of brass, held in

the hand, be placed between a charged body and an electroscope, the latter will exhibit no signs of inductive action, showing that the earth-connected brass plate limits the field in the direction of the electroscope. If, however, the plate be insulated, then its interposition will not interfere with the inductive action of the charge on the electroscope. Similarly, a plate of glass will have little effect unless its thickness is considerable, compared with the distance between the charged body and the electroscope. In this case the result will be modified by the relative inductive capacities of glass and air; for example, if (as is actually the case) the inductive capacity of glass be greater than that of air, that is, if induction takes place more easily through glass than through air, then the effect will be increased by the interposition of the glass. In this connection it is well to notice why the inductive effect of a charged conductor on any other conductor increases as the distance between the two diminishes. It is not because induction does not take place as well through a great thickness of the intervening medium as through a small thickness, but because the portion of the field subtending the surface of the conductor subjected to induction rapidly increases as this distance decreases. Moreover, as in the case of the non-continuous conductor considered above, some of the lines of force which would otherwise pass outside the conductor are gathered up so as to terminate on the surface bearing the attracted induced charge, and this effect also increases as the distance between the inducing and induced charges diminishes.

As an example of the effect of a conducting enclosure, the behaviour of an electroscope placed in a tin can may be noticed. An electroscope so placed is quite unaffected by charges external to the can, and this will be found to be the case even when the can is perforated with a large number of holes—in fact an enclosure of perforated sheet zinc or wire netting, or a glass case covered with strips of tinfoil, is quite as effective a screen as a conducting enclosure with continuous walls. Professor W. E. Ayrton has taken advantage of this fact in the construction of his improved

form of Gold-leaf Electroscope. This instrument is shown in Fig. 12. Special precautions indicated in the figure are taken to ensure good insulation of the leaves, and the case is covered with tinfoil, leaving a few narrow gaps through which the leaves may be observed. The leaves are thus protected from all inductive action from without (not directed on the cap of the instrument), and the case being earth-connected limits the field of the leaves when they are charged, and thus prevents their divergence being affected by inductive action on external neutral bodies. It is also important to notice that the interior of a hollow conductor is screened, not only from the inductive action of neighbouring charges on other conductors, but also from the inductive action of its own charge, if it possess one. Faraday illustrated this in a very striking manner. He constructed a wooden cube of 12 feet edge, covered it with tinfoil, insulated it, and went inside, taking his most delicate electroscopes with him. The cube was then strongly charged, but the electroscopes were not in the least affected. This result may perhaps be explained thus: consider an insulated conducting body inside a positively charged hollow conductor; the negative electricity in the former may be considered to be equally attracted in all directions by the surrounding positive charge, and the resultant force acting on it is therefore zero. Similarly, the positive may be considered to be repelled equally in all directions, and is likewise in equilibrium. Hence the body shows no induced charge. If, however, it be connected to earth, the line of connection is one

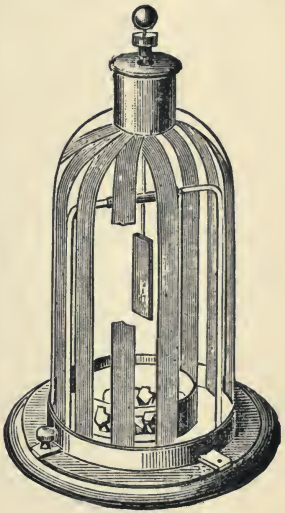


Fig. 12.

along which the positive electricity may pass to earth, leaving the negative electricity bound by the external charge.*

These effects are readily illustrated by the following experiment. A metal can is placed on a block of paraffin to insulate it and charged positively, say by the inductive action of an excited ebonite rod. A smaller can, held by an ebonite handle, is now placed inside this one, touched with the finger, and then removed. On testing, it will be found to be negatively charged.

14. Faraday's Ice-pail Experiments. If a charged conductor be suspended by an insulating thread inside an insulated hollow conducting vessel which completely surrounds it (Fig. 13), the *total* induced charges will be found on this vessel, the attracted charge being on its inner surface and the repelled charge on its outer surface. Assuming the vessel to be neutral to start with, it is evident that these two charges must be equal to one another, for no charge has been communicated to the vessel; and, therefore, if the induced charges are allowed to again unite, that is, if the inducing charge is removed, the vessel will again become neutral, thus showing that the induced charges are equal and opposite to each other. This tells us nothing of the actual magnitude of these charges—we simply know that they are equal to one another and of opposite sign. It is important, however, to learn whether there is any relation between the magnitude of the inducing charge and that of the induced charges, and, if so, to determine the nature of this relation. With this object in view, Faraday

Fig. 13.

* The line of connection being of conducting material is necessarily one along which the external charge can exert no force, and, therefore, the positive electricity meets with no opposition along this path, and being repelled in every other direction, it escapes by it to earth. The negative, on the other hand, is attracted in every other direction except this, and is therefore held bound by the external charge.

performed the following experiment, which has now become historical.

An ice-pail, I (Fig. 14), is placed on an insulating stand and connected by the wire, *w*, with the cap of the electroscope, E. A positively charged body, B, suspended by

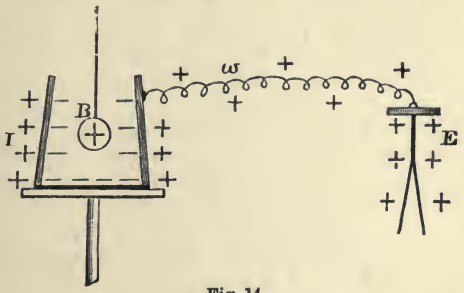


Fig. 14.

a dry silk thread, is now slowly lowered into the pail. As B gradually descends, the inner surface of I comes more and more under its inductive action, and a gradually increasing negative charge is attracted to this surface, while a corresponding positive charge is repelled to the outer surface, and thence to E. As a result of this action, the gold leaves of the electroscope begin to diverge as B is about to enter I, and this divergence slowly increases as B is lowered into the pail. When, however, B is well inside I, as shown in the figure, practically the whole* of its inductive action is concentrated on the inner side of that vessel, and the negative charge there found is consequently the total induced charge of that sign, while the positive charge repelled to the outer surface and to the electroscope represents the total induced positive charge. Hence, when this result is obtained, further lowering of B into I will not affect the induced charges, and consequently the divergence

* If the pail is not too wide compared with its depth, very few of the lines of force emanating from B pass out through the opening at the top. They are all, or nearly all, attracted up to the negatively charged inner surface of the vessel. *Cf.* Art. 13.

of the leaves of the electroscope remains unchanged. If now B be allowed to *touch* the inner surface of I, and the leaves of the electroscope be carefully watched, it will be found that they are not in the least disturbed by the contact. Further, if B be now removed, it will be found to be completely discharged. The significance of these results is important. When B touches the inner surface of I the positive charge on the former combines with the negative induced charge on the latter, and, since B is completely discharged and the external charge on I and E remains quite undisturbed, it is evident that these two opposite charges must exactly neutralise each other, that is, the negative induced charge is exactly equal in magnitude to the positive inducing charge.

Hence, we may state generally, that the induced charges are equal to one another, but of opposite sign, and the *total* magnitude of either of the induced charges is numerically equal to that of the inducing charge. The first part of this statement applies to any case of induction; that is, whenever a body is subjected to induction, the induced charges are equal and opposite. In the second part, however, the significance of the word *total* must be noted—the magnitude of the induced charges on any body is equal to that of the inducing charge only when the whole inductive action of the charge is concentrated on that body.

15. Illustrative Experiments. Let a number of metal cans of different sizes be arranged, as shown in Fig. 15, as a nest of conductors insulated from one another, and let an electroscope, E, be connected to any one of them by a wire, *w*. If a charged body, A, be now lowered into the inner can, B, the leaves of the electroscope at once diverge, no matter with which of the conductors it is connected. This illustrates an effect which is sometimes called *successive induction*. Suppose the charge on A to be positive; then it acts inductively on the inner can, attracting a negative charge to its inner surface, and repelling a positive charge to its outer surface; this repelled charge in turn acts inductively on the next can, with a result indicated in the figure, and by this successive action induced charges are separated on each

conductor. It will be noticed that the inductive action on any one conductor is due to the *algebraic sum* of the charges in its interior; for example, in the case of the conductor C, there are three *equal* (Art. 14) charges in its interior, and the effective charge, that is, the algebraic sum of the three, may be represented by that on A, or by that on the outer surface of the first can. Hence the induced charges on the second can are equal to those on the first can, and each numerically equal to the charge on A. Similarly, the induced charges on any one of the conductors are each numerically equal to that on A.

If the electroscope be connected to any one of the conductors, and another *interior* to that one be touched with

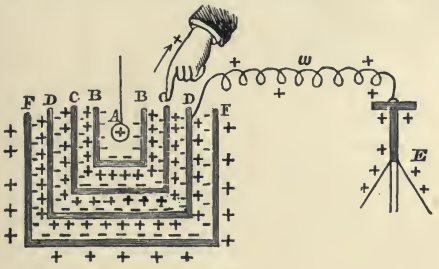


Fig. 15.

the finger, it will be found that the electroscope is not affected when A is lowered into the inner can, thus showing that the earth-connected conductor limits the field of induction. This result is evident from Fig. 15; when C is touched the algebraic sum of the charges inside D is zero, owing to the fact that the odd positive charge on C has escaped to earth. If, however, a conductor external to that connected with the electroscope, for example, E, be touched, then the inductive action of A is at once indicated by a divergence of the leaves of the electroscope.

The fact that the inductive action on any hollow conductor is due to the algebraic sum of the charges in its interior has an important application.

In Art. 5 we have seen that when electrification is pro-

duced by friction, the rubber and the substance rubbed become oppositely electrified. We are now able to show by experiment that the charges so produced are not only opposite in sign, but *equal in magnitude.* Two metal cans, A and B, are arranged as shown in Fig. 16, and insulated from each other. The inner, A, is lined with fur, and an ebonite rod, R, fits closely into it. The outer can, B, is connected with an electroscope. On rotating R, electrical separation takes place, the fur becoming positively and the rod negatively electrified. If these two charges are unequal, then the excess of the greater will act inductively on B, and the leaves of the electroscope will diverge. Experiment, however, shows that no such divergence takes place, thus proving that the algebraic sum of the charges in the interior of B is zero, that is, the charges produced on the rubber and the body rubbed are equal in magnitude, but of opposite sign. If, however, R be removed, it carries the negative charge with it, and the positive charge in A, acting inductively on B, causes an immediate divergence of the leaves. On replacing R, the leaves at once collapse, showing directly that the charge on A is equal and opposite to that on R.

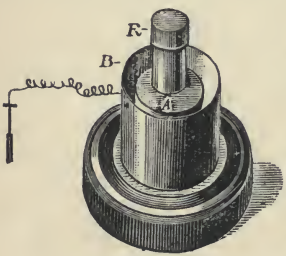


Fig. 16.

16. The Electrophorus. It is often desirable to be able to charge a conductor with electricity of a given sign. In Art. 4 we have seen that an insulator charged by friction does not readily yield up its charge to a conductor placed in contact with it, and cannot therefore be conveniently used for this purpose, but in Art. 11 it is explained how inductive action may be applied to effect the charging more satisfactorily than direct contact. Thus, although a metal can is not appreciably charged by touching or rubbing it with a charged ebonite rod, it can be easily charged by induction. The rod is placed in the interior of the can and

the latter touched with the finger; as a result, the can at once receives a charge equal in magnitude but opposite in sign to that on the rod, and this is effected *without diminishing the charge on the rod*, so that, theoretically, the latter may be used to charge any number of cans in the same way. In practice, however, the charge on the rod gradually escapes owing to defective insulation, but under favourable circumstances a large number of charges could be obtained without again exciting the rod. This is the principle of the electrophorus. As usually constructed the instrument consists, as shown in Fig. 17, of a disc of ebonite,* E, and a plate of brass, B, of slightly smaller diameter, attached to an insulating glass handle. The ebonite is sometimes fixed in a brass base called the sole, but this is not an essential part of the instrument, and is frequently represented by a coating of tinfoil on the under surface of the ebonite or by the table on which the disc rests. The action of the instrument is indicated in Fig. 18. The ebonite is excited by rubbing it with fur, and a strong negative charge is thus developed on its surface. The brass plate is now laid on the surface of the ebonite, and the charge on the latter, acting inductively on the plate, attracts positive electricity to its lower surface and repels negative to its upper surface. The brass is now touched with the finger and the repelled negative goes to earth, leaving the positive charge on the plate bound by the negative on the ebonite. If the brass plate be now removed, this positive charge distributes itself over its surface, and may be communicated by contact to the body which it is desired to charge, or applied to any other purpose.

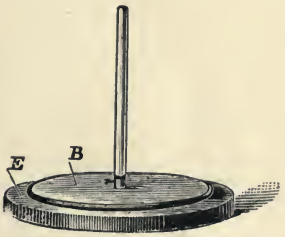


Fig. 17.

* *Theoretically* any insulating substance would answer the purpose, but ebonite gives the best results. Cakes of sulphur, sealing-wax, and other resinous substances, are sometimes used.

It should be noticed that when the brass plate is laid on the ebonite, it does not receive a charge by conduction, but is acted on *inductively*. This is due to the fact that neither surface being truly plane the points of actual contact are comparatively few, and the total area of contact is very small. Hence, only an inappreciable fraction of the charge

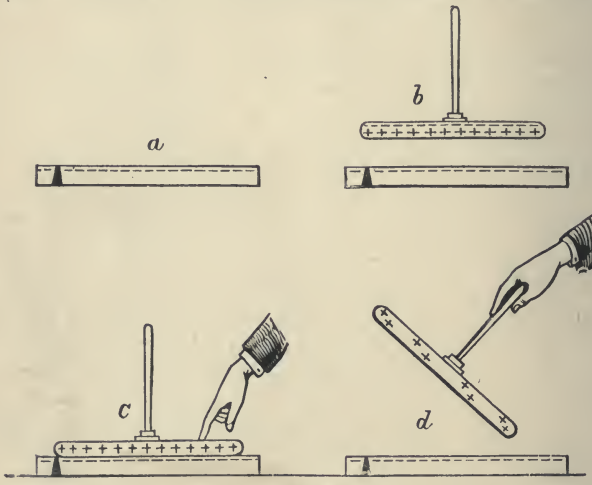


Fig. 18.

on the disc can be directly communicated to the brass, for the ebonite, being an insulator, gives up only that portion of its charge which is spread over the surface of actual contact.

To do away with the necessity of touching the plate with the finger each time it is charged, a brass pin connected with the sole is sometimes let into the ebonite disc in the way shown in Fig. 18. The brass plate, when laid on the ebonite, makes contact * with the point of the pin, and is thus connected to earth as required.

* Actual contact is not essential; the repelled negative from the brass will readily spark across to the point of the pin if the distance is not too great. See Art. 17.

In the brief explanation just given the action of the *sole* is not dealt with, but the complete explanation will now be readily understood. Suppose the ebonite to be excited, and let the charge on its surface be represented by — 20. This charge acts inductively in all directions, and there will be an equal and opposite induced charge on the surface of the earth-connected conductors in its neighbourhood. The sole being the nearest of these conductors will possess the greater portion of this charge, and the remainder will be, say, on the ceiling of the room. Let + 16 denote the induced charge on the sole and + 4 that on the ceiling. On placing the *insulated* brass plate on the ebonite neither of these charges will be disturbed, but on connecting it to earth it is put in connection with the sole, and the total inductive action of the charge on the ebonite is now concentrated on these two conductors. Hence the four units originally appearing on the ceiling are now to be found on the brass plate, and this conductor being now nearer than the sole, the greater portion of the induced charge will lie on it. Hence the distribution of the induced charge may now be represented by, say, + 15 on the brass plate and + 5 on the sole, that is, 11 units have passed from the latter to the former, and these combined with the four originally found on the ceiling make up the 15 it now possesses. On removing the plate the 15 units are removed with it, and the induced charge on the sole gradually increases until, when the plate is some distance away, the original distribution of 16 on the sole and 4 on the ceiling again obtains. When the plate rests on the ebonite the 15 units are distributed over its *lower* surface; if it be slowly raised vertically, the charge gradually spreads over the upper surface, and when removed beyond the influence of the negative charge both surfaces are equally charged.

Hence, when the plate of the instrument is touched a positive charge flows *out* of the sole, and when the charged plate is removed a positive charge flows *into* the sole. These effects are readily illustrated by experiment. Place the charged disc of an electrophorus on the cap of

an electroscope. On placing the plate on the ebonite no effect is produced, but on touching the plate the leaves at once diverge widely, showing that a portion of the charge in the sole is now free and would pass to earth if communication offered.* If the plate be now removed the leaves will again collapse,† showing that the charge just liberated is now recalled into the sole.

It thus appears that, by the action of the electrophorus, a theoretically unlimited number of charges can be obtained from the single charge on the ebonite disc. This at once shows that electricity cannot, in itself, be a form of energy, for if it were, we could, by the action of this instrument, obtain an almost inexhaustible supply of energy from the finite quantity represented by the charge on the disc. If, however, the positively charged plate be held over a number of pith balls it attracts them vigorously, and may be thus made to do a considerable amount of work. The plate therefore possesses energy, and a question arises as to where the energy which it receives each time it is charged comes from. This question is readily answered. When the positively charged plate lies on the negatively charged ebonite, each attracts the other; hence in removing the plate work has to be done against this force of attraction, and the energy it possesses when separated from the ebonite is equivalent to the work done in effecting the separation, just as in lifting a weight from the ground the potential energy it possesses at any height is the equivalent of the work done in lifting it to that height. In the one case, work is done in moving electricity against electric force, and the potential energy of the charge may be called electric potential energy; in the other case, the work is done in moving matter against the attraction of the earth, and the

* If at this stage the disc and plate be removed together without discharging the electroscope, the latter will be found to be charged with positive electricity.

† On trying this experiment it will be found that occasionally the leaves collapse and re-diverge. This is due to a partial escape of the freed positive charge from the edges of the leaves. On removing the plate an equivalent amount of negative is repelled to the leaves and causes the re-divergence. See Art. 12.

potential energy of the weight is what is commonly called potential energy of position. If the electrical potential energy be expended in doing mechanical or chemical work, in producing heat, or is in any way transformed, its transformation is subject to the general law of the Conservation of Energy.

In the above, it should be noticed that the electric potential energy of the charge on the brass plate is the equivalent of the work done in separating it from its inducing charge; hence, in considering the potential energy of any charge, we may take it to be the equivalent of the work that would have to be done to separate it from its induced charge, wherever that exists, or the equivalent of the work that either would do in attracting the other up to it.

17. The Electric Spark. If the knuckle be presented to the charged plate of an electrophorus, it will be found that, on bringing it within, say, half an inch of the edge of the plate, a bright spark passes between the knuckle and the plate, and at the same time a sharp crack accompanying the spark is plainly heard, and a peculiar pricking sensation is experienced. On testing the plate after the passage of the spark, it will generally be found to be completely discharged. This spark discharge is due to the break-down of the intervening air space, under the strain produced by the attraction between the charge on the plate and the induced charge on the knuckle. These charges unite, and the potential energy due to their separation is transformed into the heat, light, and sound of the spark.

CHAPTER III.

DISTRIBUTION OF ELECTRICITY ON CONDUCTORS.

18. The Charge resides on the Outer Surface of a Charged Conductor. In Art. 3 it has already been indicated that the charge on an electrified body is distributed over the surface of that body. In the case of charged conductors the charge is found on the *outer* surface only; for example, if a charge be given to a hollow conductor no charge can be detected on its inner surface. This statement is readily established by a number of simple experiments. If a brass ball suspended by a silk thread be placed in contact with the inner surface of a charged metal can, it will be found on removal to possess no charge. This shows that no free charge resides on the inner surface of the vessel, for when in contact the surface of the ball formed a portion of this surface, and would therefore have removed a corresponding portion of the charge, had any such existed. The same fact may be more directly illustrated thus. Let an electroscope be placed inside a tin box, and let its cap be connected with the inner surface of the box. On charging the latter, the leaves of the electroscope, viewed through a small opening in the side of the box, show no divergence even when the box is very strongly charged.

It is not essential for the electroscope to be connected with the inner surface, for it will be seen on consideration that the inner surface is connected with the outer surface by the material of the box, and therefore the same effect will be produced whether the connection be made inside or outside, so long as the electroscope remains within the hollow con-

ductor. Another experiment, first made by Biot, illustrates this fact very neatly. An insulated brass ball, A (Fig. 19), is charged, and two hollow hemispherical conductors, B and C, of somewhat larger diameter, are then closed over it by the help of the insulating handles attached to them. So long as contact does not take place between the ball and the enclosing shell formed by B and C, the charge remains on the former, but immediately contact is made the charge passes to the outer surface. On removing B and C and testing them they will be found to be charged, while the ball may be shown to be completely discharged. This

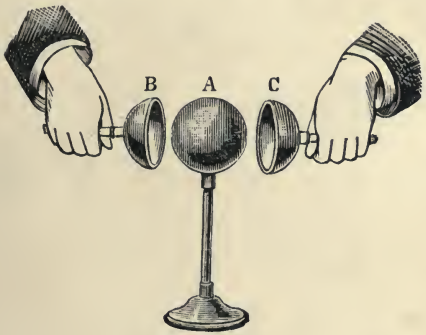


Fig. 19.

is Biot's form of the experiment, but it may be more readily performed by lowering the charged body into a rather deep metal can—as in Faraday's ice-pail experiment. So long as the ball does not touch the vessel it retains its charge, but if contact is made, then it will be found that the vessel is charged and the ball completely discharged, thus showing that a charge cannot remain on a conductor whose surface forms a portion of the inner surface of a hollow conductor. The result of this experiment has an important practical application. If one conductor is required to *completely* deliver up its charge to another, this latter should be made in the form of a hollow vessel and insulated;

then on placing the former *inside* it and in contact with its inner surface, the total charge at once passes into the outer conductor. This may be repeated as often as is desirable, and a considerable charge may thus be accumulated in a single conductor.

Many other experiments have been devised in support of the fact that no charge resides on the inner surface. Faraday's net experiment should be mentioned. A net of linen gauze is mounted on an insulating stand, as shown in Fig. 20. The string, *ss*, is of silk, and is used to turn the net inside out. The net, extended as in the figure, is strongly charged. On testing with a proof-plane and an electroscope the charge is found to be accumulated on the outer surface, but none can be detected in the interior. The

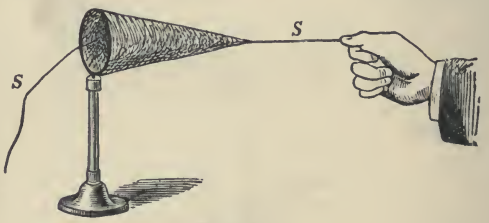


Fig. 20.

net is now turned inside out, but the charge is still found on the outer surface, only having thus changed surfaces.

In applying the above law as to the distribution of the charge on the outer surface only, care must be taken that the proper interpretation is given to the term "outer." The ordinary meaning of the word is not always the one applicable—for the purpose of this law the outer surface of any conductor is the surface nearest the induced charge of opposite sign. For example, if a metal pail be charged, the induced charge will, under ordinary circumstances, be on objects external to it, and, therefore, the charge will be on what we usually call its outer surface; but if a slightly smaller pail be placed inside it and connected to earth, then the induced charge will be found chiefly on this pail, and

the charge on the outer vessel will lie on what is usually called its inner surface, but which is now its electrical "outer" surface, that is, the surface nearest the induced charge.

From a consideration of this result, we shall be able in a general way to explain why the charge always resides on the "outer" surface of a conductor. Evidently the charge always distributes itself so as to be as near as possible to its induced charge, that is, in such a way as to possess minimum potential energy. Just as objects always tend to fall towards the earth and to rest at the lowest attainable level, so electricity passes to its position of lowest attainable potential energy on the outer surface of the conductor. For example, if mercury be poured into a vessel full of water, it passes through the water and takes up a position at the bottom of the vessel, between the water and the vessel; similarly, if a charge be given to the interior of a hollow conductor, it passes to the outer surface, and takes up its position between the conducting surface and the insulator surrounding it. Hence, if we choose, we may say that the charge of a conductor resides not on the conductor at all, but on the surface of the insulating medium adjacent to its "outer" surface.

Another way of expressing the same fact is to say that the charge on a conductor takes up the position of lowest potential energy.

19. Distribution of the Charge on the Outer Surface of a Conductor. The general law given at the end of the preceding article may be applied to the consideration of this question. If all points on the outer surface of a conductor are positions of equal potential energy for the charge, then the distribution of the charge over that surface will be uniform. If, however, there are portions of the surface where the potential energy of the charge would be less than on other portions of the surface, then the charge will accumulate on those portions, until the distribution of the opposite induced charge is such that all points on the surface of the conductor are positions of equal potential energy for the charge. In this case the distribution of the

charge on the surface will not be uniform, but will vary with the form of the surface and with the position and arrangement of earth-connected conductors in its neighbourhood.

We shall only consider how the form of the conductor affects the distribution, assuming all other conductors to be removed to a distance. On a spherical conductor the distribution is uniform, as indicated by the dotted line in Fig. 21 (*a*). On a conical conductor, such as shown at (*b*), the distribution is indicated in a general way in the figure; the charge accumulates at the pointed end, where the curvature of the surface is greater than at other points. Similarly, in the case of the circular plate (*c*) the charge accumulates at the edges where the curvature is greatest, and, in general,

Fig. 21.

experiment shows that the distribution varies with the curvature of the surface, that is, the greater the curvature at any point the greater will be the quantity of electricity at that point, provided the surface be *convex* at the point and the latter not within a reentrant hollow.

These facts may be experimentally verified in the following manner. Let a large spherical conductor, such as shown in Fig. 21 (*a*), be well insulated, and then strongly charged. By means of a proof-plane we can now test the distribution of the charge over the surface of the conductor. The proof-plane is laid on this surface at any point, and the charge received is communicated to an electroscope. The divergence of the leaves, which may be noted on a small scale mounted behind them, roughly indicates the magnitude of this charge. The charge which the proof-plane receives at any point on the surface of the conductor is, however, proportional to the quantity of electricity distributed over the

surface at that point. Hence, by comparing the charges to be obtained at different points of the surface, the distribution over the whole surface may be investigated. In this case of a spherical surface, it will be found that the charge to be obtained from any point on the surface is the same, showing that the distribution is uniform.

If a conical conductor, such as shown at *b*, be charged, and the distribution of the charge tested, it will be found that a much greater charge can be obtained from the pointed end than from the other end. Similarly, the distribution on any surface may be determined, and it will be found that the charge always tends to accumulate at points, corners, and edges, leaving the "flatter" portions of the surface less heavily charged, and the reentrant portions almost or quite free from charge.

20. Surface Density. The surface density for the surface of a charged conductor is measured by the quantity of electricity distributed over unit area of surface. Thus, if a charge, Q, be uniformly distributed over a surface of area, S, then the surface density (usually denoted by σ) is given by—

$$\sigma = \frac{Q}{S}.$$

If the charge is not uniformly distributed, then $\frac{Q}{S}$ gives the *average* surface density. The surface density *at any point* on a surface is determined by the ratio $\frac{q}{s}$, where q is the small charge distributed over s, a very small area of surface which contains the given point.

The surface density from point to point on the surface of a conductor varies in the same way as the distribution; that is, the greater the quantity of electricity distributed at any point, the greater the surface density at that point. The dotted lines in Fig. 21 may thus be taken to indicate the variation of the surface density from point to point on the surface of the conductors there represented.

21. The Action of Points. From the preceding articles we have learnt that the charge on a conductor tends to

accumulate at the points, corners, and sharp edges where the curvature of the surface is greatest. Thus, if a needle point project from the surface of a spherical conductor, the charge accumulates on this point, and the surface density there becomes very great. If the conductor is sufficiently charged, the density may become so great that the molecules of air in contact with the point receive a charge. These molecules are then repelled from the point, and their places taken by another set, which are in turn charged and repelled. Every molecule of air repelled from the point thus carries away with it a minute charge, and, until the surface density falls too low, there is a constant rain of charged molecules streaming away from the point. Hence, there is a continual discharge of electricity, sometimes called a *convection discharge*, from the point, and, in consequence, the charge gradually decreases, and will continue to do so until the surface density on the point is too small to charge the molecules of air in contact with it.* For the same reason a similar convection discharge takes place from the sharp edges of a charged conductor. This discharging action of points is readily illustrated by a number of experiments.† If a pin point be soldered to the cap of an electroscope and the instrument be charged, it will be found that it rapidly loses its charge —the divergent leaves quickly close up and soon indicate complete discharge. Similarly, if a pointed wire be attached to the plate of an electrophorus, it will be found that on charging the plate no spark can be drawn from it; but if the point of the wire be at once presented to a steady flame, the "wind" due to the convection discharge will blow the flame aside. The discharging action of sharp edges can be illustrated by a very simple but somewhat striking experiment. If a strongly charged ebonite rod be held over the cap of an electroscope for a few seconds and then slowly removed, the divergent leaves will be seen to gradually collapse, and finally to open out as widely

* It must be remembered that the air is an insulator, and that its molecules do not readily receive a charge.

† See also Art. 23.

as before, but with a charge of opposite sign. The leaves at first diverge with negative electricity, inductively repelled by the charged rod. During the short time that the rod is held in position this repelled charge partially escapes from the very thin edges of the gold leaf. On slowly removing the rod the leaves gradually collapse as the bound positive charge on the cap is slowly liberated, and finally re-diverge with the excess of positive electricity corresponding to the negative which has escaped.

This action of points has some important applications. In the illustration just given the electroscope is charged by induction—the repelled induced charge escapes from the thin edges of the leaves, leaving the instrument charged with electricity of the sign opposite to that inducing charge. Similarly, if a pointed wire be attached to the cap of the instrument, and the inducing charge held for a short time over the point, the attracted induced charge will escape, leaving the instrument charged with electricity of the same sign as that of the inducing charge. Hence, if we wish to "collect" the charge from a charged body, it is only necessary to present a point or row of points attached to a conductor to it—the charge induced on the points flows as a convection discharge on to the surface of the charged body, neutralising the charge there found, and leaving the conductor charged with electricity of the same sign. The final effect is thus the same as if the points directly collected the charge from the charged body. This effect may be illustrated by a few simple experiments. Let one end of a wire be attached to the cap of an electroscope, and the other end, carried in a split ebonite penholder, presented to any charged body—the leaves at once diverge, showing inductive action, and on removing the wire it will be found that the instrument is permanently charged. Similarly, if the point of a needle held in the hand be moved several times, close to the surface, from one end of a charged ebonite rod to the other, it will be found on testing that the rod is almost completely discharged. Again, if a large needle, connected to the cap of a very delicate electroscope, be carefully insulated in the open

air,* it will be found that the leaves of the instrument will, in general, gradually diverge. The needle point "collects" the charge from the air around it, and the divergence of the leaves indicates the nature and extent of the electrification of the air at the point where the needle is placed. When this electrification is very great, the convection discharge often gives rise to a luminous glow diverging from the discharging point. This effect is often seen from the points of ship's masts, flagstaffs, etc., and has been called St. Elmo's fire. It may be seen on a small scale in a dark room from a needle point attached to a strongly charged conductor; for example, to the newly charged plate of an electrophorus, or better, the prime conductor of an electrical machine.

Owing to this action of points and sharp edges it is necessary to have the corners and edges of all conductors intended to retain charges carefully rounded and polished. The apparatus must also be kept scrupulously clean, for every particle of dust acts as a point, and therefore admits of escape of the charge.

* Some distance above the ground.

CHAPTER IV.

FRICTIONAL ELECTRICAL MACHINES.

22. The Frictional Electrical Machine. A frictional electrical machine is a mechanical arrangement for generating electricity by the friction of two suitable substances, and for collecting and storing the electricity so produced. Such a machine consists essentially of three parts, the *generator*, consisting of the body rubbed and the rubbers, the *collecting*

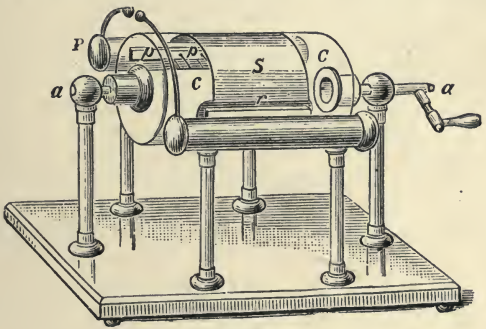


Fig. 22.

combs, and the *prime conductor* in which the charge collected by the combs accumulates. A very simple form of machine is shown in Fig. 22, which represents the essential parts of the ordinary cylinder electrical machine. The generator consists of a glass cylinder, C, capable of rotation about its axis, *a a*, and a rubber, *r*, made of a leather-covered pad coated with a mixture of an amalgam of zinc and mercury with grease. The rubber presses against the surface of

the cylinder, and on rotation the friction between the glass and the amalgam causes the surface of the former to become positively electrified, and that of the latter negatively. As the cylinder rotates the electrified portions leave the rubber behind, and other portions previously unelectrified come under its influence. Before again returning to the rubber the electrified surface of the glass passes close to the row of sharp-pointed wires which constitutes the collecting comb, *pp*. These points, acting in the way explained in Art. 21, "collect" the charge from the surface of the glass, which is thus rendered neutral and ready to be again electrified by the friction of the rubber. Connected with the comb is the prime conductor, P, an insulated conductor in which the charge collected from the cylinder accumulates. S is a silk flap attached to the rubber, and extending over the surface of the cylinder nearly to the collecting comb. Its use may be explained in a general way as follows. As the cylinder revolves the positive charge produced on its surface is separated from the corresponding negative charge on the rubber, but, owing to their mutual attraction, the former tends to slip back, over the surface of the glass, towards the rubber. The silk flap, however, acting as a rubber, has a small negative charge produced in it, and the attraction of this charge for the positive electricity on the glass is just sufficient to prevent any appreciable loss by reunion of the separated electricities. Fig. 23 illustrates a more usual form of electrical machine. It is known as the Plate Electrical Machine, and is in principle exactly similar to the one just described. A circular plate of glass

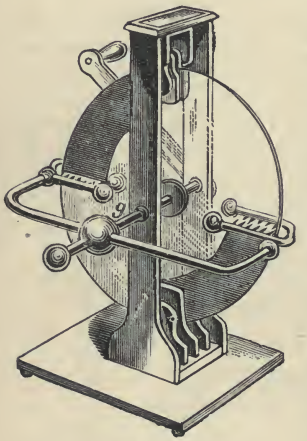


Fig. 23.

takes the place of the cylinder. The rubbers, two in number, are placed at the upper and lower extremities of the vertical diameter of the plate, each one being made up of two pads so arranged that the plate runs between them and consequently has both sides excited. The combs, having to collect from both sides of the plate, are bent round it in the way shown in the figure, and are placed at the extremities of its horizontal diameter. The silk flaps and prime conductor * perform the same function as in the cylinder machine. The action of the machine is readily understood. Following a portion of the surface of the plate from the lower rubber round through a complete revolution, we notice that as it leaves the rubber the glass is positively charged, and comes round to the right hand comb, where it is neutralised by the rain of negatively charged air molecules from the points; thus rendered neutral, it next comes under the influence of the top rubber, again becomes positively charged, and in passing the next comb is again neutralised, ready to be again excited by the friction of the bottom rubber. The continual discharge of negative electricity from the points of the combs leaves the prime conductor positively charged, and the magnitude of this charge increases as the machine is worked until the loss by defective insulation and convection discharge from its surface is equal to the gain from the combs. To reduce this source of loss as much as possible every part of the conductor is carefully rounded and polished, and the machine should be thoroughly clean and dry.

As generally worked the rubbers of the machine are earth-connected, that is, they are not insulated, but directly connected with the framework of the machine.† In this way no charge accumulates on the rubbers. If, however, they are insulated, then the negative charge accumulates, and finally becomes sufficiently great to prevent the positive

* The prime conductor is here attached to the frame of the machine, and is insulated by the glass stem *g*, which should be well varnished and kept perfectly clean and dry.

† In Fig. 22 the rubber *r* is attached to the insulated conductor shown in the front of the figure.

electricity produced on the glass from reaching the combs, and thus the machine ceases to work. If rubber and prime conductor are both insulated, but connected together by a conducting wire, then the negative of the rubber neutralises the positive charge induced in the prime conductor, and, however hard the machine may be worked, no sign of electrification can be detected in either rubber or prime conductor, thus proving that the quantities of electricity produced by friction are equal, but of opposite sign.

23. Experiments with the Electrical Machine. Let the machine be worked until the prime conductor is charged, then, on presenting the knuckle or any conducting body to the prime conductor, a spark at once passes and the conductor is discharged. If the machine be worked in a dark room the action of the points on the collecting combs may be noticed. A bright luminous glow or brush will be seen to diverge from each point, indicating that there is a continuous convection discharge from the points on to the surface of the glass plate. Similarly, if a sharp-pointed rod or wire be attached to the prime conductor, it will be found impossible to get a spark of any length from the machine, but in the dark, a bright brush discharge may be seen to emanate from the pointed end of the wire. Also, if the point of a needle held in the hand be presented to the charged prime conductor of the machine, a silent discharge takes place from the point neutralising the charge on the conductor. If the discharging point in either of these experiments be directed towards a candle-flame, the "wind" of the discharge will blow the flame aside and may extinguish it. The electric whirl shown in Fig. 24 is a favourite illustration of the discharging action of points. It conclusively proves that the discharge is due to the repulsion of molecules of air from those

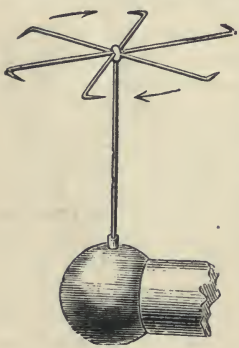


Fig. 24.

points, for in this arrangement the arms carrying the points are lightly pivoted on a central axis, and the reaction of the repelled molecules drives them round in a direction opposite to that in which the pointed ends of the wire are bent.

The repulsion between similarly electrified bodies is strikingly shown by attaching a tassel of light thread, strips of paper, hair, etc., to the prime conductor. On working the machine the tassel opens out, each individual thread or hair repelling and being repelled by every other one.

Henley's quadrant indicator is a simple application of this effect. As shown in Fig. 25, it is often attached to the prime conductor of electrical machines, to indicate the magnitude of the charge here collected. The pith-ball *b*, carried by a light lever, is repelled from the similarly charged brass rod, B, to which it is attached, and the angular distance through which it is repelled may be read off on the circular scale, *ss*, and roughly indicates the state of electrification of the conductor.

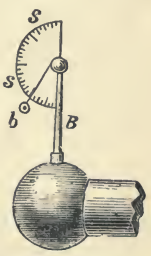


Fig. 25.

If a person stand on an insulating stool, that is, a stool with legs of glass or other insulating material, and grasp the prime conductor of the machine, he will practically form a part of the prime conductor, and will therefore gradually become charged as the machine is worked. Sparks can be drawn from any portion of his person, just as they may be drawn from the prime conductor, and by placing his finger near a gas jet, the gas may be lit by the spark which passes. A person thus charged will not retain a charge for many seconds after the machine stops working, for electricity escapes from the point of every hair and fibre of the clothing.

CHAPTER V.

ELECTRICAL POTENTIAL AND CAPACITY.

24. Electrical Potential. In Fig. 26 let A represent a positively charged body, and *a* and *b* any two points on the line AB. If a positive charge be placed at *b*, it will experience a force of repulsion due to the charge on A, and before it can be moved from *b* up to *a* this force of repulsion must

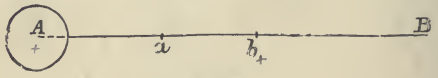


Fig. 26.

be overcome through the distance *ba*, and, therefore, work must be done. The work so done has its equivalent in the potential energy gained by the charge, in virtue of the change in its position relative to the charge on A. At *a* the potential energy of the charge is greater than at *b*, and if we consider *unit positive charge*, the difference for the two positions is called *the difference of the electrical potentials at the two points.* This difference of potential is due to the charge at A, and its value will therefore depend on the magnitude of this charge; hence, when we speak of the difference of potential at any two points, we imply the existence of a charge to which the difference of potential is due. The difference of the potentials at any two points *due to a given charge* may, therefore, be defined as the equivalent of the work done in moving *unit positive charge* from one point to the other against the electrical force *exerted by the given charge.* A familiar illustration may help the student to realise the above explanations. If a pound-weight is

lifted from a chair to a table, work is done, and the gain of potential energy by the weight is equivalent to the work done. In this case, the work is done against the force of attraction exerted by the earth, and the potential energy gained by the weight in virtue of the change of its position relative to the earth may be called the difference of the gravitation potentials due to the earth at the two points considered. So far, we have only considered difference of potential, but it is often convenient to speak of the electrical potential *at any point* due to a given charge. Now just as in Mechanics we can speak of the potential energy of a pound-weight, at any level, as the equivalent of the work done in raising it from *the surface of the earth* to that level, so in Electrostatics we may speak of the electrical potential due to a given charge, at any point, as equivalent to the work done in bringing a unit positive charge from the earth to that point *against the electrical force* exerted by the given charge. This is evidently not the absolute potential at the point, but merely the difference between the potential due to the given charge at the point considered and at a point on the earth.

If the charge at A be taken as negative instead of positive, *then no work would have to be done* in bringing unit positive charge up to *a*, but the force of attraction which the charge exerts *would do work* in so bringing it up. Hence, the potential at *a* is said to be negative, for the unit charge, instead of having, as it were, to be pushed up to *a* against the force of *repulsion* exerted by the charge on A, is pulled up to *a* by the force of attraction due to that charge.

If the potential, at a point in the neighbourhood of two or more charged bodies, be considered, then the total potential at that point due to these charges is the sum of the potentials due to each of the charges taken separately, each with its proper sign, according to the sign of the charge.

Difference of potential may be defined in another way. Just as in Hydrostatics difference of level is the condition of flow of a liquid from one point to another, so in Electrostatics

difference of potential is the condition of flow of electricity from one point to another. It must, however, be noticed that a liquid flows from a high level to a low level, in virtue of the greater potential energy it possesses at the higher level, and this difference of potential energy per unit mass of liquid is the true analogue of difference of electrical potential, so that this method of defining difference of potential is practically identical with that given above. Further, although difference of level (implying difference of potential energy) is the condition of flow of a liquid from one point to another, this flow will not take place unless some path is open by which it can take place. Similarly, flow of electricity between two points at different potentials will not take place, unless these points are connected by conducting material through which the electricity can pass. From this it follows that no difference of potential can exist between any two points on a conductor, for if such a difference exist for an instant, a flow of electricity at once sets in and immediately establishes equality of potential.* Hence, all points on and in a conductor are at the same potential called *the potential of the conductor.* This potential may be defined as the equivalent of the work done in bringing a unit positive charge from the earth up to any point on the surface of the conductor.

The potential at any point in the interior of a hollow conductor is also the same as that of the conductor, for in bringing a unit positive charge up to any point in the interior, it must first be brought up to the outer surface, and then all the work to be done on it is done, for there is no electric force in the interior of the conductor, and therefore no more work will be done in moving it from the surface to any point in the interior. A neutral conductor is said to be at zero potential, for it offers no resistance to the approach of a unit positive charge.

Since difference of potential is the condition of transfer of electricity from one point to another, it follows that, if

* This applies to electrostatics. In current electricity there is a constant supply maintaining the difference of potential and a *current* tending to equalise this difference.

two conductors at different potentials are put in contact electricity will flow from the one at the higher potential to the one at the lower potential until equality of potential is established. Similarly, if two conductors at considerably different potentials are placed *near* each other, electricity *tends* to pass from one to the other through the insulating air-gap separating them. If the difference of potential is sufficiently great, the electricity may break through this gap, producing a spark, and the potentials of the two conductors are at once equalised. The electric spark may be illustrated hydrostatically. Two communicating tubes, A and B (Fig. 27), are separated by a thin membrane, *m*. Mercury is poured into A up to a certain height, and then into B up to a higher level. The liquid now tends to pass from B to A, and the membrane, *m*, resisting the passage, is in a state of strain. If more mercury be poured into B this state of strain increases, and finally the membrane bursts, and the difference of level in the two tubes is at once equalised. If this experiment be performed, it will be noticed that after the break-down of *m* the mercury oscillates rapidly up and down in the tubes, and finally comes to rest at the same level in each. It has recently been shown that an exactly similar phenomenon accompanies the electric spark: the duration of the spark is probably considerably less than the millionth part of a second, but in this short time the electricity oscillates to and fro across the spark path a large number of times before equality of potential is established.

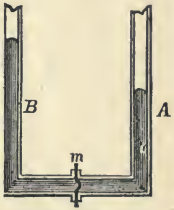


Fig. 27.

25. Potential Energy of a Charge. In the preceding article we have considered the electrical potential at a point due to a given charge. This must not be confounded with the electrical potential energy of a charge placed at that point. The electrical potential at any point due to a given charge is the work that would have to be done in bringing *unit positive charge* from the earth up to that point *against the electrical force exerted by the given charge.* The potential

energy of a charge placed at the point considered is the equivalent of the total electrical work done in accumulating the charge at that point. Thus the potential of a charged conductor is the equivalent of the work necessary to bring a unit positive charge from the earth up to the conductor, against the electric force exerted by the charge of the conductor, but the potential energy of the charge on the conductor is the equivalent of *all* the electrical work done in charging the conductor.

Consider a conductor charged with a positive charge Q to a potential V. That is, the quantity of electricity with which it is charged is denoted by Q, and the work that would have to be done to bring a positive unit of electricity from the earth up to its surface when so charged is denoted by V. Hence, the quantity of work necessary to bring Q units of electricity from the earth up to the conductor, *supposing its potential to remain constant at* V during the process, is given by QV. This, however, is evidently not the work done in charging the conductor *up to potential* V, with Q units of electricity, for during this process the potential is not constant at V, but rises from zero to V *in proportion as the charge is increased*, the latter value being acquired when the charging is complete.

The *average* value of the potential during the process of charging is therefore $\frac{V}{2}$, and the work done is the same as if the potential remained constant at this value during the process. Hence the work done in charging is given by $\frac{1}{2}QV$, and this therefore expresses the potential energy of a charge Q at potential V. If Q be expressed in the electrostatic units of quantity (Art. 6), and V in *ergs per unit of quantity*, then the energy of the charge is expressed in *ergs*.

26. Equipotential Surfaces. An equipotential surface is the locus of all points having the same potential. Imagine a charge to be concentrated at a point A (Fig. 28), then all points at the same distance from the charge have the same potential, that is, the equipotential surfaces in this case are a series of concentric spheres having their centre at the point A. The potential at each of the spherical surfaces is

different, but for all points on any one surface it is the same; that is, work has to be done to move a positive unit of electricity from one surface to another, but no work is done in moving it from any point on a given surface to another point on the *same* surface. From this it follows that the lines of force are at all points normal to the equipotential surfaces, for if no work is done in moving electricity from one point to an adjacent one on an equipotential surface, then the direction of motion must be everywhere perpendicular to the lines of force. In Fig. 28

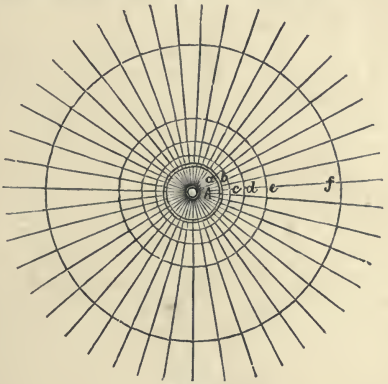


Fig. 28.

the traces of the equipotential surfaces surrounding the point A are shown as concentric circles, and the lines of force in the plane of the paper are shown as straight lines radiating from the centre A. The number of equipotential surfaces surrounding any charge is of course infinite, but it is convenient in drawing them to represent them so that unit quantities of work must be done in conveying unit quantity of electricity from any one surface to the next. When so drawn the distance between consecutive surfaces gradually increases as the distance from the charge increases. For the greater this distance the weaker the force exerted by the charge, and therefore the greater must be the distance through which it has to be overcome to do unit work.

If a charge be accumulated on a spherical conductor its action at all *external* points is the same as if the charge were accumulated at the centre of the conductor. Hence the equipotential surfaces surrounding a charged sphere are concentric spherical surfaces having the centre of the sphere as their common centre. The surface of the sphere is also an equipotential surface, for with statical electricity there can be no difference of potential between any two points of a conducting body. Hence, for a given charge the actual potential of the conductor will depend upon its radius. For example, in Fig. 28, if the surface of the conductor coincide with the equipotential surface *a,* the potential of the conductor *and of all points inside it* will be that of surface *a*; if, however, it coincide with *b*, *c*, *d*, or any other surface, then its potential will be that of the surface with which it coincides. The potential of all points external to the conductor is the same as if the charge were concentrated at its centre, but the potential of all points in the interior is the same as that of the conductor itself.

The fact that the surface of a conductor must be an equipotential surface may be applied to explain electrostatic induction. In Fig. 7, C represents a positively charged conductor, and the potential at A due to the charge on C is therefore greater than the potential at B due to the same charge. As a consequence of this difference of potential between the points A and B on the conductor AB, positive electricity flows from A to B, until equality of potential is established; that is, the end B becomes positively charged and the end A is left negatively charged. The potentials at the points A and B are now equal, because the potential at A due to the charge on C is diminished by the potential due to the negative charge there induced, and the potential at B due to C is increased by the potential due to the positive charge induced at that point. Similarly, equality of potential is established all over the surface of the conductor, for the charge induced at any point is such that the algebraic sum of the potentials at that point due to the electrification on A and the charge on C is the same for all points on the surface.

The fact that the charge always resides on the outer surface of a conductor may also be explained in terms of potential. In Fig. 29, let A represent a hollow spherical conducting shell of some thickness. If a positive charge be given to the inner surface, the potential of that surface will be greater than the potential due to this charge at the outer surface. Hence, since the inner and outer surfaces are connected by conducting material, the charge will at once pass to the outer surface, and no charge will be found on the inner surface.

Fig. 29.

27. Electrical Capacity. In the preceding article it has been noticed that the potential produced in a spherical conductor by a given charge varies with the size of the conductor. For example, in Fig. 28 if a charge Q be given to a conductor of the same diameter as the equipotential surface *c*, a lower potential would result than if the same charge were given to a conductor of the same diameter as the surface *a*; that is, the potential of a spherical conductor due to its own charge decreases with the size of the conductor. Further, it can be shown by experiment that if the same charge be given to any two conductors differing in size or form, different potentials will be produced in these conductors. For example, if we charge two conductors of different size or form with equal charges, and then present the one to the other, a spark will pass, indicating that they are at different potentials.

These facts may be expressed by saying that different conductors have different *electrical capacities*, and *the electrical capacity of a conductor* may be defined as *the ratio of the charge given to the conductor to the potential produced in the conductor by that charge.* That is, if a charge Q given to any conductor produce in it a potential V, then the capacity of the conductor is given by—

$$C = \frac{Q}{V},$$

where C denotes the capacity of the conductor. Another

way of expressing this definition is to say that *the capacity of a conductor is measured by the quantity of electricity necessary to raise the conductor to unit potential.* Thus, if a charge of 10 units of electricity raise the potential of a conductor to 5 units, then the capacity of the conductor is given by the ratio $\frac{10}{5} = 2$, that is, the conductor requires 2 units of electricity to raise it to unit potential. Similarly, if a conductor at potential V receive a charge Q, raising the potential to V′, then $C = \frac{Q}{V' - V}$, that is, the capacity of a conductor is measured by the quantity of electricity necessary to raise its potential by unity. Thus, if a charge of 10 units of electricity raise the potential of a conductor from 2 to 7, then the capacity of the conductor is given by $\frac{10}{7-2} = \frac{10}{5} = 2$, that is, the conductor requires 2 units of electricity to raise its potential by unity.

These ideas will be more easily realised by considering a simple illustration. If the same quantity of water be poured into tubes of different bores, the level of the water in each tube will be different, and the capacity of any tube may be conveniently defined as the quantity of water necessary to raise the level of the water in the tube through unit distance. This definition of the capacity of the tube relative to quantity of water and level is analogous to the definition of electrical capacity relative to quantity of electricity and potential.

It can be shown mathematically that the potential due to a charge Q, at any point at a distance r from the charge, is given by $\frac{Q}{r}$. Hence, if a charge Q be given to a spherical conductor of radius R, the potential of the conductor will be the potential at any point of its surface due to the charge Q, the latter being supposed to act from the centre of the conductor. That is, the potential of the conductor is given by $V = \frac{Q}{R}$, where Q denotes the charge

and R the radius. From the relation given above however we have $V = \frac{Q}{C}$, where Q denotes the charge and C the capacity of the conductor—hence, *in the case of a spherical conductor the capacity is measured by the radius of the conductor.* Thus, if V be expressed in ergs per unit of quantity, and Q in electrostatic units of quantity (Art. 6), then the radius of a spherical conductor in centimetres will express the capacity of that conductor.

From the ratio $C = \frac{Q}{V}$ we have $Q = VC$; that is, if a conductor of capacity C be charged to potential V, then the quantity of electricity with which it is charged is given by $Q = VC$. Thus, if a spherical conductor of radius 10 cms. be charged so that 10 ergs of work have to be done in bringing a positive unit from the earth up to its surface, that is, to potential 10, then the charge on the conductor is given by—

$$Q = 10 \times 10 = 100 \text{ electrostatic units of quantity.}$$

If two conductors charged to different potentials are brought into contact, they take up a common potential and share the combined charge in direct proportion to their capacities; for example, if two conductors of respective capacities 3 and 5 are made to share their charges, then one takes $\frac{3}{8}$ and the other $\frac{5}{8}$ of the combined charge. To determine the common potential which two or more conductors take on sharing their charges, we must remember two things—first, that the total quantity of electricity is the same before and after sharing; second, that the capacity of two or more conductors in contact with one another is the sum of the individual capacities of the conductors. Thus, if two conductors of capacities C_1 and C_2 be charged to potentials V_1 and V_2 respectively, then on being placed in contact they will take up a common potential V given by—

$$C_1V_1 + C_2V_2 = (C_1 + C_2)V;$$

or

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}.$$

The expression $C_1V_1 + C_2V_2$ gives the quantity of electricity before contact, and $(C_1 + C_2)V$ expresses the *same* quantity after contact and redistribution of the charge. If the conductors be separated they will still retain the potential V, and the charges on them will be in the ratio $C_1 : C_2$; that is, the charge on the conductor of capacity C_1 will be $\frac{C_1}{C_1 + C_2}$ of the total charge on both conductors, and the charge on that of capacity C_2 will be $\frac{C_2}{C_1 + C_2}$ of the total charge.

The quantitative comparison of potential is made by means of instruments called *electrometers*. Most electrometers are instruments of very elaborate construction, so that here we

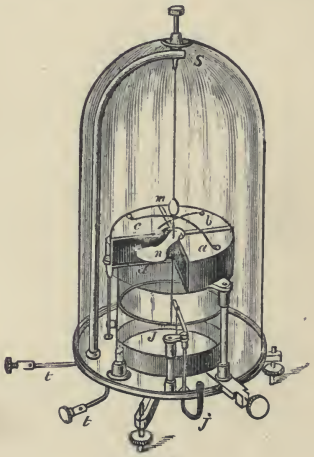


Fig. 30.

can only give a brief description of a simple form of the one most generally used. Sir William Thomson's *Quadrant Electrometer* consists essentially of four quadrantal boxes (such as might be obtained by cutting a shallow cylindrical

brass box into four quadrants) and a light needle of aluminium foil, suspended by a fine silver wire, so that it hangs inside these boxes. Fig. 30 shows three of the quadrants, *a*, *b* and *c*, and the needle, *n*, the fourth quadrant, *d*, being removed to show the needle.

The four quadrants are mounted horizontally on insulating glass stems, and the suspension wire of the needle is attached, as shown in the figure, to a suspension head, S. The principle of action of the instrument is simple. The needle *n* is charged to a comparatively high constant potential, by connecting it to the inner coating of a charged Leyden jar :* thus charged, it is attracted by the neutral brass quadrants ; but if it lie, as in Fig. 31, symmetrically along one of the lines of separation of the quadrants, the forces of attraction are in equilibrium, and the needle is not displaced. If, however, the four quadrants are divided into two pairs by connecting *opposite* quadrants (*a* to *c*, and *b* to *d*), the needle may be deflected by charging these pairs to unequal potentials.

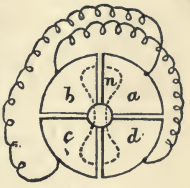


Fig. 31.

For example, if the needle be charged positively, and the quadrants *a* and *c* positively, and *b* and *d* also positively, but to a lower potential than *a* and *c*, then, since the capacity of each pair of quadrants is the same, the charge on *b* and *d* is less than on *a* and *c*, and therefore the force of repulsion between *b* and *d* and the needle is less than between *a* and *c* and the needle. As a consequence, the needle is deflected towards *b* and *d* by a couple equal to the difference between the couples due to the charges on *b*, *d*, and *a*, *c*. This difference depends ultimately on the potential of the needle and the difference of potential between *b*, *d*, and *a*, *c*, and therefore the deflection depends upon the potential of the needle and the difference of potential between the pairs of quadrants.

If the potential of the needle be maintained at a *constant high* value, and if the deflection be small, it can be shown

* See Art. 28.

that *the difference of potential between the two pairs of quadrants is proportional to the deflection.* Thus, if one pair of quadrants be connected to a charged conductor, the conducting system made up of the conductor and the two connected quadrants will be at a potential very slightly less than that of the conductor before connecting it with the quadrants. Now, if the other pair of quadrants be connected to earth, or *earthed*, the difference of potential between the quadrants will practically be that of the charged conductor, and the deflection of the needle will be proportional to that potential. Similarly, with any other charged conductor, a deflection proportional to its potential may be obtained, and the ratio of the two deflections is thus a measure of the relative potentials of the two conductors.

For convenience the Leyden jar, with which the needle is connected, is arranged in the base of the instrument. It consists of a glass vessel, J (Fig. 30), coated outside with tinfoil, and, when in use, half filled with strong sulphuric acid. The acid being a conductor serves as the inner coating of the jar, and also helps to secure good insulation by absorbing all moisture. The needle is connected to this inner coating by means of the piece of wire to which it is attached, and which is of sufficient length to dip into the acid. The opposite quadrants are connected by short pieces of fine copper wire, and the brass rods *t*, *t*, which are carried outside the glass case covering the instrument, are connected to two *adjacent* quadrants, and thus serve to make contact with the two pairs of quadrants; they are called the *terminals* of the instrument. Another similar brass rod, *j*, communicates, by means of a short length of platinum wire suspended from it, with the acid in the jar, and its terminal thus represents the knob of the jar. As in the torsion balance, the couple deflecting the needle is, in the position of equilibrum, balanced by the opposing couple due to the torsion on the wire, which, to secure the necessary sensitiveness, must be very fine. As the deflection is in general very small, the lamp and scale method illustrated in Fig. 32 is chosen. The light from the lamp passes through a

small circular aperture just below the scale, and, passing slightly upwards, falls on a small concave mirror, *m*, attached to the wire carrying the needle. It is thence reflected back on to the scale, where an image of the aperture is formed, and if a fine wire be stretched vertically across the slit the image of the wire enables the position of the reflected "spot" of light to be accurately read off on the scale. When the needle is deflected even through an extremely small angle, the *spot* on the scale moves through an appreciable distance, and for small deflections the angle of deflection may be taken as proportional to the deflection of the spot as measured by

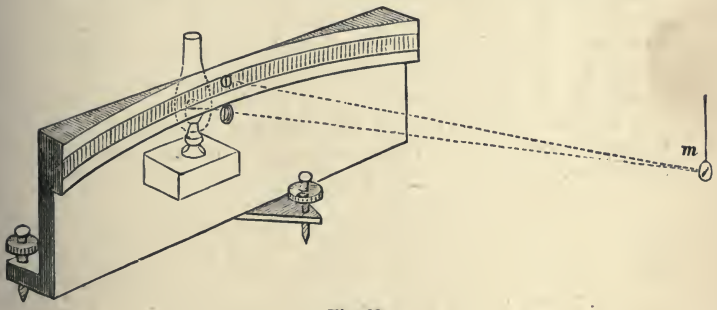


Fig. 32.

the divisions on the scale. Fig. 32 shows the scale and lamp adjusted to the mirror *m*, but the student can appreciate all the details of the arrangement only by seeing and working with the apparatus.

The electrometer in some form or other is *the* measuring instrument in electrostatics. Its use in comparing potentials has already been referred to, but it may also be used in the measurement of capacity and electrical quantity. As an example of the method of measuring capacity, let it be required to determine the capacity of a given conductor, say a tin can. A spherical conductor of known radius * is

* The diameter of the sphere should be about equal to the largest dimension of the given conductor.

carefully insulated and connected to one of the terminals of the electrometer, the other terminal being put to earth. This conductor is then charged until the needle of the electrometer experiences a considerable deflection, and the corresponding deflection of the spot on the scale is read off and noted down. The given conductor is now insulated and put in contact with the charged spherical conductor. A division of the charge then takes place, and the potential of the compound conductor takes a value lower than that first indicated by the electrometer. The reduced deflection of the spot is then read off and noted for comparison with the initial deflection. Let d_1 and d_2 denote these deflections in the order obtained, and let V_1 and V_2 denote the potentials to which they correspond, that is, the initial potential of the charged spherical conductor, and the potential of the system made up of the two conductors in contact. Then, if Q denote the charge given to the spherical conductor, C_1 its capacity, and C_2 the required capacity of the given conductor, we have—

$$Q = V_1 C_1 \qquad (1)$$

also

$$Q = V_2 (C_1 + C_2) \qquad (2)$$

for $(C_1 + C_2)$ is the capacity of the compound conductor made up of the two conductors in contact. Hence, from (1) and (2) we have—

$$V_1 C_1 \quad V_2 (C_1 + C_2),$$

that is,

$$\frac{V_1}{V_2} = \frac{C_1 + C_2}{C_1}.$$

But

$$\frac{V_1}{V_2} = \frac{d_1}{d_2},$$

and therefore

$$\frac{C_1 + C_2}{C_1} = \frac{d_1}{d_2},$$

that is,

$$\frac{C_1}{C_2} = \frac{d_2}{d_1 - d_2}.$$

But C_1 is known, for it is given by the radius of the spherical conductor, also d_1 and d_2 have been noted, therefore C_2 can be calculated from this relation.

Now that potential and capacity can be measured, it is evident that quantity can also be measured, for, to determine the magnitude of the charge on any conductor, it is only necessary to determine its potential, V, and capacity, C, and then to apply the relation $Q = VC$.

In the description given above of the method of measuring capacity, it has been assumed that the capacity of the quadrants of the electrometer is negligible, compared with the capacities of the conductors employed. This is not always the case, but it is always easy to determine the capacity of the quadrants, and to include it in working out the relation between C_1 and C_2. A simple method of effecting this determination is as follows. Let two spherical conductors of capacities C_1 and C_2, measured by their radii, be charged to the same potential,* and their potentials compared when joined successively to the quadrants of the electrometer. Let c denote the capacity of the quadrants of the instrument, V the common initial potential of the two conductors, and V_1 and V_2 the potentials assumed by these conductors when connected with the quadrants; then, if d_1 and d_2 denote the deflections corresponding to these potentials, we have—

$$V_1 (C_1 + c) = VC_1$$

and

$$V_2 (C_2 + c) = VC_2$$

that is,

$$\frac{V_1 (C_1 + c)}{V_2 (C_2 + c)} = \frac{C_1}{C_2}.$$

But

$$\frac{V_1}{V_2} = \frac{d_1}{d_2},$$

and therefore

$$\frac{d_1}{d_2} \cdot \frac{C_1 + c}{C_2 + c} = \frac{C_1}{C_2}.$$

* This is readily done by charging them in contact.

or

$$c = \frac{C_1C_2\,(d_2 - d_1)}{d_1C_2 - d_2C_1}.$$

In this relation C_1 and C_2 are known, d_1 and d_2 are given by the experiment, and c is therefore completely determined.

28. Electrical Condensers. In Fig. 33 let A represent a spherical conductor of radius R_1, to which a positive charge Q is given. The potential of the conductor due to this charge is $\frac{Q}{R_1}$. Let this conductor be surrounded by a hollow conducting shell, B, of radius R_2. This shell is connected to earth, and the space between the conductors is supposed to be filled with air. The positive charge Q acting inductively across the layer of air between A and B induces a negative charge, $-Q$, on the inner surface of B, and repels a positive charge equal to Q to earth. The potential of A is now due to its own charge *and to the charge on B.* The potential due to its own charge is $\frac{Q}{R_1}$ and that due to the charge on B in $\frac{-Q}{R_2}$;* hence, the potential of A is $\frac{Q}{R_1} - \frac{Q}{R_2}$, that is, its original potential $\frac{Q}{R_1}$ is now diminished by a quantity equal to $\frac{Q}{R_2}$. This result shows that the capacity of A has been *increased* by surrounding it with the earth-connected conductor B, for the same quantity of electricity, Q, raises it to a lower potential than at first, and, therefore, a greater quantity would now

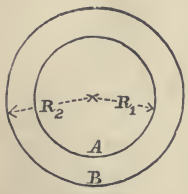


Fig. 33.

* The potential of B is $\frac{-Q}{R_2}$, and therefore the potential of all points within it is $\frac{-Q}{R_2}$. Art. 24.

be required to raise it by unit potential. The capacity of A by itself is R_1, but after surrounding it with B, it is greater than this, and its value may be determined from the fact that the charge Q raises it to potential $\frac{Q}{R_1} - \frac{Q}{R_2}$, that is, if C denote the increased capacity we have—

$$\frac{Q}{C} = \frac{Q}{R_1} - \frac{Q}{R_2},$$

for $\frac{Q}{C}$ also expresses the potential of the conductor. This gives—

$$\frac{1}{C} = \frac{1}{R_1} - \frac{1}{R_2}$$

or

$$C = \frac{R_1 R_2}{R_2 - R_1}.$$

From the fact that the capacity of A is increased by the presence of the earth-connected conductor B, this arrangement of conductors is called an electrical *condenser*. A greater quantity of electricity must now be given to A to raise it to a given potential, that is, the arrangement of the conductors A and B as a *condenser* enables A to receive a greater charge for a given rise of potential than it can receive as a simple conductor. For example, if A be connected to the prime conductor of an electrical machine it will be charged to the potential of the prime conductor, but the magnitude of the charge will be much greater when it forms a condenser with B than when it is charged as an ordinary conducting body.

It is not at all necessary that a condenser should have the form indicated in Fig. 33. Any two conductors separated by a layer of insulating substance will act as a condenser. One conductor must be insulated, and the other must be earth-connected—the former receives the charge, the latter by the reaction of its induced charge diminishes the rise of potential due to the charge, and admits of greater accumulation of electricity than if no

earth-connected conductor were present. A familiar form of condenser is the *Æ*pinus air condenser shown in Fig. 34.

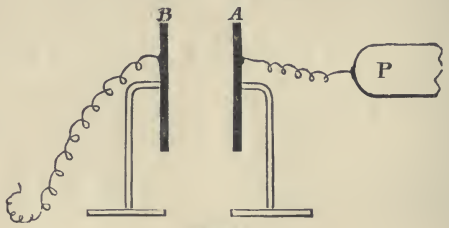


Fig. 34.

This consists of two metal discs, A and B, mounted on insulating stands. They are set parallel to each other, and the layer of air between them forms the separating insulating medium. Imagine A connected to the prime conductor P of an electrical machine, and B connected to earth. Then A receives a positive charge, and its potential gradually rises. At the same time a negative charge is induced on B, and *the negative potential at A* due to this charge also increases.

Hence, the rise of potential in A, due to the charge coming from P, is retarded by the *negative* potential, due to the induced charge on B, and thus a large quantity of electricity will flow into A before it rises to the potential of P. This action of the condenser may also be seen in this way. Imagine B to be removed, and A connected with the prime conductor; then electricity will flow into A, and it will quickly attain the potential of P. If B be now placed in position a negative charge is induced in it, and the potential at A due to this induced charge will reduce the potential of A to a value below that of P, and, as a consequence, a further flow of electricity will take place from P to A, until equality of potential is again established.

From the relation, $C = \frac{R_1 R_2}{R_2 - R_1}$, obtained above, it is

evident that the capacity of the spherical air-condenser there considered increases as the difference $(R_2 - R_1)$ diminishes; that is, the thinner the layer of air between the plates of the condenser, the greater the capacity. Also, if R_1 and R_2 are nearly equal, and therefore large compared with the difference between them $(R_2 - R_1)$, then we may write $C = \frac{R^2}{d}$, where R is the mean of R_1 and R_2 and d the thickness of the layer of air between the plates. But $4 \pi R^2$, being the area of the surface of a sphere of radius R, is approximately equal to the surface area of either of the plates of the condenser; that is, $4 \pi R^2 = S$, or $R^2 = \frac{S}{4 \pi}$, where S denotes the area of the surface of one of the plates of the condenser. Hence, substituting this value for R^2 in $C = \frac{R^2}{d}$ we have $C = \frac{S}{4 \pi d}$.

It can be shown that this result is roughly true for a condenser of any form, so long as d is small compared with the other dimensions. Hence we see that the capacity of such a condenser varies directly as the surface area of its plates, and inversely as the thickness of the layer of air between the plates; that is, the larger the plates and the closer they are together the greater will be the capacity of the condenser they form.

Further, it is found by experiment that the capacity of a condenser varies with the nature of the insulating medium or *dielectric* separating its plates. For example, if the capacity of a condenser with air as dielectric be C, then the capacity of the *same* condenser with, say, shellac as dielectric will have a different value, C_1, and with paraffin as dielectric another different value, C_2, and so on. This variation of capacity with the nature of the dielectric employed depends upon the *specific inductive capacity* of the dielectric; and *the ratio of the capacity of a condenser with some given substance as dielectric, to the capacity of the same condenser with air as dielectric, is a measure of the specific inductive*

capacity of the given substance. Hence, if the capacity of an air condenser be taken as $\frac{S}{4\pi d}$, then the capacity of that same condenser with a substance of specific inductive capacity K as dielectric is given by $\frac{K S}{4\pi d}$; that is, the capacity of a condenser of given form and dimensions increases with the specific inductive capacity of the dielectric between its plates.

There are several simple forms of apparatus constructed to illustrate the action of a condenser. If a small square of tinfoil be pasted on the middle of one surface of a pane of glass it will act as an insulated conductor, but its capacity is very small, that is, it can take only a very small charge. This may be shown by transferring the charge from the prime conductor of a machine to the tinfoil by means of a proof-plane. At the first contact a small spark passes, indicating the transfer of the charge from the proof-plane to the foil; at the second contact the spark is much feebler, and after a few times no spark can be seen or heard, indicating that the foil has been raised to the potential of the proof-plane, and that no further transfer can be effected. If, however, another similar piece of foil be pasted on the other side of the pane, directly opposite the first piece, and connected to earth, the two pieces of tinfoil act as a condenser, and a much larger charge may now be given to the first. If the charging be effected as before by means of the proof-plane, it will be found that a very great number of contacts have to be made before the cessation of the spark indicates that the charging is complete. The two pieces of foil are now charged, the one positively and the other negatively; but if either be touched with the finger no discharge takes place, because each charge is, at it were, bound by the attraction of the other. Hence a condenser cannot be discharged by touching or "earthing" either of its plates, but if the two plates be connected by any conducting material, discharge at once takes place, that is, the two opposite charges unite and neutralise each other. If

the discharge takes place through the human body, for example, if the plates of a condenser be connected by placing one hand on one plate and the other on the other plate, the discharge takes place through the arms and shoulders, and a convulsive muscular jerk, commonly called the electric "shock," is experienced. This is more or less severe according to the energy of the discharge.

A very familiar form of condenser is called the Leyden Jar. It consists, as shown in Fig. 35, of a glass jar having an inner and outer coating of tinfoil. These coatings cover the bottom and sides up to about three-quarters of the height of the jar, and act as the plates of the condenser. To exclude dust, and for convenience in use, the mouth of the jar is closed by a lid made of dry well-varnished wood, and contact is made with the inner coating by means of a brass rod, which passes through the centre of the lid. This rod terminates above in a rounded knob and carries below a loose piece of brass chain, the lower end of which rests on the tinfoil. To charge the jar the inner coating is put in connection with the prime conductor of an electric machine, that is, the knob of the jar is connected to the prime conductor or is put in direct contact with it, and the outer coating is connected to earth, for example, by being held in the hand. The inner coating thus receives, say, a positive charge, and an equal negative charge is induced on the inner surface of the outer coating, the corresponding positive charge being repelled to earth. These charges in the jar exert mutual attraction, and must therefore reside on the surfaces of the coatings nearest the glass, or on the inner and outer surfaces of the glass itself where they are as near as possible to each other. That this is actually the case may be shown experimentally by means of a jar with movable coatings (Fig. 36). The jar is charged in the usual way and placed upon an insulating stand, for example, on a sheet of dry varnished glass. The inner coating is now

Fig. 35.

lifted out by means of an insulating loop of silk thread or ribbon and placed on the sheet of glass. The glass jar is now lifted out of the outer coating and also laid on the glass. If either of the metallic coatings be now removed and tested by the electroscope, it will be found to possess no charge whatever; but if the jar be put together again as before, it will be found to be as strongly charged as at first. This shows that the charge may be located on the surfaces of the glass which forms the dielectric of the condenser. If a Leyden jar be charged and placed upon an insulating stand, either coating may be connected to earth, for example, by touching it with the hand, without discharging it, because the charges are mutually bound, and neither can escape because of the attraction exerted by the other. If, however, the two coatings be connected, discharge at once takes place; for example, if the little finger be placed on the outer coating, and the thumb be gradually brought near the knob of the inner coating, the strain in the air-gap separating the two will ultimately become too great, and a spark will pass, producing an oscillatory discharge of the jar and a convulsive shock in the muscles of the hand.

Fig. 36.

Similarly, if a jar be charged and laid on the table, and a person standing on the floor present his knuckle to the knob, spark discharge at once takes place through table, floor, and the person of the experimenter. A safer and more pleasant method of discharging a jar is by means of the discharging tongs of Fig. 37, or by a bent piece of stout copper wire held in the hand.

The capacity of a Leyden jar,* as of any other form of

* The Leyden Jar being a condenser in which the thickness of the glass is small compared with its other dimensions, its capacity is approximately given by $C = \frac{KS}{4\pi d}$, where S denotes the area of either coating, K the sp. ind. cap., and d the thickness of the glass.

condenser, varies with its size: the larger it is the greater is its capacity, and for a given size the capacity is inversely proportional to the thickness of the glass jar, and directly proportional to the specific inductive capacity of the kind of glass used. It is not convenient to construct or work with very large jars, so when a jar of very large capacity is required, the expedient of uniting several small jars together is adopted. The outer coatings are all connected together, generally by placing them in a box lined with tinfoil, and the inner coatings are all joined by short brass rods passing from knob to knob. This arrangement of jars is called a *battery* of jars, and if n jars are so arranged, they are in reality equivalent to one jar n times as large, and consequently of n times the capacity of one of the components of the battery. The capacity of a jar is also increased by using thin glass, but the glass must not be too thin, or it may break down under the strain due to the mutual attraction of the charges.

Fig. 37.

If a jar be strongly charged, and then discharged, it will be found that if allowed to stand for some time a second small spark may be obtained by discharging it again. This charge which thus collects in the jar on standing is called the *residual charge*. It is due to the gradual recovery of the glass from the strain produced by the stress to which it is subjected while the jar is charged. Hence the more highly the jar is charged and the longer it remains charged before being discharged, the greater will be the residual charge. The magnitude of this charge also varies with the nature of the dielectric: an air condenser, for example, exhibits no residual charge, the air appearing to recover instantaneously from the strain to which it is subjected.

The condensing electroscope is an application of the principle of the condenser which is sometimes very convenient. The instrument shown in Fig. 38 is an ordinary gold-leaf electroscope, with a somewhat larger cap than usual. The cap C acts as the insulated plate of a condenser,

and another similar brass disc, D, acts as the other plate. This disc is usually attached to an insulating handle, *h*, and is carefully varnished, so that when laid on the cap of the electroscope the two plates form a condenser with a very thin layer of dielectric made up of the varnish and a film of air. If it be required to test the electrification of a body of very large capacity, but at a very low potential, the ordinary electroscope would be of no use. For, on placing the body in contact with the cap of the instrument, the common potential of the system would be slightly less than the initial potential of the body, and the capacity of the leaves is so small that the quantity of electricity necessary to charge them to this low potential is not sufficient to cause any appreciable divergence. If, however, a condensing electroscope be used with its plates C and D in position as a condenser, and the body be placed in contact with C, while D is connected to earth, the plate C will receive a very large charge. The common potential of the system will, however, be much lower than the initial potential of the body, and consequently the charge in the leaves is not sufficient to cause any divergence. When the condenser is charged, let the body be removed, and the plate D slowly raised. As D is raised, the potential of C rises, owing to the recession of the opposite charge on D, and electricity flows down to the leaves, and ultimately produces divergence. When D is entirely removed, the capacity of C is so much reduced that the charge it received *as the plate of a condenser* is sufficient to raise it and the gold leaves to a very high potential, and thus the latter re-

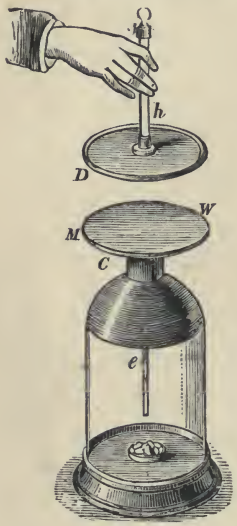


Fig. 38.

ceive a sufficient charge to produce divergence by mutual repulsion.*

The *potential of a condenser* is the potential of the insulated plate when the other is earth-connected, or, more generally, it is *the difference of potential* between the two plates. The *capacity* of a condenser is therefore equal to the quantity of electricity necessary to produce unit difference of potential between its plates. If V be the potential of a condenser, and C its capacity, then the charge in it is given by $Q = VC$, and the potential energy of that charge or the energy of discharge is, as in the case of a simple conductor, given by $\frac{1}{2}QV$.

It will be noticed that the capacity of a condenser depends upon three things. 1. The size of the plates—the greater the size the greater the capacity. 2. The thickness of the dielectric—the greater the thickness the less the capacity. 3. The specific inductive capacity of the dielectric—the greater the specific inductive capacity the greater the capacity.

The capacities of two condensers may be compared experimentally by the following method, due to Faraday.† Let the condensers be, for example, two Leyden jars of capacities C_1 and C_2, and let the one of capacity C_1 be charged to potential V. The quantity of electricity with which this jar is charged is given by $Q = C_1 V$ (1); and if it be made to share its charge with the other jar by putting the two knobs in contact, no electricity will be lost, but the common potential of the two jars after contact will be less than V. Let this common potential be denoted by v, then, as in Art. 27, we have—

$$Q = (C_1 + C_2)\, v, \qquad (2)$$

for during contact the two jars form one condenser of capacity $(C_1 + C_2)$.

* The charge on the leaves depends upon *their* capacity and the potential to which they are raised ($Q = CV$). Their capacity is very small; hence, for an appreciable charge, the potential must be high.

† This method is identical with that given on p. 80 for comparing the capacities of two conductors.

Hence, from (1) and (2) we have—

$$C_1 V = (C_1 + C_2)\, v,$$

or

$$C_1 (V - v) = C_2\, v,$$

that is,

$$\frac{C_1}{C_2} = \frac{v}{V - v}.$$

This relation expresses the required ratio of the capacities of the two condensers in terms of the potentials V and v. Now these potentials can be compared by means of the Quadrant Electrometer, and an experimental comparison of the capacities of any two condensers can thus be made.

To compare the potentials V and v by means of the Quadrant Electrometer, it is only necessary, after adjusting the instrument, to earth one terminal, that is, to connect it to iron gas or water pipes by means of a wire, and to connect the knob of the jar to the other terminal. The outer coating is earthed, and the deflection of the needle is then proportional to the potential of the inner coating, that is, to the potential of the jar as a condenser.

If therefore the deflection of the spot, given by the jar of capacity C_1, before sharing the charge with the other jar, is denoted by D, and the deflection given by *either* jar after the sharing of the charge by d, then—

$$\frac{V}{v} = \frac{D}{d},$$

and

$$\frac{C_1}{C_2} = \frac{v}{V - v} = \frac{d}{D - d}.$$

Compound condensers of very large capacity may be made by joining together a large number of simple condensers in the way indicated in the battery of Leyden jars. For example, if a large number of sheets of tinfoil and paraffined paper be laid alternately on one another, a

condenser may be formed by connecting together the 1st, 3rd, 5th, etc., sheets of foil as one plate of the condenser, and the 2nd, 4th, 6th, etc., sheets as the other plate. The sheets of paraffined paper act as the dielectric; but in the better forms of condensers, thin sheets of mica are used instead.

CALCULATIONS.

THE potential at any point at a distance r from a charge q is $\frac{q}{r}$ when the intervening medium is air, and $\frac{q}{Kr}$ when the intervening medium has a specific inductive capacity K. (Art. 24.)

If a conductor be charged with a quantity of electricity Q to a potential V, then the potential energy of the charge is given by

$$\tfrac{1}{2} Q V = \tfrac{1}{2} V^2 C = \tfrac{1}{2} \frac{Q^2}{C}.$$ (Art. 25.)

If Q be expressed in electrostatic units of quantity (Art. 6), and V in ergs per unit of quantity, then the energy of the charge is expressed in ergs.

The electrical capacity of a conductor is defined as the ratio of the charge given to the conductor to the potential to which the conductor is raised by that charge. That is,

$$C = \frac{Q}{V},$$

where Q denotes the quantity of electricity constituting the charge, V the potential to which the conductor is raised, and C the capacity of the conductor. (Art. 27.)

In the case of a spherical conductor of radius R, the potential to which it is raised by a charge Q is given by—

$$V = \frac{Q}{R}.$$ (Art. 24.)

But from the relation just given we have—

$$V = \frac{Q}{C}.$$

Therefore,

$$C = R.$$

That is, the capacity of a sphere is measured by its radius. (Art. 27.)

From the ratio $C = \frac{Q}{V}$ we have $Q = V C$.

That is, if a conductor of capacity C be charged to a potential V, then the quantity of electricity with which it is charged is given by $Q = V C$. (Art. 27.)

Similarly, if a quantity of electricity Q raise the potential of a conductor from V to V′, then the capacity of that conductor is given by—

$$C = \frac{Q}{V' - V}. \qquad \text{(Art. 27.)}$$

If two conductors of capacities C_1 and C_2 be charged to potentials V_1 and V_2, then on being placed in contact they will take up a common potential V, given by—

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}. \qquad \text{(Art. 27.)}$$

That is, for any number of conductors—

$$V = \frac{\Sigma (C V)}{\Sigma (C)}.$$

After contact the charge on the conductor of capacity C_1 is $\frac{C_1}{C_1 + C_2}$ of the total charge on *both* conductors, and the charge on that of capacity C_2 is $\frac{C_2}{C_1 + C_2}$ of the total charge. The *ratio* of the charges on the two conductors is therefore $C_1 : C_2$, and each conductor has the same potential V.

The capacity of an *air condenser* (that is, a condenser in which air is the insulating medium between the plates) formed of two concentric spherical surfaces of radii R_1 and R_2 is given by—

$$C = \frac{R_1 R_2}{R_2 - R_1}. \qquad \text{(Art. 28.)}$$

If V denote the potential of a *condenser*, and C its capacity, then the charge in it is given by $Q = V C$, and the energy of this charge is $\frac{1}{2} Q V = \frac{1}{2} V^2 C = \frac{1}{2} \frac{Q^2}{C}$.

Examples II.

1. The potential at a point 10 cm. from the centre of a charged sphere of 5 cm. radius is 1 C.G.S. unit. Find the energy of the charge on the sphere.

Here, if Q denote the charge on the sphere, we have—

$$\frac{Q}{10} = 1, \text{ or } Q = 10 \text{ C.G.S. units of quantity.}$$

Hence, the potential of the sphere is given by—

$$\frac{10}{5} = 2 \text{ C.G.S. units,}$$

and for the energy of the charge we therefore have—

$$E = \frac{1}{2} Q V = \frac{1}{2} . 10 . 2 = 10 \text{ ergs.}$$

2. A charge of 50 C.G.S units raises the potential of a spherical conductor from 10 to 15 units. Find the radius of the conductor.

The capacity of the conductor is from the definition given by—

$$C = \frac{50}{15 - 10} = \frac{50}{5} = 10,$$

and since the capacity of a spherical conductor is measured by its radius, the radius of the given conductor is 10 cm.

3. A spherical conductor of 5 cm. radius, and charged to a potential of 100 units, is placed inside a larger uncharged spherical conductor of 10 cm. radius, and made to touch its inner surface. Find the potential to which the larger conductor is raised.

Here, *all* the charge from the smaller sphere passes to the larger (Art. 18). Hence we have—

$$5 \times 100 = 10 \times V,$$

where V denotes the required potential of the larger sphere.

That is,

$$10 V = 500,$$

or

$$V = 50 \text{ C.G.S units.}$$

4. The smaller conductor of question (3), after having been discharged by contact with the inner surface of the larger conductor, is taken out and connected with the latter by a long thin wire. Find the common potential which the conductors attain, and the charge on each.

Here, if V denote the common potential of the conductors, we have—

$$V (5 + 10) = (50 \times 10) + 0.$$

That is,

$$15 V = 500,$$

or

$$V = \frac{500}{15} = 33{\cdot}3 \text{ units of potential.}$$

The charge on the larger conductor =

$$\frac{10}{15} \times 500 = 333{\cdot}3 \text{ units of quantity,}$$

and that on the smaller =

$$\frac{5}{15} \times 500 = 166{\cdot}6 \text{ units of quantity.}$$

5 A spherical air condenser, the radii of whose spherical surfaces are 8 cm. and 10 cm. respectively, is charged so that the difference of potential between its plates is 80 units. Find its charge.

The capacity of a condenser is given by—

$$C = \frac{10 \times 8}{10 - 8} = 40,$$

and since $Q = V C$ we have—

$$Q = 80 \times 40 = 3200 \text{ units of quantity.}$$

6. A Leyden jar charged to a potential of 200 units made to share its charge with a spherical conductor of 20 cm. radius. The potential of this conductor is then found to be 100 C.G.S. units. Find the capacity of the Leyden jar and the energy of its discharge before and after sharing its charge with the spherical conductor. Also find the *energy* lost when the charge is shared.

If C denote the capacity of the jar, we have, since no electricity is lost during the sharing of the charge—

$$200 C = 100 C + (100 \times 20),$$

or

$$100 C = 2000,$$

that is,

$$C = 20 \text{ units.}$$

The charge of the jar before sharing with the spherical conductor is given by—

$$Q = 20 \times 200 = 4000 \text{ units of quantity,}$$

and the energy of its discharge would therefore be—

$$\frac{1}{2} \times 4000 \times 200 = 400000 \text{ ergs.}$$

After sharing with the spherical conductor, the jar retains a charge equal to—

$$\frac{20}{40} \times 4000 = 2000 \text{ units of quantity,}$$

and the energy of its discharge would now be—

$$\frac{1}{2} \times 2000 \times 100 = 100000 \text{ ergs.}$$

The energy lost when the charge is shared will be given by the difference between the energy of the charge in the jar before sharing with the spherical conductor, and the sum of the energy of the jar after sharing, and the energy of the charge on the spherical conductor. This latter charge is equal to $\frac{20}{40} \times 4000 = 2000$ units of quantity. Hence its energy is $\frac{1}{2} \times 2000 \times 100 = 100000$ ergs, and we therefore have—

$$\begin{aligned} \text{Energy lost} &= 400000 - (100000 + 100000) \\ &= 400000 - 200000 \\ &= 200000 \text{ ergs.} \end{aligned}$$

7. A conductor of capacity C is charged to a potential V. Show that the energy of its charge is given by $\frac{1}{2} V^2 C$.

8. Four equal charges, each of 10 units, are placed one at each of the four corners of a square of 5 cm. side. Find the potential at the centre of the square and at the middle point of either of the sides.

9. A sphere of 10 cm. diameter charged with 50 C.G.S. units of electricity is placed in contact with an insulated tin can, and the potential of the conductors after contact is found to be 5 C.G.S. units. Find the capacity of the tin can.

10. Two equally charged spherical conductors of radii a and b are connected by a long thin wire. Find the ratios of their common potential after contact to the potential of each before contact.

11. Find an expression for the energy lost when a charged Leyden jar is made to share its charge with another exactly equal and similar jar. In what sense is the energy lost? what becomes of it?

12. The thickness of the air layer between the two coatings of a spherical air-condenser is 2 cm. The condenser has the same capacity as a sphere of 120 cm. diameter. Find the radii of its surfaces.

13. Two small equal insulated metal spheres are charged with quantities of electricity in ratio 3 to 7 respectively, and, when placed at a distance from each other equal to several times their diameter, they are found to repel each other with a certain force: they are then made to touch each other and afterwards separated to 3 times their previous distance. Compare the force now exerted between them with that exerted before they were brought in contact.

14. Give the theory of the Leyden jar and show how to charge it.

A quantity of electricity = 5 is conducted into the interior of a Leyden jar of surface = 2; and a quantity of electricity = 6 is conducted into the interior of a similar jar of surface = 3. Compare the heat developed by discharging each.

15. What is the law of attraction and repulsion between small bodies charged with electricity?

If a number of insulated bodies, some charged positively and some negatively, be suspended within an insulated tin canister, what will be the condition of the outside of the canister, and under what circumstances will it possess no charge?

16. Explain the electrical terms *potential*, *capacity*, and *specific inductive capacity*.

Assuming that the quantity of electricity produced by a plate machine is proportional to the number of turns of the disk, explain how the capacities of two condensers may be compared.

17. If a metal ball, hung by a dry silk thread, be made to touch the inside of an electrified metal jar and then carried to an electroscope, the electroscope is not affected; but, if the electroscope is connected with the inside of the jar by a wire, it receives a charge. Account for the difference between the two results.

18. A, B, and C are three Leyden jars, equal in all respects. A is charged, made to share its charge with B, and afterwards to share the remainder with C—both B and C being previously without charge. The three jars are now separately discharged. Compare the quantity of heat resulting from each discharge with what would have been produced by the discharge of A before any sharing of its charge.

19. It is usually said that two portions of electricity attract or repel each other with a force that is inversely proportional to the square of the distance between them. Does it follow that the attraction or repulsion between two electrified bodies is always in the inverse ratio of the square of their distance? If not, show why not: and explain generally the conditions under which the law of inverse squares does or does not apply to the force between two electrified bodies.

20. How would you compare the electric capacities of two Leyden jars? A Leyden jar is charged and caused to share its charge with another jar whose linear dimensions are double its own. How is the charge distributed, and how is the potential related to the original potential?

21. An insulated conductor, elongated in shape, is placed near a charged sphere. How would you investigate the distribution of the electricity on its surface?

Describe some method of charging two bodies so that they may contain equal quantities of electricity of the same kind.

22. Two small equal insulated spheres, placed 10 centimetres apart, are charged respectively with 5 and 20 units of electricity. The spheres are made to touch and then replaced; what are the forces between them before and after contact respectively?

23. Two spheres, of diameters 9 and 3, are connected by a long thin wire, and 144 units of electricity are shared between them. Compare their charges; and describe how to study the distribution of electricity on their surfaces when they are brought nearer together. From which sphere would a brush discharge first occur? Give a reason.

24. Given a large fixed insulated sphere, how would you proceed to test its charge (i.) for sign; (ii.) for approximate amount? In what units would you express the quantity of electricity on it?

***29. Electric Force exerted by a Charged Sphere at a Point near its Surface.** If Q denote the charge on the sphere, and σ the surface density of the charge, then we have—

$$Q = 4\pi r^2\sigma,$$

where r denotes the radius of the sphere.

For all points external to the sphere the charge acts as if it were a point-charge at the centre of the sphere, and the electric force at a point just outside the sphere, that is, at a distance r from its centre, is given by—

$$f = \frac{Q}{r^2} = \frac{4\pi r^2\sigma}{r^2} = 4\pi\sigma.$$

That is, the electric force exerted by a charged sphere on a point infinitely near its surface is $4\pi\sigma$ where σ denotes the surface density of the charge on the sphere. It can be shown that this result is true not only for a sphere, but for for any *closed* conductor. The electric force inside a closed conductor is zero (Art. 13); hence, for a point just outside the electrified surface the electrical force is $4\pi\sigma$, and for a point just inside the surface it is zero; that is, as the point

passes through the electrified surface the force changes by $4\pi\sigma$.

***30. The Electric Force exerted by a Charged Plane Surface of Indefinite Extent at a Point near it.** If a small plate be charged the surface density of the charge will be greater at the edges than at the central parts of the plate. If, however, the plate be very large the surface density over a small area near its centre will be practically uniform. Let this density be denoted by σ, then it can be shown that the force exerted at a point infinitely near this area is $2\pi\sigma$; this force will be of opposite sign on opposite sides of the plate: that is, on one side it will be $+2\pi\sigma$, and on the other $-2\pi\sigma$ (for a positive unit of electricity would be repelled in opposite directions, on opposite sides of the plate), and, consequently, as the point passes through the electrified surface the force changes by $4\pi\sigma$, just as in the case considered in the preceding article.

By the following *guard-plate* arrangement it is possible to charge a small plate so that the surface density of the charge is uniform. Let a small circular plate be cut centrally out of a larger plate and slightly reduced in size so that it fits freely, without contact, into the ring left in cutting it out of the larger plate. If this plate and ring be connected together by a fine wire and the arrangement insulated and charged, the surface density of the charge on the small plate will be uniform when it is in the same plane as the ring. The presence of this ring prevents increase of surface density at the edge of the ring, and for this reason is called a *guard-ring*.

***31. The Attracted-disc Electrometer.** The *absolute* measurement of difference of potential, that is, its direct determination in ergs per unit of quantity, can be effected by means of Thomson's *Attracted-disc Electrometer*. The principle of this instrument consists in balancing the attraction between two parallel discs at different potentials, and at a known distance apart, by a weight or other force which can be expressed in absolute units of force. The first instrument of this kind was used by Sir William Snow-Harris; the essentials of this instrument are shown in

Fig. 39. The parallel discs are shown at *a* and *b*, and the attraction between them, when at different potentials, is measured by weights placed in scale-pan P, which, when empty, exactly balances the disc *b*, when there is no electrical attraction between it and the disc *a*. The attraction between *a* and *b* is thus measured in absolute units of force, and as

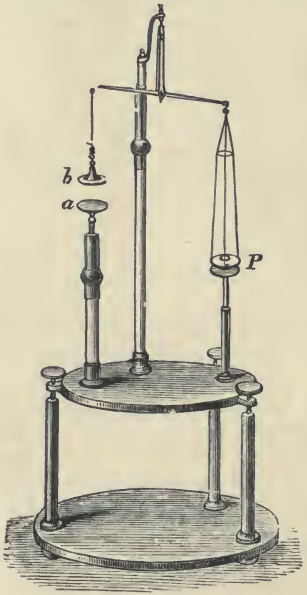


Fig. 39.

this attraction can be expressed in terms of the difference of potential between *a* and *b*, it is evident that this difference can be expressed in absolute C.G.S. units of potentials.

The modern form of the instrument is due to Sir William Thomson; the balance part of Snow-Harris' form is replaced by the arrangement indicated in Fig. 40. The attracted-disc, C, surrounded by a *guard-ring*, B, is suspended by

wires, in the way shown, from one end of a metal beam, L, pivoted at a by a wire stretched between two metal supports. This beam carries a counterpoise Q, at its other end, and the weight of Q, together with torsion on the suspending wire, is more than sufficient to support C in the plane of the guard-ring, so that to bring it down to this position it is necessary to apply a downward force, which can be determined by placing weights on C until the indicator attached to it shows that B and C are on the same plane. This indicator consists of a fine black hair (w) stretched across the forked end of the lever L; when C is in position this

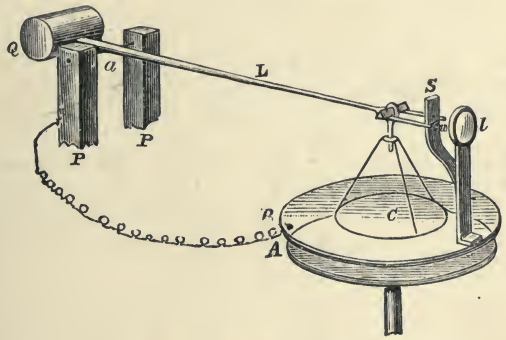


Fig. 40.

hair lies between two fine black dots on the enamelled surface of the metal strip S, and the lens l is provided to facilitate the accuracy of this adjustment by allowing the operator to observe a clear magnified image of the fine hair and dots. The plate A, which is considerably larger than C, is carried by an insulating support attached to a micrometer screw arrangement, by which the distance between A and C can be varied, and the amount of this variation exactly measured.

Before describing how the instrument is used, it will be necessary to show briefly how the force of the attraction between two plates at different potentials can be expressed

in terms of their difference of potential. In the arrangement of Fig. 40, let the plate C be charged to potential V, and let the surface density of the charge be denoted by σ. If A be now connected to earth it remains at zero potential, but a charge of opposite sign to that on C is induced on it, and the surface density of this charge will be denoted by $-\sigma$. The difference of potential between the plates is therefore V, that is, V units of work would be done in conveying unit quantity of electricity from A to C, hence if f denote the *electric force* in the space between the plates, and d the distance between the plates, we have—$fd = V$, for the field of force between the plates is uniform. But the electric force between the plates is given by $4\pi\sigma$, for a positive unit of electricity would be repelled downwards by C with a force $2\pi\sigma$, and it would be attracted downwards by A with a force $2\pi\sigma$, hence the total force which it would experience would be $4\pi\sigma$ downwards. Giving this value to f, we get $4\pi\sigma d = V$, or $\sigma = V/4\pi d$.

Now each unit of electricity on C is attracted downwards by A, with a force $2\pi\sigma$, and if S denote the area of C the charge on it is $S\sigma$, and the total force with which A attracts it down is $S\sigma \times 2\pi\sigma = 2\pi S\sigma^2$. If this force be denoted by F we have—

$$F = 2\pi S\sigma^2,$$

But,

$$\sigma = V/4\pi d.$$

Therefore,

$$F = \frac{2\pi S V^2}{16\pi^2 d^2} = \frac{S V^2}{8\pi d^2},$$

and

$$V = \sqrt{\frac{8\pi d^2 F}{S}} = d\sqrt{\frac{8\pi F}{S}}.$$

That is, if F is constant, the difference of potential between the plates is proportional to their distance apart.

We can now describe the method of using Thomson's Attracted-disc Electrometer; the plate C is always adjusted by sighting the indicator to the plane of the guard-ring, but the adjustment is made by moving A up or down until the force of attraction between the plates is equal to the known

force necessary to bring C into the sighted position, that is, the distance between the plates is varied so that the force of attraction between them always has a constant known value. Hence, one way of using the instrument would be as follows: let the conductor, whose potential is to be measured, be connected to C and the guard-plate,* and let A be earthed. Then let the position of A be adjusted until C occupies the sighted position and distance d between the plates C and A measured. The required potential of the conductor will then be given by the relation obtained above—

$$V = d\sqrt{\frac{8\pi F}{S}},$$

where d, F and S (the area of C) are known. This direct method, however, does not give good results, because it is difficult to measure d accurately, so the following indirect method has to be adopted.

The plate C and the guard-ring are maintained at a constant potential, and the plate A is first connected to earth and adjusted in position until C is in position. The reading of the micrometer screw attached to the pillar supporting A is then taken, and the connection between A and the earth broken. The plate A is next connected to the conductor whose potential has to be measured, and again adjusted for the equilibrium of C. The reading of the micrometer-screw is again taken, and the *difference* of the two readings noted; this difference gives the distance through which the plate A has been moved in effecting the second adjustment. Now, if V denote the constant potential at which the plate C and guard-ring are maintained, and v the potential to be measured, then we have, since the potential of A for the first adjustment is zero—

$$V = d_1\sqrt{\frac{8\pi F}{S}}, \qquad (1)$$

* The plate C and the guard-plate are connected by a wire joining the guard-ring, with the metal supports on which the lever L is pivoted.

where d_1 denotes the distance between the plates when the first adjustment is made. Similarly we have—

$$V - v = d_2 \sqrt{\frac{8\pi F}{S}}, \qquad (2)$$

where d_2 denotes the distance between the plates when the second adjustment is made. Hence, subtracting (2) from (1) we get—

$$v = (d_1 - d_2) \sqrt{\frac{8\pi F}{S}},$$

where v denotes the required potential, and $(d_1 - d_2)$ is accurately given by the micrometer screw, as the *difference* of two readings. The factor $\sqrt{\frac{8\pi F}{S}}$ may be determined once for all as a constant of the instrument.

***32. Specific Inductive Capacity.** In Art. 28 it is explained that the specific inductive capacity of a given substance is measured by the ratio of the capacity of a condenser with that substance as dielectric to the capacity of the same condenser (or an exactly similar one) with air as dielectric. Hence, the problem of measuring the specific inductive capacity of any substance resolves itself into one of comparing the capacities of condensers. The first measurements of specific inductive capacity were made by Faraday. He constructed two similar spherical condensers, each of which consisted of an inner spherical brass ball and an outer spherical shell, made in two halves, so that the space between might be readily filled with any given dielectric. To the inner sphere was attached a thin brass rod carrying a small brass knob, and this rod, insulated by a plug of shellac, passed through the outer shell and formed the pole by which the condenser could be charged—that is, it performed the same function as the knob of a Leyden jar. In one of these condensers air was the dielectric, and into the other different substances whose specific inductive capacities were to be determined were successively introduced. In each case the capacities of the two condensers

were compared by the method given in Art. 27, and the specific inductive capacity of the substance used as dielectric determined.

Since Faraday's time many new determinations of specific inductive capacities have been made. Modern experimenters have endeavoured to effect improvements mainly in two directions. Firstly, in the forms of condensers used and the methods of comparison. Guard-plate and cylindrical condensers of adjustable capacity are used in preference to spherical condensers, and null methods of comparison of capacity are employed instead of the simple, direct method of Art. 27. Secondly, efforts are made to eliminate the effect of what is known as *electrical absorption.* When a condenser is charged to a given potential it is found that the charge which it takes tends to increase with the time of charging. It was at first thought that the charge was absorbed by the dielectric—hence the term *absorption*—but it has been shown that the effect results from a change in the state of the dielectric, which requires time to take place, and which is analogous to the "set" which a wire takes when subjected to torsion.* The existence of this effect evidently complicates the question of comparing capacities; for the capacity of a condenser will vary with the time of charging, and the amount of this variation will also depend on the nature of the dielectric. To avoid this complication methods of comparison have been adopted in which the condensers are charged only for an extremely small fraction of a second, so that the effects of absorption are reduced to a minimum.

* The recovery of the dielectric from this electrical "set" explains the existence of the *residual charge*; see Art. 28.

CHAPTER VI.

INDUCTION ELECTRICAL MACHINES.

***33.—Introductory.** In induction electrical machines the supply of positive and negative electricity is the result of continuous inductive action between the charged moving parts of the machine. The electrophorus described in Art. 16 is a simple form of induction machine supplying only positive electricity; its action, however, is not continuous, and the manipulations of the operator take the place of the automatic action of a machine. A careful consideration of the following experiment will enable the reader to grasp the principles involved in the action of induction machines. Let two metal vessels, A and B, be insulated and charged, A positively and B negatively, and let a brass ball, C, suspended by a silk thread, be lowered into one of them, say A. If C be now, while inside A, connected to earth for an instant, it becomes negatively charged by the inductive action of the charge on A, and may be raised out of A, carrying its negative charge with it. While still negatively charged, let it be lowered into B and allowed to touch the inner surface of that conductor; *it at once gives up its charge* to B (Art. 18), *and the negative charge on B is thereby increased.* The ball C is now uncharged; but if connected to earth for a moment while it is still inside B, it will acquire a *positive* charge, as the result of the inductive action of the negative charge on B, and this charge may be transferred to A by removing C from B, and putting it in contact with the inner surface of A. The operations just described may evidently be repeated indefinitely, *and at each operation the initial*

charges on A and B are increased. This accumulative action will go on until the conductors A and B are charged to such a potential that their loss of charge, owing to imperfect insulation, is as great as their gain from *the carrier* C. If the motion of C were controlled by machinery the arrangement just described might be considered as a form of induction machine, in which the conductors A and B represent the positive and negative poles respectively, and if these conductors were furnished with knobs nearly in contact a spark would pass between them when the difference of potential between the knobs became sufficiently great. After the passage of this spark, however, the action of the arrangement would, in general, cease, for the conductors A and B would be almost completely discharged. We shall now proceed to consider a simple modification of the above apparatus, from which the possibility of continuous action will be more easily realised; but, before going further, the reader should be careful to fully understand the principles of action involved in the above arrangement. These are, first, the inductive action of the charged conductors A and B, on the carrier C, *when the latter is connected to earth*; second, the gradual increase of the initial charges on A and B by the successive transfer of the induced charges from C.

In the arrangement of Fig. 41, let the conductors A and B be represented by two short arcs of brass tube split along the inner edge, and the carrier C by the small brass ball C, attached to the end of an insulating rod pivoted at O, the centre of the circle, of which the axes of the tubes A and B, are arcs. The ball C, when rotated, passes freely through the tubes A and B, the insulating arm passing through the split on the inner side of these tubes. To effect the necessary contacts with the earth and the conductors A and B, two sets of springs, *a*, *b* and *e*, *e*, are provided; *a* and *b* make contact with A and B respectively, and *e* and *e* both with the earth. Let A receive a small positive charge, and imagine C to rotate in the direction indicated by the arrow. In passing through A the carrier C first touches the spring *a*, but has not yet received any

charge to give up; it next touches *e*, and is for an instant connected to earth; the positive charge on A now acts inductively on it, and, as a result, C leaves A with a negative charge induced on it. This charge is carried round towards B, and when C touches the spring *b*, which is on the inner surface of B, it gives up all its charge to this conductor. The charge on B is now increased, and when C touches the spring *e* in B it is acted on inductively by

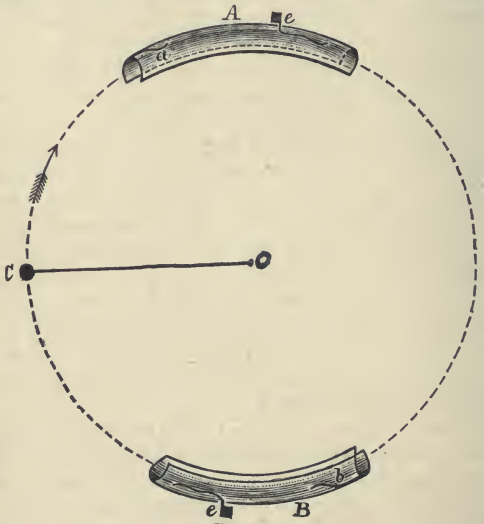


Fig. 41.

this increased charge, and the induced charge on C is carried round to the spring *a*, where it is delivered up to A. This action goes on until A and B are charged as highly as the circumstances of their insulation will permit. If these conductors are provided with knobs which can be brought near each other by means of sliding rods to which they are attached, successive sparks may, under favourable conditions, be made to pass between these knobs, for, after the passage of one spark, although A and B are nearly

discharged, a sufficient residual charge in general remains in each to maintain the action of the machine, and the conductors A and B are again quickly charged up to the necessary difference of potential to give the spark discharge between the knobs. In machines constructed on this principle, however, it is not usual for the parts here represented by A and B to form the poles between which spark discharge takes place. In this relation let us consider a further modification of the arrangement of Fig. 41. Let the carrier C, and the insulating arm which carries it, be replaced by an insulating rod pivoted through its centre at O, and bearing at *each* end a carrier C. Further, let the springs *e*, *e*, be insulated and connected with one another, instead of being connected to earth. The rod pivoted at O is supposed to revolve in the same way as that bearing the single carrier C, and the electrical action of the arrangement is exactly similar to that described above; when one end of the revolving rod is in contact with the spring *e* in A, the other end is in contact with the spring *e* in B, and it is evident that the inductive actions of the charges on A and B work together to induce a negative charge in the end of the rod near A, and a positive charge on the other end near B. If now a gap be made in the connection between *e* and *e*, it will be seen that, if the inductive action is strong enough, a spark may pass across this gap, and in most induction machines this is where the spark discharge takes place. When the machine is starting to work, the connection between *e* and *e* should be continuous; but as the charges on A and B accumulate, and the inductive action on the carriers becomes stronger and stronger, a break may be made in the connection, and spark discharge take place across this break, without interfering with the action of the machine.

Sir William Thomson, in 1867, constructed a small apparatus known as *Thomson's Replenisher*, which works in exactly the same way as the arrangement just described. The construction of the instrument is sufficiently explained by the Figs. 42 and 43, which are lettered to correspond with the diagrammatic figures given above. The darkly

shaded parts are of ebonite, and the rest of metal. The springs *e*, *e*, are connected by the strip of brass M, which runs round the ebonite base, on which the spindle for rotating the carrier C C rests. There is no gap in this connection between the springs *e*, *e*, for the instrument is not required to produce sparks. It is used in Thomson's electrometers to maintain the Leyden jar at a constant potential—one of the *inductors*, A, B, is connected to the inner coating of the jar, and the other to earth; and when the potential of the jar shows signs of falling a few turns

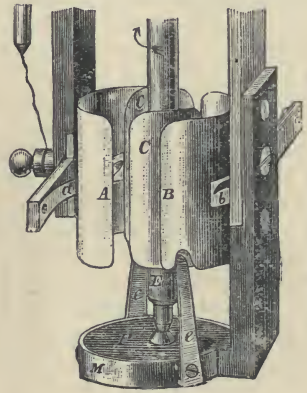


Fig. 42.

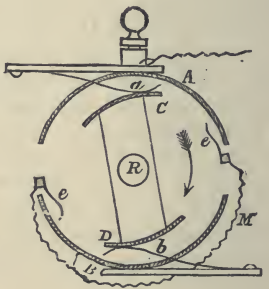


Fig. 43.

of the carrier quickly restores it to its original value, which is indicated by a suitably constructed gauge.

The principles involved in the action of the usual forms of induction machines are completely illustrated by the simple apparatus just described, but it is somewhat difficult at first sight to pick out, in the modern forms of the machine, the parts corresponding to the several parts of the arrangements described above. Attention to the following points will be of some service to the reader, in this respect. It is evident that the apparatus of Fig. 41, will work more efficiently if we have a number of carriers, instead of only one or two. Now, the most convenient way

of arranging a number of insulated carriers, for revolution round a central axis, is to fix a number of metal studs round a circle, near the edge of a circular glass plate which can be rotated on an axis, passing through its centre. Hence a varnished glass plate with a ring of metal studs fixed to the glass in a circle near its edge, is the characteristic feature of an induction or *influence* machine, and constitutes the carrier system of the machine. In some cases, however (for example in the first forms of the Holtz machine) a plain glass plate without any metal studs on it, forms the carrier system; and, on the other hand, in the

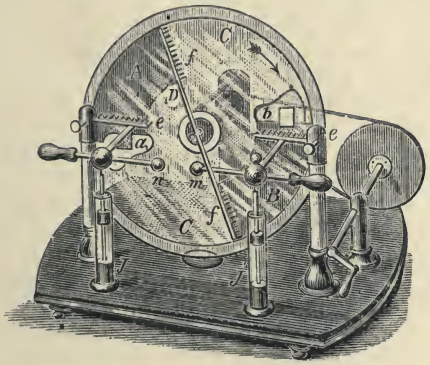


Fig. 44.

most recent form of the machine (the Wimshurst machine) the studs on the plates act the parts of the inductors A and B as well as that of carriers. The springs *a*, *b*, are represented by points, or fine wire brushes, while the springs *e*, *e*, are in all cases represented by combs of fine points, similar to those used in frictional machines.

***34. The Holtz and Voss Influence Machines.** The Holtz machine, shown in Fig. 44, was the first form of machine that came into general use, as a substitute for frictional machines. Its construction will be readily understood from the figure, which is lettered in accordance with the notation used above. The inductors A, B (usually called *armatures*),

are pieces of paper fastened to the back of the fixed plate of glass, shown at the back of the figure. The pointed tongues *a*, *b*, of these armatures, which protrude through openings in the plate, represent the springs *a*, *b*, of Fig. 41, and are the means of electrical communication between the armatures of the carriers. The smaller glass plate C, C, in front is the "carrier" plate of the machine, and its rotation is effected by means of the strap and pulley arrangements shown. In most forms of the Holtz machine, this carrier plate is a plain plate of varnished glass, having no metal studs fixed on it; the surface of the plate acts as a continuous carrier, much in the same way as if a ring of tin foil were pasted round it, near the outer edge. In front of this plate are arranged the combs, *e*, *e*, which communicate with one another across the spark gap *m n*. To work the machine a small charge is given to one of the armatures, the knobs *m*, *n*, are put in contact, and the carrier plate C, C, is rotated; in a short time it will be found that it is much more difficult to effect the rotation, and if the room is darkened, brushes of light will be seen diverging from the teeth of the combs. If the knobs *m*, *n*, be now separated, sparks will at once break across the gap. When the difference of potential between A and B is very great, then the action of the single pair of combs *e*, *e*, on the rapidly rotating plate may not be sufficient to admit of complete inductive action at each point of the plate, or on each carrier on the plate, as it passes the comb. To remedy this, a second pair of combs connected by a continuous insulated conductor D, is used, and the armatures extended so that this pair of combs may be placed, as at *f*, *f*, in such a position that it may complete the action of the pair *e*, *e*. A machine provided with this second pair of combs may evidently be started without *m* and *n* being in contact.

The small Leyden jars J, J, are connected to the combs in order to increase their capacity, and thus a larger charge can be accumulated on them, before spark discharge takes place across *m*, *n*, and when this spark does take place it is much louder and brighter than without the jars.

The Voss machine is a slight modification of the Holtz

machine. The carrier-plate is usually furnished with a ring of metal studs as carriers, but the characteristic of the machine is that it is self-starting; small brushes of fine wire are fixed on the combs, so as to touch the carrier studs as they pass, and the friction between them causes the latter to become very slightly charged, and thus the initial charge is obtained. The other details of the machine are similar to those of the Holtz.

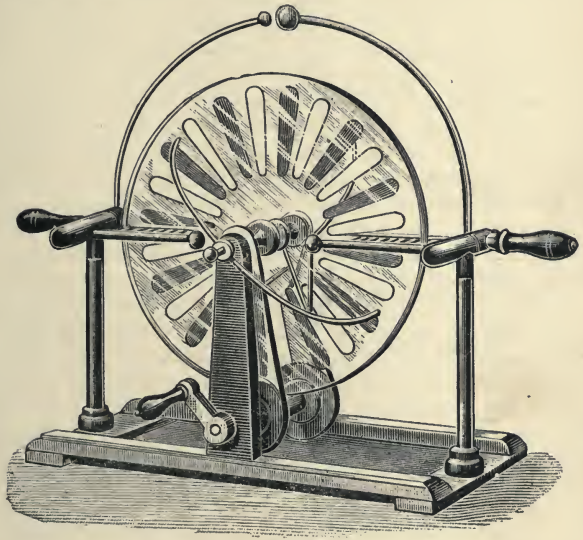


Fig. 45.

***35. The Wimshurst Machine.** This is the most recent form of influence machine. A simple form is shown in Fig. 45.

In construction it is very simple and is practically a simplified Voss machine. Instead of having one fixed and one movable plate, both plates of this machine are movable; but they are made to rotate in opposite directions. These plates, which are exactly similar, are plates of ordinary window glass, varnished and carrying a series of strips of

metal fixed in a ring round the plate, with their lengths arranged radially. These strips serve as carriers and as armatures, the carriers of one plate being in succession armatures for the carriers on the other plate.

The arrangement of brushes in this machine is however somewhat different from that adopted in the older forms. The combs are arranged as in the Voss machine, with small brushes attached to them to give the initial charge to the carriers, but instead of the tongues of the armatures there are the two *uninsulated* conductors shown in the figure, one for each plate, and each carrying a small brush of fine brass wire at each end. These conductors are arranged at right angles to each other, and at an angle of about 45° with the combs. By this arrangement each carrier is connected to earth four times in each revolution of the plate, and at each contact receives a charge by the inductive action of the charge on the carrier directly opposite it on the other plate. In this way the metal strips on the plates perform the functions of both carriers and armatures, and from the method of action it is evident that the plates of the machine should be made to revolve in opposite directions; for the charges on the carriers of one plate being due to the inductive action of those on the carriers of the other plate, the sign of the charge of one set of carriers must be opposite to that of the other set, and therefore one plate should be revolving towards one comb, say that at which positive electricity is collected, and the other plate towards the other comb. Similarly it will be seen that, on each plate, the sign of the charges of all the carriers *above* the combs must be opposite to those on the carriers below the combs. This last result, however, obtains in all forms of influence machine; in the simple machine of Fig. 41 the charge on the carrier reverses each half-revolution.

The machine shown above has only two plates, but compound machines are now made with four, six, eight and more plates. These machines can be made to give sparks several inches in length and are very convenient for general use; for, unlike the frictional machines, they can be made to work under almost any atmospheric conditions.

PART II.

MAGNETISM.

CHAPTER VII.

FUNDAMENTAL PHENOMENA.

36. Natural Magnets. The name *magnet* was applied at a very early date to pieces of a mineral found in Magnesia in Asia Minor. These specimens of what is now known as *magnetite*, or magnetic iron ore ($F_3 O_4$), were found to possess the property of attracting pieces of iron and steel, and at a later date it was found that when properly suspended by a fine thread they took up a definite position, always pointing approximately north and south.

Magnetite is found in abundance in Spain, Sweden, and various parts of North America, but not always in the magnetic condition. If a specimen exhibiting magnetic properties be examined, it will be noticed that its power of attracting pieces of iron or steel varies for different portions of its surface, being most marked at two regions loosely called the *poles* of the magnet. These poles are generally placed at two extreme points, and the region midway between them is found to be devoid of the power of attraction exercised by the other portions of the surface. Thus, if a magnet be rolled in iron filings and then lifted out, the filings will be found adhering thickly to the poles and the neighbouring

portions of the surface; but they gradually thin off as the distance from the poles increases, and midway between the poles no filings are found (Fig. 46).

Dr. Gilbert, who first carefully observed these properties of the magnet, called the region midway between the poles the equator of the magnet, and the imaginary line joining the poles the *magnetic axis*. This latter term is now frequently used, but the former is seldom seen.

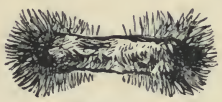


Fig. 46.

If a magnet is freely suspended, it is found that it sets itself so that the plane of its *magnetic axis* and the point of suspension lies approximately north and south. Hence, in order that this phenomenon of *polarity* may be most advantageously exhibited, the magnet should be suspended so that its magnetic axis may be horizontal, that is, the point of suspension should be near the "equator" of the magnet; if the point of suspension were near either pole, the magnetic axis might be vertical, or nearly so, and the magnet would give no indication of polarity.

If two magnets be suspended at a distance from each other, so as to set themselves north and south, it will be found that if the north-pointing end of one be brought near the north-pointing end of the other, mutual repulsion will be exhibited. Similarly, if the south-pointing pole of one be brought near the south-pointing pole of another, repulsion ensues; that is, *similar poles repel one another*. If, however, the north-pointing end of one be brought near the south-pointing end of the other, mutual attraction results; that is, *dissimilar poles attract one another*. It will be noticed that these laws of magnetic attraction and repulsion are perfectly general, and similar in nature to the corresponding laws of electrostatic attraction and repulsion.

Iron and steel have been mentioned above as being attracted by magnets, and they are the only substances on which magnetic attraction is at all appreciable. Many other substances, notably nickel and cobalt, are feebly attracted, while others, such as copper and bismuth, are

feebly repelled. In fact, Faraday has shown that all substances may be divided into two classes—the *paramagnetic*, or *magnetic*, which are attracted, and the *diamagnetic*, which are very feebly repelled. In this book we shall only have to deal with the strongly magnetic substances, iron and steel.

From what has been said it will be seen that a magnet possesses the following properties:—

(1) *Marked attraction for the magnetic substances iron and steel.* This is the most important case of the general law, that all substances are affected by magnetic force—*paramagnetic* substances being attracted and *diamagnetic* substances repelled. In all cases except iron and steel, and, in a less degree, nickel and cobalt, the forces of attraction and repulsion are very feeble.

(2) *Polarity.* That is, a magnet possesses poles, or regions where the power of attraction (or repulsion) is a maximum, and which, when the magnet is freely suspended, tend to point north and south. Further, these poles are not identical in properties—similar poles, that is, poles which point in the same direction, repel one another, whereas dissimilar poles attract one another.

This magnetic duality constitutes what is called *polarity.* In addition to possessing these properties, a magnet is able to communicate them to a piece of hard steel. For example, if a piece of watch-spring be rubbed with the pole of a magnet, it will be found to possess all the properties of a magnet, and will even be able to communicate them to another piece of steel.

Hence, if we wish to test whether a given piece of ore or steel is a magnet or not, we may proceed in the following way:—

(1) Dip the piece in iron filings. If it be a magnet the filings will adhere to it, and the points where they are most thickly clustered will be poles of the magnet. If it be not a magnet, the filings will not adhere to it.

(2) If the piece be found to be a magnet, suspend it by a single silk fibre so that its magnetic axis is horizontal. It will then set magnetically north and south, and the end

pointing towards the north will be the north-pointing pole. This pole is sometimes marked with a file mark so as to be readily recognised.

Instead of suspending the magnet as here described, it is often more convenient to use a piece of magnetised watch-spring as a test magnet. This magnet is readily suspended by a silk fibre, and by simply presenting either pole of the magnet under examination to, say, the north-pointing pole of the test magnet, the nature of the former can be at once determined; if it be also a north-pointing pole, the two poles will repel one another, but if it be a south-pointing pole, then mutual attraction will result. This use of a test magnet or needle to determine the poles of another magnet is very common, but the pole of the magnet under test must not be held too near the pole of the needle, for, even with similar poles, the attraction of the magnet for the steel may be greater than the repulsion between the poles, and attraction instead of repulsion will result.

Natural magnets are, more or less, curiosities. They are seldom or never seen in experimental work, artificial steel magnets (Art. 38) being used instead.

In explanation of the fact that a magnet, when freely suspended or pivoted, points north and south, Dr. Gilbert assumed that the earth itself must act as a large magnet, having its poles near its geographical north and south poles. The attraction of these poles for the dissimilar poles of the suspended magnet causes the latter to set in a north and south direction, the north-pointing end being dissimilar to the magnetic north pole of the earth by which it is attracted, and the south-pointing pole dissimilar to the magnetic south pole of the earth. Subsequent investigation has confirmed this explanation of Dr. Gilbert's, and terrestrial magnetism, or the magnetism of the earth, is now an important branch of the science of magnetism. The north-pointing pole of a magnet is usually for shortness called the *north pole*, and the south-pointing pole the *south pole*: thus the so-called north and south poles of a magnet are respectively *dissimilar* to the magnetic north and south poles of the earth.

37. Magnetic Induction. If the pole of a magnet be brought near a short piece of steel wire, the latter is at once attracted by the magnet, and may be lifted in the way shown at *a* in Fig. 47. Let another short piece of

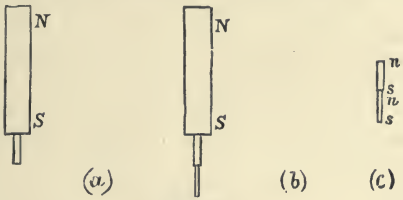


Fig. 47.

steel be now brought near the lower end of the piece clinging to the magnet; this second piece is also attracted, and, if the magnet be strong enough, it will remain hanging to the first piece as shown at *b*. If the first piece be now gently detached from the magnet, it will be found that the lower piece still clings to it, and, if the two pieces of steel be tested, each will be found to be a magnet with poles as shown at *c*. It will be noticed that these poles are arranged relative to one another and to the magnet NS, so that dissimilar poles are in contact, and therefore exert mutual attraction.

If this experiment be modified, as indicated in Fig. 48,

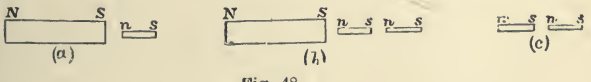


Fig. 48.

by arranging the magnet and the pieces of steel on a table so that they are not allowed to come into contact with one another, it will be found that the result is exactly the same as before, only that the magnetisation produced in the pieces of steel is somewhat more feeble than in the first case. Thus, in the cases illustrated by *a* and *b* of Fig. 48, if iron filings be sprinkled over the pieces of steel, they will adhere in tufts to the ends of the pieces, and behave in precisely the

same way as when sprinkled over a small magnet obtained by the arrangement of Fig. 47. On removing the magnet NS, each piece of steel will be found to possess all the properties of a magnet.

If the same experiments be performed with pieces of soft iron instead of steel, a very important difference is noticed in the results. In the cases illustrated by *a* and *b* of Figs. 47, 48, the *same* effects are produced: the pieces of iron become magnetised, but immediately the magnet NS is removed, they lose their magnetic properties almost entirely.

Thus a piece of steel, when placed in contact with or near a magnet, becomes *permanently* magnetised, but a piece of iron under the same conditions becomes only *temporarily* magnetised. On removing the magnet the iron loses its magnetism almost entirely; the slight amount remaining is called *residual magnetism*, and varies with the quality of the iron, from a very small percentage for the best soft iron to quite an appreciable quantity for harder qualities.

These effects are due to what is called *magnetic induction*. A magnet acts inductively on magnetic bodies (steel or iron) placed near it, and the attraction exercised by magnets on magnetic bodies is a direct result of this inductive action. Thus, in Figs. 47, 48 (*a*), the south-pointing pole, S, of the magnet NS, acts inductively on the piece of steel or iron, and induces a *dissimilar* pole, *n*, in the *nearer end* of the piece, and a *similar* pole, *s*, at the *farther end*. As a result of the attraction between S and *n*, the steel or iron is attracted to the magnet, and the force of attraction may be sufficient to overcome the weight of the piece of metal. In steel the effect of induction is permanent and the induced poles persist, but in iron the effect is more or less transient, and the induced poles persist only while the metal is actually subjected to induction.

Thus we see that, when a magnet attracts a magnetic substance, induction precedes attraction, which is really the attraction between two dissimilar poles—the pole which is presented to the substance, and the unlike pole which it induces at the nearest point of that substance. This fact

may be emphasised by the following experiment. Take two magnets similar in size and strength, and place them with their dissimilar poles together as shown in Fig. 49. If either end of the combined magnets be presented to a piece of iron or steel, little or no attraction is exhibited.

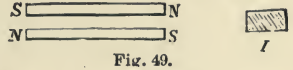


Fig. 49.

The inductive effect of one pole, N, is *neutralised* by the opposite inductive effect of the pole S, and the piece of magnetic substance, I, experiences no attraction. This result may also be explained by saying that the pole N of one magnet is *neutralised* by the equal and opposite pole S of the other, and thus the combined magnets have no external inductive effect at all.

Magnetic induction, like electrostatic induction, is due to the action of the medium, but the property of the medium on which it depends is evidently different in the two cases. Thus in electrostatics we have to do with that property of the medium which determines the action of two electrically charged bodies on one another, but in magnetism we have to do with that property which determines the action of two magnetic poles on one another.

It is possible to obtain some idea of the mechanism of magnetisation by induction by considering the following experiment.

Let a thin glass tube a few inches in length be loosely filled with steel filings and closed at both ends. If the tube be now laid on a table, and the pole of a magnet drawn over it from end to end, it will be noticed that the superficial layer of filings immediately under the pole experiences magnetic induction, and under its influence the filings arrange themselves regularly with their longer axes parallel to the length of the tube. Thus, before being subjected to the action of the magnet, the filings are arranged irregularly with their axes in any direction, as shown diagrammatically in Fig. 50 (*a*).

Under the influence of the magnet, however, each individual filing becomes magnetised, and as a result of the mutual attraction and repulsion between their poles and

the pole of the magnet, they arrange themselves as shown in Fig. 50 (*b*).

The depth of the layer of filings affected will vary with the strength of the magnet, but even with a very strong magnet the disturbed layer is comparatively shallow.

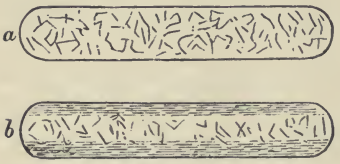


Fig. 50.

If the magnet be drawn along the tube several times, always in the same direction, and so as to bring every portion of the surface of the enclosed filings under the influence of the pole, it will be found on testing that the tube of filings possesses all the properties of a feeble magnet. If, however, the tube be shaken so as to disarrange the particles, it loses its magnetic properties even although the individual steel filings retain their magnetism.

If the molecules in a bar of steel be supposed to act in the same way as the individual filings in the tube, we have a partial explanation of the fact, stated above, that a piece of steel may be magnetised by the inductive action of a magnet, and we also see why a bar of steel may be magnetised by rubbing it with a magnet. From this it appears that magnetisation is accompanied by molecular arrangement, and that the magnetic properties of the body magnetised are due to this arrangement; but what constitutes the magnetisation of individual molecules is a question on which we cannot here enter. From what has been said it is clear that the difference between steel and iron as magnetic bodies lies in the fact that in steel the molecular arrangement accompanying magnetisation may be permanent, but in iron it persists only while constrained by the inducing magnet, and when the latter is removed most of the molecules take up their normal position.

Another point of difference between iron and steel is that, while soft iron is easily magnetised by magnetic induction, steel, and especially hard steel, is influenced with difficulty

For example, if a light magnet be suspended by a fibre, and a piece of hard steel be held near one pole, the attraction resulting from the inductive action is not very strong; but if a piece of soft iron be substituted for the steel, the attraction is much more marked. This effect is very conveniently shown by arranging a piece of soft iron and a piece of hard steel on each side of the pole of the magnet as shown in Fig. 51. If the iron and steel be placed at equal distances from the pole, the attraction for the former will be the greater, and the magnet will be deflected towards it. By removing the iron to a greater distance from the pole, the two attractions may be balanced against one another, and the needle will remain in its original position. This shows that the inductive attraction for the steel is less than for the iron, that is, soft iron is more susceptible to magnetic induction than hard steel. This difference is, however, practically the same as the one already mentioned, and both are included in the statement that in soft iron the molecular arrangement constituting magnetisation is easily set up, and as easily lost when the constraining force is removed.

Fig. 51.

38. Artificial Magnets. In the preceding articles we have noticed that when steel is subjected to the inductive action of a magnet it becomes permanently magnetised.

Advantage may be taken of this fact to convert a bar of hard steel into a magnet. Thus, if a strip of steel be rubbed from end to end, always in the same direction, with the pole of a magnet, it acquires magnetic properties identical with those of the magnet. For example, if AB (Fig. 52) represent the steel strip, and if it be rubbed always in the direction AB with the north pole of a magnet, it will also become a magnet having a north pole at the end A and a south pole at the end B. This method of magnetisation, where the piece of steel to be magnetised is rubbed from

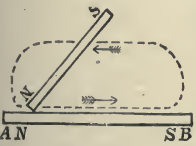


Fig. 52.

end to end with one pole of a magnet, is called magnetisation by *single touch*. The magnetising magnet must be drawn over the steel always in the same direction, the rubbing end following the path indicated by the dotted line and arrows of Fig. 52. If the piece of steel be thick, both sides should be rubbed, first one side, then the other. A similar but more effectual method of magnetisation is by the method of *divided touch* indicated in Fig. 53. The steel bar to be

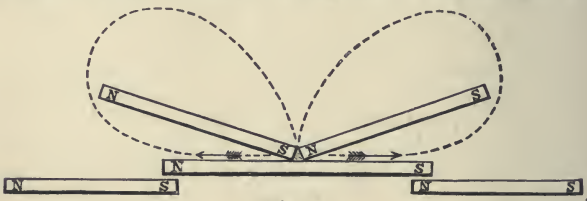


Fig. 53.

magnetised is laid with its extremities on the dissimilar poles of two magnets arranged as shown in the figure. It is then rubbed by the dissimilar poles of two other magnets, from the middle outwards, the rubbing ends of the magnets following the paths indicated by the dotted lines and arrows. A modification of this method is known as magnetisation by *double touch*. A piece of wood or cork is placed between the rubbing ends of the magnets, and these ends are then moved together from the middle to either end, back to the other end, and so on backwards and forwards, finishing at the middle.

It will be noticed from the figures that in all the methods each pole of the magnet produced is unlike that of the magnetising magnet with which it was last touched. This result is evidently due to the effect of magnetic induction on the molecules of the steel; for example, in Fig. 52, as the magnet is drawn over the bar, a result similar to that illustrated by Fig. 50 in the case of a tube full of iron filings is produced; the molecules arrange themselves in regular order with their magnetic axes parallel to the length of the bar, and with their south poles pointing towards the north pole of the magnetising magnet which has just passed

over them. The final result of this is, that most of the molecules in the surface layer of the steel strip are arranged with their south poles pointing towards B, and their north poles pointing towards A. This may be represented diagrammatically by Fig. 54.

It will be noticed that the molecular magnets are arranged with dissimilar poles in contact, and, if these poles are equal, there will be mutual neutralisation, except at the ends A and B, where the free north poles of the end molecules constitute a north pole, N, at A, and the free south poles at B constitute the south pole S. If the molecular magnets are exactly equal, so that there is exact neutralisation at all points in the length of AB, then the magnet will exhibit magnetic properties only at the ends; for example, if dipped into iron filings these will adhere only at the ends.

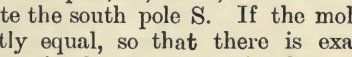


Fig. 54.

This, however, is never the case with an actual magnet—the filings adhere most thickly at, or near, the ends, but they also adhere to the sides at all points except near the middle of the magnet, the arrangement of the filings indicating that the attraction diminishes from the poles to the equator. This may be explained by saying that the molecular magnets are about equal at the middle of the magnet, but that they diminish in strength as we approach the ends. Thus in AB the effect of this would be that, in the right-hand half, the south poles of the molecular magnets would be stronger than the adjacent north poles. Similarly, in the left-hand half, the north poles would be stronger than the adjacent south poles. Hence, if the magnet be tested by a test needle, the right-hand half will exhibit south polarity through the greater part of its length, the maximum effect being at the end B, and the left-hand half will similarly exhibit north polarity.

It is thus evident that the amount of *free* or unneutralised magnetism at any section of the magnet varies with the position of the section considered, being greater the nearer the section is to either end of the magnet. In Fig. 55 let

NS represent a magnet, and at a number of points taken at equal distances along its length let perpendiculars be erected, each proportional to the amount of free magnetism at the section corresponding to its position on the magnet. If the perpendiculars are drawn to the left of the magnet for north magnetism, and to the right of it for south magnetism, then the curve *nos* drawn through their extremities may be said to indicate the *distribution* of free magnetism on the magnet. Further the *areas Non* and *Sos* indicate the *amounts* of free magnetism distributed on the corresponding halves of the magnet.

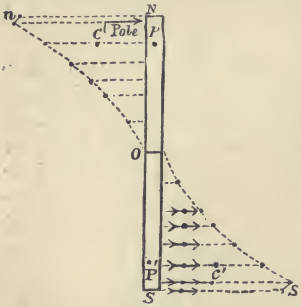


Fig. 55.

The *poles* of a magnet are sometimes defined as the points at which the free north and south magnetism of the magnet may be supposed to be concentrated, so as to produce the same external magnetic field, but such points cannot, in general, be found. In Fig. 55, if C and C′ denote the centroids of the figures *Non* and *Sos*, then the poles of the magnet may for some purpose be assumed to be at P and P′ the feet of the perpendiculars from C to the axis of magnet. In the case of thin bar magnets, these points are very near the ends of the magnet, so that for general purposes the poles of a long bar magnet are assumed to be at the ends of the magnet, though in reality they are so placed that the actual length of the magnet is slightly greater than the distance between its poles.

The molecular theory of magnetisation described above is further supported by the fact that if a steel magnet be broken in two, each piece is a complete magnet. For example, if the magnet AB (Fig. 56) be broken into the two pieces AC, C′B, then

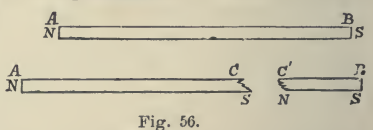


Fig. 56.

each piece is a magnet with poles as indicated. This is evidently just what we should expect. At the end C of the piece AC all the south poles of the end molecules are freed from the neutralisation of the north poles of the end molecules at C′ of the piece BC′. Hence C becomes a free south pole, and C′ a free north pole. Similarly, if a magnet be broken into any number of pieces, each piece is found to be a complete magnet.

When a magnet is irregularly magnetised, it may exhibit what are known as *consequent poles*, that is to say, poles lying between the terminal ones of the magnet. Fig. 57 shows a magnet with consequent poles at *n* and *s*. The magnet practically consists of three smaller magnets joined end to end, so that similar poles are united at *n* and *s*.

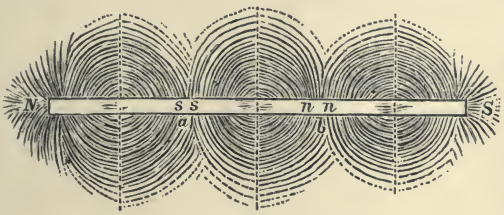


Fig. 57.

Magnets with consequent poles may be produced either by accidental irregularity in magnetisation, or purposely. For example, by touching a bar of steel at several points with the same pole of a strong magnet, consequent poles of polarity opposite to that of the magnet pole are produced at the points touched.

To show that the magnetisation may only affect the surface layer of molecules, let a piece of steel be magnetised by lightly stroking with another magnet, and then placed for a short time in strong nitric acid. The acid eats away the outer layers of molecules, and the steel will be found to have to some extent lost its magnetic properties.

Artificial magnets are made in various forms, the bar magnet and horse-shoe magnet being the commonest. Bar magnets are usually in pairs ; Fig. 58 shows a pair arranged in a box with their dissimilar poles adjacent and united by

loose pieces of soft iron called *keepers*. These keepers serve to maintain the magnetic state of the magnets; their function will be more fully explained in Art. 40.

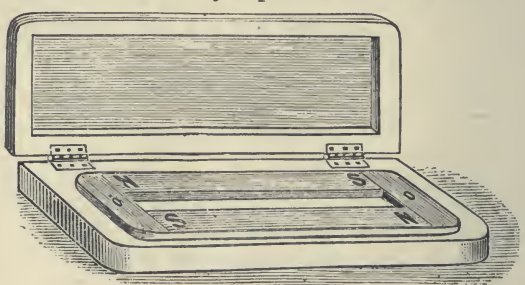


Fig. 58.

The *horse-shoe* magnet has the form shown in Fig. 59. It is practically a bar magnet, bent so as to bring the two poles near to one another; thus constructed, it is more convenient for attracting pieces of iron, for the attraction of both poles can be utilised simultaneously. When not in use, a keeper of soft iron is placed across its poles.

Fig. 59.

Since the surface layers of the steel are generally the more highly magnetised, there is evidently very little advantage in using steel bars of great thickness; and, with a given mass of steel, more powerful magnets can be made by making several thin bar magnets of the required length, and uniting them as a *compound magnet*, than in magnetising the steel as one thick bar of the same length. Fig. 60 shows the simple forms of a compound bar magnet, and a compound horse-shoe magnet; all the north poles of the component magnets are at N, and constitute the north pole of the compound magnet. Similarly, all the south poles of the component magnets at S constitute the south pole of the compound magnet.

A useful piece of apparatus, called the magnetic needle,

consists of a strip of carefully magnetised hard steel pivoted at its centre on a hard steel point. The strip is usually of the shape shown in Fig. 61, and at its centre is inserted a conical cap of agate or glass, which serves to pivot it on the steel point, and allows it to turn freely with little or no friction. When properly balanced and pivoted, the needle sets itself north and south, that is, it sets in what is called the *magnetic meridian* of the place where it is suspended (Art. 44). The *compasses* sold by opticians are simple forms of this instrument, the needle being balanced over a card indicating the points of the compass. The *mariner's compass* is a more elaborate instrument, in which the compass card is attached to the needle, so that the north point on the card always indicates the magnetic north.

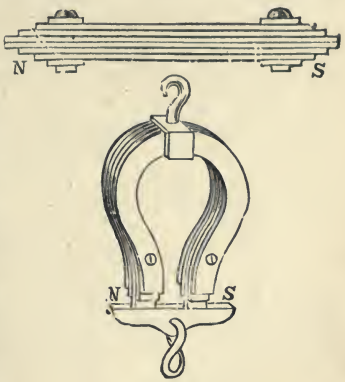


Fig. 60.

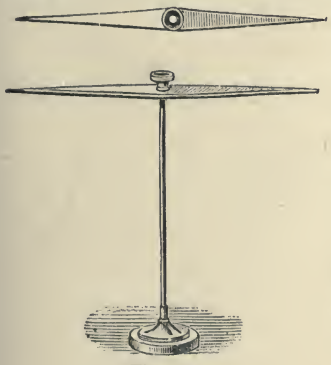


Fig. 61.

The properties of a magnet or of a magnetic body are greatly influenced by heat. Thus, if a strip of steel be magnetised and slowly raised to a red heat, it gradually loses its magnetic properties, and at a bright red heat it entirely loses them, and does not recover them on cooling. Similarly, if a piece of iron be heated to a bright red heat, it is not attracted

by a magnet. These facts support the molecular theory of magnetisation, and are evidently due to the molecular changes produced by the influence of heat.

39. Laws of Magnetic Attraction and Repulsion. The general law that like poles repel, and unlike poles attract, has already been given, and we have now to consider the quantitative expression of this law. This is exactly similar to the electrostatic law given in Art. 6. Thus, if two magnetic north poles of strengths m and m' are separated in air by a distance, d, the force of repulsion exerted between them is given by—

$$f = \frac{m\,m'}{d^2}.$$

This involves the definition of unit pole, which may be given thus: A magnetic pole is of unit strength when it repels another similar pole placed at unit distance from it in air with unit force. If distance be measured in centimetres and force in dynes, then the strength of the pole is given in C. G. S. magnetic units. A north pole is usually considered positive, and a south pole negative; hence, when $m\,m' / d^2$ is positive, the force is one of repulsion, and when negative it is one of attraction.

This law is more generally expressed by writing—

$$f \propto \frac{m\,m'}{d^2}, \text{ or } f = \frac{1}{\mu} \cdot \frac{m\,m'}{d^2},$$

where μ is a constant of a definite value for any given medium, but differing for different media. This constant may be called the *magnetic inductivity* of the medium. The magnitude of this constant for any medium depends upon the facility with which *magnetic* force acts through that medium, just as the value of the constant κ referred to in Art. 6 depends upon the facility with which *electric* force acts through the medium. If μ is taken as unity for air the value of f in any other medium is given by $f = m\,m'/\mathrm{M}\,d^2$ where M corresponds to K of Art. 6, and is evidently the ratio of the magnetic inductivity of the air to that of the given medium. This constant is known as the *magnetic permeability* of the medium.

40. The Magnetic Field. The term *magnetic field* in magnetism is analogous to *electric field* in electrostatics The magnetic field due to any magnet is the whole space in which magnetic force due to the magnet is appreciable. Similarly, the magnetic field due to any system of magnets is the whole space in which magnetic force due to the system is appreciable.

The magnetic force at any point in a magnetic field, or the *intensity of the field* at any point, is determined in magnitude and direction by the resultant force which a unit *north* pole would experience if placed at that point. Thus, in Fig. 62, if NS represent a magnet with poles of strength m and $-m$, the intensity of the field at a point P may be determined thus. The force at P due to the pole N is given by $\frac{m \times 1}{NP^2}$ and acts in the direction NP; let it be represented by PR. Similarly, the force at P due to the pole S is given by $\frac{-m \times 1}{SP^2}$ and acts in the direction PS; let it be represented by PQ. Then, by the parallelogram of forces, the *resultant* force at P due to the magnet NS is represented by PT. The magnitude and direction of this resultant force determines the intensity of the magnetic field due to the magnet NS at the point P.

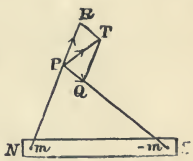


Fig. 62.

By conventions similar to those explained in Art. 7 a magnetic field may be mapped out by lines of magnetic force, a line of magnetic force being, in general, a curve such that the direction of the tangent at any point gives the direction of the resultant magnetic force at that point. The magnitude of the force at any point may also be measured by the number of lines of force per unit area round the given point in a plane at right angles to the direction of the line of force passing through that point.

In Fig. 62, PT is the direction in which a unit *north* pole would be urged; hence a unit *south* pole would be urged with an equal force in the opposite direction. Similarly, a

north pole of strength m would be urged in the *direction* PT with a force of magnitude equal to m *times* that represented by PT, and a *south* pole of strength $-m$ would be urged with equal force in an opposite direction.

Hence, if a very short magnet be placed at P, its north pole will be urged along PT, and its south pole will experience a nearly equal force in the opposite direction. That is, the force of translation, tending to move the magnet from the position P, will be very small, but the magnet will set itself in the line PT, that is, parallel to the direction of the resultant magnetic force at the point where it is placed.

Hence, if a small magnet be placed at different points in

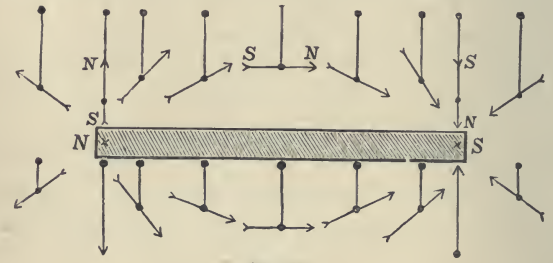


Fig. 63.

the neighbourhood of a large magnet, it will set itself along the resultant direction of the magnetic force at each point, that is, its length will be a tangent to the line of force passing through the point where it is placed. This experiment may readily be performed by magnetising a very short strip of steel watch-spring, and suspending it at different points in the neighbourhood of a large magnet. Fig. 63 shows roughly the positions for different points in the plane of the paper. This experiment may, however, be modified so as to give much more satisfactory results. Let a sheet of thin glass or thin pasteboard be placed over a magnet, and let iron filings be dusted over it It will be noticed that as the filings fall on the card, they at once come under the inductive action of the magnet, and each, becoming a magnet, sets itself parallel to the direction of

the resultant force at the point where it is placed. On gently tapping the card, the filings thus arrange themselves in continuous curves, and roughly map out the lines of force in the plane of the card.

Fig. 64 shows the general arrangement of the filings over

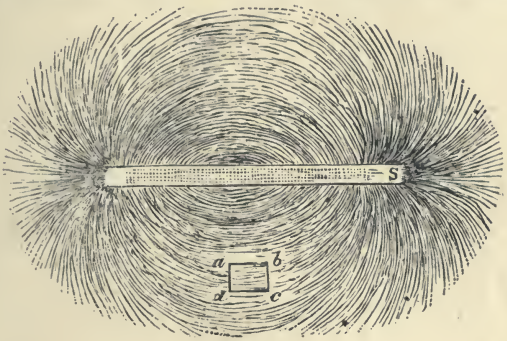


Fig. 64.

an ordinary bar magnet, and the student may readily verify the fact that the direction of the magnetic force at any point is tangential to the curve passing through that point. Since the intensity of the magnetic field is defined with

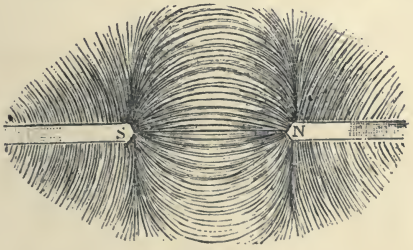


Fig. 65.

reference to a unit *north* pole, the lines of force are supposed to run from N to S, that is, in the direction in which a unit north pole would travel.

Fig. 65 shows the general direction of the lines of force

between two dissimilar poles of two bar magnets, and Fig. 66, for a horse-shoe magnet, illustrates the same thing.

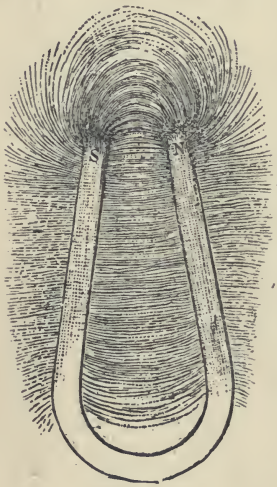


Fig. 66.

The distribution of the lines of force between two similar poles is shown by Fig. 67, and the student will find it instructive to draw the force diagram (cp. Fig. 62) for different points in this field, so as to verify the general directions of the lines.

It must be remembered that the above figures only show the lines of force *in the plane of the card*, that is, they represent longitudinal *sections* of the field. All longitudinal sections are, however, exactly similar, so that the figures completely represent the field if we remember that the lines of force surround the magnet on all sides, and that the above figures represent their disposition in *any* longitudinal section. It must further be remembered that lines of force have no

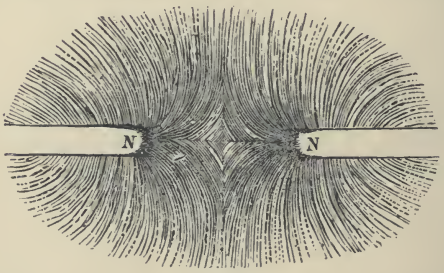


Fig. 67.

real existence—they are merely graphical representations of the direction of the magnetic force at any point in a

field of force, and the experiments with the filings are merely magnetic devices for indicating to the eye the general direction of these lines, just as a chain hanging vertically indicates the direction of the force of gravitation.

Magnetic induction may be said to act indifferently through all substances except magnetic substances; that is, for most substances, the value of μ (Art. 39) is much the same, but for magnetic substances it has a higher value. Thus, if a piece of soft iron be placed between two unlike magnetic poles, nearly all the lines of force pass through the iron. This is readily shown by the help of iron filings. If a short piece of soft iron, of about the same breadth and thickness as the magnets, is placed between the unlike poles of two bar magnets as shown in Fig. 68, the distribution of the lines of force, obtained in the usual way by means of filings, shows that the magnetic induction takes place chiefly through the piece of iron, the lines of force being as it were gathered up by it, so that the magnetic field is to some extent limited by the dimensions of the iron.* If a piece of wood, brass, glass, etc., be placed between the poles, the normal distribution of Fig. 65 is not sensibly disturbed. If a soft iron *ring* be placed between the poles, then the distribution shown in Fig. 69 is obtained, and it will be noticed that the space within the ring is, as it were, screened from the magnetic

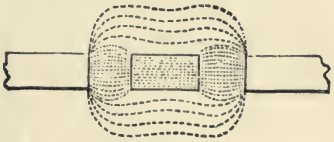


Fig. 68.

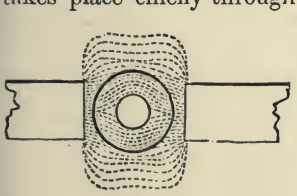


Fig. 69.

* The filings in the space over the piece of iron will not arrange themselves in lines of force, in fact, on tapping the card the space will probably be free from filings. This is because nearly all the lines of force are in the *iron*, and therefore there is no field where the filings lie. These results are more correctly explained by saying that the magnetic field which obtains is the *resultant* field due to the two inducing poles *and the two induced poles* of the piece of iron,

induction of the poles. Most of the lines of force pass through the iron of the ring, and in the space within there is no magnetic field, so that if a small magnet were placed there it would set itself indifferently in any position. Hence, if a magnet is suspended inside an iron tube or an iron enclosure of any kind, it is effectively screened from the influence of any magnetic field in which it is placed.

This property of soft iron explains why *keepers* of soft iron are placed on magnets to assist in maintaining their magnetic condition. The end molecules of the magnets have their interior extremities fixed by the attraction of adjacent molecules, but their free extremities experience no fixed attraction, and for this reason their equilibrium is not very stable and is easily disturbed. Hence, if a magnet be handled roughly or knocked about in any way, it gradually loses its magnetic properties. When, however, a keeper is placed on a pair of magnets as in Fig. 58, or across the poles of a horse-shoe magnet, the inductive action of each of the poles is concentrated on the other, and the force of attraction between the poles is greatly increased. The free ends of the molecules are fixed by this mutual attraction, and the molecular arrangement of the magnet is not now so easily disturbed. For the same reason, a sheet of iron placed between a magnet and a magnetic needle shields the latter from the inductive action of the magnet—the lines of force take the course indicated diagrammatically in Fig. 70, and do not pass across the iron.

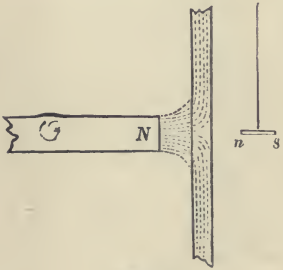


Fig. 70.

Steel acts in much the same way as iron, but not so effectively; the lines of force pass much more readily through soft iron than through steel, and the softer the iron the greater is its *permeability*.

The effect produced on the external field of a horse-shoe

magnet by placing a keeper across its poles is readily seen by testing the distribution of the lines of force when the keeper is first off and then on. Fig. 66 shows the first case, and Fig. 71, giving the second case, shows that when the keeper is on there is, as we should expect, little external magnetic field. The same thing may be illustrated by holding the poles of a horse-shoe magnet near a suspended test needle. The needle is at first deflected from its normal position, but on putting the keeper across the poles it at once returns to that position.

Fig. 71.

From a consideration of Figs. 64-67 it will be seen that in general the intensity of a magnetic field varies, both in magnitude and direction, at different points in the field; that is, the directions of the lines of force and the number crossing unit area are different at different points. If, however, we consider a portion of the field whose dimensions are small compared with its distance from the poles of the magnetic system to which the field is due, the intensity of the field may be considered to be uniform throughout this small area. For example, in the magnetic field due to the magnet NS (Fig. 64), the field in the space *a b c d* may be considered *uniform* in *direction* and *magnitude*, that is, the lines of force are practially *parallel equidistant straight lines*. For this reason the magnetic field due to the earth at any point on the earth's surface is said to be uniform; the intensity of the earth's magnetic field is of course different at different places, but at any one place it is practically uniform, and may be represented by parallel straight lines equidistant from one another.

41. Action of a Magnet in a Uniform Magnetic Field. We have already learnt in Art. 36 that, when a magnet is placed in a magnetic field, it tends to set itself parallel to the direction of the lines of force at the point where

it is placed. Hence, when a magnet is freely suspended in a uniform magnetic field, it at once sets itself with its magnetic axis parallel to the lines of force. Thus, let NS (Fig. 72) represent a magnet freely suspended in a uniform magnetic field, and let the strength of its poles be denoted by m. Then if I denote the intensity of the field, the north pole N will experience a force mI in the direction indicated in the figure, and the south pole S will experience an equal force in the opposite direction.

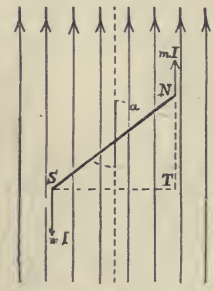


Fig. 72.*

Hence for any position in which the magnet is not parallel to the lines of force, it experiences a couple urging it into that position. The *moment* of this couple is equal to $m\text{I} \times \text{ST}$, and its magnitude evidently depends upon the position of the magnet—it has a minimum value equal to zero when the magnet is parallel to the lines of force, and a maximum value equal to $m\text{I} \times \text{SN}$ when the magnet is at right angles to the lines of force. For any intermediate position, where the axis of the magnet makes an angle α with the direction of the lines of force, the moment experienced by the magnet is $m\text{I} \times \text{ST}$. But $\frac{\text{ST}}{\text{SN}} = \sin \alpha$, or $\text{ST} = \text{SN} \sin \alpha$, and if SN, the length of the magnet, be denoted by l, we have $\text{ST} = l \sin \alpha$. The moment of the couple tending to place the magnet parallel to the lines of force is thus given by $ml\text{I} \sin \alpha$. In this expression m and l are two constants of the magnet, and their product ml is known as the *magnetic moment* of the magnet.

This important constant is usually denoted by M, and may be defined briefly as the product of the strength of either pole of the magnet into the distance between the poles, or more satisfactorily as the maximum moment which the magnet can experience in a field of unit intensity

* The direction of the lines of force indicated by the arrow-heads is the direction in which a *north pole* would be urged. Art. 40.

This latter definition is applicable whatever be the form of the magnet or the distribution of its magnetism.

It is evident from what has been said, that, if a freely suspended magnet be deflected from its position of equilibrium in a magnetic field, it at once experiences a couple urging it back into this position, and as a result it oscillates backwards and forwards, just as a pendulum oscillates when disturbed from its position of rest. This oscillation of the magnet is isochronous, that is, the time of each oscillation is the same, and it can be shown that, for a given magnet, the time of a single oscillation depends upon the magnetic moment of the magnet, and the intensity of the field in which it oscillates. The greater the moment and the stronger the field, the shorter the time of oscillation, and the mathematical investigation shows that, for a magnet of given size, form, and mass,* the time varies inversely as the square root of the magnetic moment and of the intensity of the field; that is, if t denote the time of one oscillation, M the magnetic moment, and I the intensity of the field—

$$t \propto \frac{1}{\sqrt{\mathrm{M}}}, \text{ and } t \propto \frac{1}{\sqrt{\mathrm{I}}},$$

or combining the two relations—

$$t \propto \frac{1}{\sqrt{\mathrm{M\,I}}}, \text{ or } t = \frac{k}{\sqrt{\mathrm{M\,I}}},$$

where k is a constant. Now, if an oscillating magnet makes n vibrations per second, then the time of one oscillation (t) is given by $t = \frac{1}{n}$ second, that is, $n = \frac{1}{t}$. But we have just found $t = \frac{k}{\sqrt{\mathrm{M\,I}}}$

$$\therefore n = \frac{1}{t} = \frac{\sqrt{\mathrm{M\,I}}}{k}, \text{ and } n^2 = \frac{\mathrm{M\,I}}{k^2} = \frac{1}{k^2}(\mathrm{M\,I}),$$

that is, the *square* of the number of oscillations, which

* The time of vibration also varies directly as the square root of the *moment of inertia* of the needle, and the value of this constant depends upon the size, form, mass, and suspension of the needle. If K denote the moment of inertia of the needle its time of oscillation (forwards and backwards) is given by—

$$t = 2\pi \sqrt{\frac{\mathrm{K}}{\mathrm{M\,I}}}.$$

the magnet performs in a given time in any magnetic field, is directly proportional to the strength of the field, and to the magnetic moment of the magnet. Hence, if the *same* magnet be caused to oscillate in two different magnetic fields, it is evident that the intensities of the fields will be directly proportional to the square of the number of oscillations performed by the magnet in each field, in a given time. For example, if in one field the magnet make 10 oscillations per minute, and in the other 12 oscillations per minute, then the intensity of the first field is to that of the second as $10^2 : 12^2$, that is, as 100 : 144 or as 1 : 1·44.

42. Action of one Magnet on Another. In Fig. 73 let NS and *ns* represent two magnets in the same horizontal plane. NS is supposed to be fixed, and *ns* to be pivoted at O so as to move freely in a horizontal plane. Let m and μ denote the respective strengths of the poles of the magnets, then, considering the action of the poles of NS on the pole *s* of *ns*, we have that the pole N attracts it with a force represented by *sa* and equal to $\frac{m\mu}{(\mathrm{N}s)^2}$, and the pole S repels it with a force represented by *sb* and equal to $\frac{m\mu}{(\mathrm{S}s)^2}$. The resultant action of the magnet NS on the pole *s* is therefore represented by *sc*; and by an exactly similar construction, the resultant action on the pole *n* may be represented by *nf*. Under the action of these forces, the magnet *ns* will take up a position of equilibrium in which the moments of the forces represented by *sc* and *nf* round O are equal, that is, $sc \times Op = nf \times Oq$.

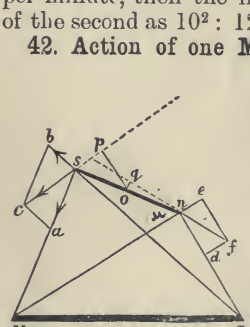


Fig. 73.

If the magnet *ns* is not pivoted at O, but perfectly free to move in the plane of the paper, then it is evident that the action of the forces *sc* and *nf* must be to draw it up to NS, that is, the magnet experiences a force causing motion of translation, and not a mere *directive force* setting it in a

particular position. When pivoted at O, the resistance of the pivot prevents the translatory motion, but allows the magnet to set in the position of equilibrium indicated above. When a magnet with poles of equal strength is placed in a uniform field, it can experience no force causing motion of translation, but only a *directive couple* setting it parallel to the lines of force, for, the field being uniform, the forces acting on the poles are equal and opposite and parallel to each other. Thus, if a magnetised sewing needle be carefully placed on the surface of water, it readily sets itself on the magnetic meridian, but does not move over the surface of the water.

In the case considered above, the magnets are supposed

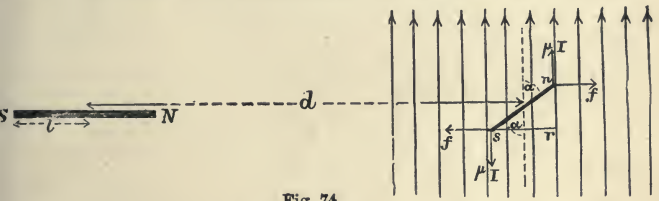


Fig. 74.

to be uninfluenced by any external magnetic field, but in practice we have always to deal with the field of the earth, and it may here be useful to consider one or two particular cases of practical importance. Let NS (Fig. 74) represent a magnet, and *ns* a small magnetic needle freely suspended in a uniform field of intensity I. Let the distance between the *centres* of the magnets be denoted by *d*, and the length of the magnet NS by 2*l*. Then, assuming the needle *ns* to be so small that both its poles may be considered to be at the distance *d* from the centre of NS, the resultant force *f*, due to the action of the poles N and S on *n*, is given by—

$$f = \frac{m\mu}{(d-l)^2} - \frac{m\mu}{(d+l)^2} = \frac{4m\mu d l}{(d^2-l^2)^2}$$*

* The force due to N on *n* is $\frac{m\mu}{(d-l)^2}$ and that due to S is $\frac{m\mu}{(d+l)^2}$. If the needle *ns* is small, these forces practically act in the same straight line, and, since they are opposed to one another, their resultant is equal to their difference and acts in the direction of the greater, that is, the resultant force is one of repulsion.

where m and μ are the strengths of the poles of magnets NS and ns respectively. Now if l is small compared with d, the denominator $(d^2—l^2)^2$ is practically equal to $(d^2)^2$, and therefore—

$$f = \frac{4\,m\,\mu\,d\,l}{d^4} = \frac{4\,m\,\mu\,l}{d^4}.$$

Similarly, a force equal and opposite to f acts on the pole s, and the needle ns is thus subject to the couple ff tending to set it parallel to the length of NS, and the moment of this couple is $f \times nr$. But on being deflected from its position of rest parallel to the lines of force of the field in which it is placed, the needle at once experiences a couple of moment $\mu\,\mathrm{I} \times sr$ (Art. 41) tending to restore it to its initial position. Hence the needle is now under the influence of two opposed couples, and will be in equilibrium in the position for which the moments of these couples are equal and opposite. Hence for equilibrium we have—

$$f \times nr = \mu\,\mathrm{I} \times sr,$$

that is,

$$\frac{4\,m\,\mu\,l}{d^3} \times nr = \mu\,\mathrm{I} \times sr.$$

$$\therefore\ 4\,m\,l = d^3\,\mathrm{I} \cdot \frac{sr}{nr},$$

$$2 \cdot 2\,m\,l = d^3\,\mathrm{I}\,\frac{sr}{nr}.$$

But $2\,m\,l\,(= m \cdot 2\,l)$ is the magnetic moment of NS and $\frac{sr}{nr} = \tan\,\alpha$, where α denotes the angle through which the needle ns is deflected from its position of rest. Hence, if M denote the magnetic moment of NS, we have—

$$2\,\mathrm{M} = d^3\,\mathrm{I}\,\tan\,\alpha,$$

or

$$\frac{\mathrm{M}}{\mathrm{I}} = \frac{d^3}{2}\,\tan\,\alpha.$$

In an exactly similar way it can be shown that if NS

and *ns* occupy, relative to one another, the positions indicated in Fig. 75, then—

$$\frac{M}{I} = d^3 \tan \alpha.$$

The proof of this is left as an exercise for the student.

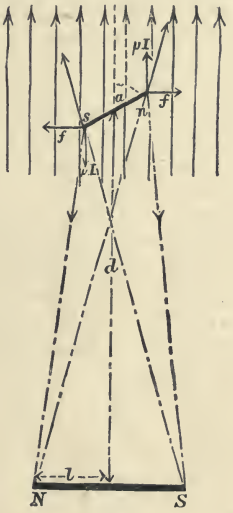


Fig. 75.

In this case the magnet NS is said to be placed "broadside on" to the needle *ns*, and not "end on," as in Fig. 74.

If *l*, the length of the deflecting magnet, be not neglected, then the formula for the "end-on" position is readily found to be—

$$\frac{M}{I} = \frac{(d^2 - l^2)^2}{2d} \tan \alpha,$$

and for the "broadside on" position—

$$\frac{M}{I} = (d^2 + l^2)^{\frac{3}{2}} \tan \alpha.$$

In practice *l* is generally too large to be neglected.

43. Intensity of Magnetisation and Magnetic Induction. If the magnetic state of a magnet is such that the quantities of free magnetism appearing on the opposite surfaces of a cross section taken at right angles to the axis at any point on the magnet are exactly equal but of opposite sign, and uniformly distributed over the surface of the section, then the magnet is said to be *uniformly magnetised.* If such a magnet be divided up into a number of pieces by planes parallel to and at right angles to the axis, then it is evident that the pole strength of each piece will be proportional to its sectional area, and the magnetic moment of each piece will be proportional to its volume, the magnetic moment of any piece being the same fraction of the magnetic moment of the original magnet as the volume of the piece is of the initial volume of the magnet. This proportionality between the magnetic moment and volume is evidently true whatever be the shape of the piece considered, and may be taken as an indication of the magnetic state of the substance of the magnet. In accordance with this view of the matter the magnetic moment per unit volume has been called the *Intensity of Magnetisation* of the substance of the magnet.

If we consider a uniformly magnetised magnet of pole strength m, length l, and uniform cross section a, then the intensity of magnetisation, $\mathbf{I}$, is given by—

$$\mathbf{I} = \frac{ml}{la} = \frac{m}{a};$$

that is, the intensity of magnetisation may be defined as the pole strength per unit area of cross section, but the definition given above is the more general.

From what has been said in Art. 36, it will be understood that uniform magnetisation, as defined above, is not found in the case of actual magnets. In the case of a uniformly magnetised magnet, the free magnetism would appear at the ends only, and the intensity of magnetisation would be the same throughout its substance. Actual magnets, on the other hand, exhibit a distribution of free magnetism along their length, and the intensity of magnetisation varies from point to point in their volume. The intensity

of magnetisation at any point in a magnet may be defined as the magnetic moment per unit volume at the given point.

Consider a long thin rod of iron or other magnetic substance placed in a uniform magnetic field in air, with its length parallel to the lines of force in the field. The rod will become magnetised by induction (Art. 40), and its ends will exhibit polarity. The number of lines of force crossing unit area of the surface separating the ends of the rod from the external medium will therefore now be greater than the number crossing unit area at the same point in the medium before the introduction of the rod; for, in addition to the lines due to the existence of the magnetic field, we have the extra lines due to the induced pole on the end of the rod.

The number of lines of force emanating from a pole of strength m in air can readily be determined from the following considerations. Imagine a north pole of strength m isolated at a point in air. The north pole of a long infinitely thin magnet may be considered as such a pole. The magnetic intensity of the field *due to this pole* at any point at a distance r from the pole is $\frac{m}{r^2}$; that is, the number of lines of force crossing *unit area* of a spherical surface of radius r having its centre at the pole is m/r^2. The area of this spherical surface is $4\pi r^2$; hence the total number of lines of force crossing it, that is, the total number of lines emanating from the pole, is $4\pi r^2 \times m/r^2 = 4\pi m$.

Hence, in the case considered above, if $\mathbf{H}$ denote the intensity of the field in which the rod of magnetic material is placed, then the total number of lines of force crossing unit area of the surface separating the ends of the rod from the medium is given by $\mathbf{H} + 4\pi\frac{m}{a}$, where m denotes the strength of the induced pole and a the area of cross section of the rod. But we have seen above that $\frac{m}{a} = \mathbf{I}$; therefore $\mathbf{H} + 4\pi \cdot \frac{m}{a}$ may be expressed as $\mathbf{H} + 4\pi\mathbf{I}$,

where **I** denotes the intensity of magnetisation at the end of the rod. The increase in intensity denoted by $4\pi\mathbf{I}$ is due to free magnetism induced on the end of the rod.

This increase in field intensity due to the presence of magnetic material in a magnetic field is more instructively considered in the following way. Imagine the rod referred to above cut across at any point by a section at right angles to its length in such a way as to leave a very narrow gap between the two cut faces. These two cut faces will exhibit equal and opposite polarity; and if the rod is uniformly magnetised, the quantity of free magnetism on these faces will be equal to the corresponding quantities on the end faces of the rod. Hence it follows that the number of lines of force crossing the gap is given by $\mathbf{H} + 4\pi\mathbf{I}$; for the field in the gap is made up of the initial field, **H**, and the field due to free magnetism on the cut faces bounding it. These lines of force crossing the gap are supposed to be continued as *lines of magnetic induction* or magnetisation in the material of the rod—that is, the flow of magnetic induction along the rod is measured by the flow of force across the gap. Further, considering free magnetism as a surface effect, wherever lines of magnetic induction leave the rod to pass into the medium, free magnetism appears at the surface of separation. In the case of a uniformly magnetised rod, the lines of the magnetic induction would pass from one end to the other, and leave it only at the ends, so that free magnetism would appear only on the end surfaces; in the case of a magnet not uniformly magnetised, the lines of induction may be supposed to leave it laterally as well as at the ends, so that free magnetism appears not only at the ends, but also as a surface distribution along the length of the bar.

The magnetic force in the magnetic material itself is defined as the force in a small cavity in the material, the cavity being so cut that there is *no free magnetism* on its surface. For example, in the case of the rod considered above, if a very narrow hole is bored in it with its length parallel to the lines of magnetisation in the material, there will be no free magnetism on the lateral surface of the

hole; and if the section of the hole is infinitely small, the amount of free magnetism on the ends will also be infinitely small, so that there will be no free magnetism on the surface of this infinitely narrow tunnel, and the magnetic force in the space gives what is called the force in the material of the bar. In the case of a very long thin rod this force in the material of the rod is evidently merely the strength of the initial field in which the rod is placed; but when the rod is thick compared with its length, another point has to be considered. It has been shown above that $4\pi m$ lines of force diverge from a pole of strength m in air. If the rod at the end of which the pole of strength m is supposed to exist were infinitely thin, then *all* these lines of force would pass through the air, and curving round to the other pole of opposite sign at the other end of the rod unite together, and might then be supposed to run through the rod as lines of induction back to their starting-point. When, however, the rod is of finite thickness, only a portion of these $4\pi m$ lines pass round through the external medium in this way: the remainder pass through the rod itself to the opposite pole, not as lines of magnetisation or induction, but as lines of force; and the number of lines thus diverted from the external path increases with the thickness of the rod.

Hence it follows that the magnetic force at any point in the material of the rod is less than the intensity of the initial field by the force at that point due to the distribution of free magnetism on the external surface of rod. This force due to the external distribution of free magnetism on the rod is sometimes called the *demagnetising force*, for it reduces the magnetic field to which the induced magnetisation of the rod is due—that is, the poles at the ends of the rod tend to magnetise the rod in the opposite sense, and thus produce partial demagnetisation. This demagnetising force is greater the greater the ratio between the thickness and the length of the rod, and experiment shows that it is only when the length is about three hundred times the thickness that the demagnetising effect is negligible.

From what has been said above it will be evident that

magnetic induction may be considered in the following way. When a magnetic substance is placed in a magnetic field existing in a given medium, a distribution of free magnetism is determined over the separating surface, and the field both external and internal to that surface is the resultant field due to the initial field, and that due to the distribution of free magnetism on the surface of separation. Further, if we consider all substances as susceptible of magnetisation, we may state the result more generally. If any portion of a medium in which a magnetic field is established is replaced by another medium, a distribution of free magnetism is determined on the surface of separation of the media, and the magnetic field at any point is the resultant of the initial field, and that due to the surface distribution of free magnetism. Also, when lines of magnetisation pass from one medium to another, positive (north-seeking) magnetisation appears on the surface of separation, if the lines pass from a more magnetic to a less magnetic medium—for example, from iron to air; and negative (south-seeking) magnetisation when they pass from the less magnetic to the more magnetic, as from air to iron. It follows from this that if a substance less magnetic than air be placed in air in a magnetic field, it will be magnetised by induction with the polarity *opposite* to that which a piece of iron would exhibit. This is the phenomenon of *diamagnetism*, and is evidently only an ordinary case of magnetic induction.

In the quantitative study of magnetic induction three quantities are of first importance. These are Intensity of Magnetic Field, **H**; Intensity of Magnetisation, **I**; and Magnetic Induction, **B**. Each of these three quantities are directed quantities—that is, they are quantities possessing both magnitude and direction. The directions of **I** and **B** are not necessarily the same as that of **H**. But in what follows we shall consider only the case where all three quantities have the same direction: the results obtained will apply to the more general case if the signs of addition and equality be taken in vectorial sense as referring to composition and resultant effect.

The first two of these quantities have already been

explained. By magnetic induction is meant the flow of induction per unit area across any section taken at right angles to the direction of flow: as explained above, this flow of induction at any section of the substance may be measured by the total flow of force across a very narrow sectional gap taken at the point considered at right angles to the direction of magnetisation.

The relation between the three quantities, **H**, **I**, **B**, may evidently be considered in two ways. When a portion of any substance is subjected to magnetic induction, quantitative results may be obtained by determining the intensity of magnetisation, **I**, produced by a given value of **H**, or by determining the value of **B** under definite conditions corresponding to given values of **H**. In both cases it is very important to note that **H** denotes, not the initial field in which the substance is placed, but the intensity of the resultant field in the interior of the substance itself—that is, the demagnetising force due to surface distribution has to be considered.

The simplest case of magnetic induction is that of a very long thin rod or wire placed in a uniform magnetic field with its length parallel to the lines of force. And this is the case of greatest practical importance. In this case, if the length of the wire is more than three hundred times its diameter, there is practically no demagnetising force; and therefore **H**, the strength of the magnetic field acting inductively on the wire, may be taken as that of the uniform field in which it is placed. The intensity of magnetisation, **I**, is given by the strength of pole per unit area of cross section, or by the magnetic moment per unit volume, and is therefore known if the magnetic moment and the dimensions of the wire are known. The magnetic induction, **B**, is given by the total flow of force per unit area across a narrow gap taken at right angles to the length of the wire—that is, as explained above, it is given by $\mathbf{H} + 4\pi\mathbf{I}$.

In considering the relation between the intensity of magnetisation, **I**, and the field, **H**, to which it is due, it is convenient to deal with the ratio **I**/**H**. This ratio evidently gives a measure of the readiness with which the substance

under induction assumes the magnetic state—that is, it gives a measure of the susceptibility of the substance to magnetisation, and is commonly called the *magnetic susceptibility* of the substance; it is usually denoted by κ, and we therefore have—

$$\frac{\mathbf{I}}{\mathbf{H}} = \kappa.$$

In the same way the ratio of the magnetic induction, **B**, to **H**, the intensity of the inducing field, is known as the *magnetic permeability* of the substance. This ratio may be denoted by M, and is the same quantity as that introduced in Art. 39. It evidently gives the ratio of the flow of force across the narrow air gap cut in the bar referred to at the beginning of this article to the flow of force across the same area before the bar was introduced. As the lines crossing the air gap are supposed to be continued through the substance of the bar as lines of induction, the ratio of **B** to **H** may be taken as a measure of the relative permeability of the substance of the bar to lines of force as compared with air.

Hence we have—

$$\frac{\mathbf{B}}{\mathbf{H}} = M.$$

But—

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I}.$$

Therefore—

$$M = \mathbf{I} + 4\pi\frac{\mathbf{I}}{\mathbf{H}};$$

and substituting κ for $\frac{\mathbf{I}}{\mathbf{H}}$, we get—

$$M = \mathbf{I} + 4\pi\kappa,$$

where M denotes the magnetic permeability of the substance subjected to induction relative to air as a standard medium, and κ denotes the magnetic susceptibility of the substance.

Experiment shows that these quantities, M and κ, for a given substance vary with the magnetic state of the

substance. Thus κ decreases as $\mathbf{I}$ and $\mathbf{H}$ increase, for $\mathbf{I}$ tends to a maximum as $\mathbf{H}$ increases; similarly κ decreases as $\mathbf{H}$ increases, for $M = \mathbf{I} + 4\pi\kappa$. Further, from the nature of the ratio which it denotes, M is evidently greater than unity for substances more magnetic than air (paramagnetic substances), and less than unity for substances less magnetic than air (diamagnetic substances). Hence it follows that κ is positive for paramagnetic substances and negative for diamagnetic substances; also, since M is taken as unity for air, κ for air must be taken as zero.

CHAPTER VIII.

TERRESTRIAL MAGNETISM.

44. The Magnetic Field of the Earth. When a magnetic needle is freely suspended at any point on the earth's surface, we have already learnt that it invariably sets in a particular position, with its magnetic axis pointing approximately north and south. This shows that at all points on the earth's surface a magnetic field exists, in which the lines of force are approximately parallel to the geographical meridian of the place. This magnetic field is supposed to be due to the earth's magnetism, but whether the earth is itself a permanent magnet, or whether its magnetism is due to some unknown cause, is, at present, an open question. The simplest explanation is to consider the earth as a permanent magnet, with its poles not far distant from the north and south poles of the earth. Assuming the distribution of magnetism to be somewhat irregular, the magnetic field that should result agrees pretty closely with observations of the actual field of the earth.

In order to quantitatively determine the magnetic field of the earth at any point, the direction and magnitude of the magnetic force at that point must be determined. The direction of the force may be found by determining (1) the vertical plane in which it acts, and (2) its direction in that plane. The intensity of the field may be determined either by measuring this magnetic force directly, or by measuring a component making a known angle with it. For example, if the magnetic force at any point is found to act in a plane represented by the plane of the paper in a direction

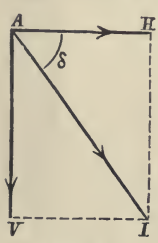


Fig. 76.

AI (Fig. 76), then the intensity of the field may be determined by measuring AI directly, or *either* of the components AH or AV. For, supposing AH to be measured and the angle HAI known, then—

$$\frac{AH}{AI} = \cos HAI, \text{ that is, } AI = \frac{AH}{\cos HAI}.$$

We shall now proceed to consider the practical determination of the earth's magnetic field.

The quantities which completely determine the magnetic field of the earth are sometimes called the *magnetic elements*. The *direction* of the magnetic force is determined by the *Declination* and the *Inclination* or *Dip*. The *Declination* at any place is the angle between the magnetic and geographical meridians of the place, the *magnetic meridian* at any place being the great circle whose plane contains the direction of the terrestrial magnetic force at that place. When the direction of the geographical meridian is known, a knowledge of the angle of declination enables us at once to find the magnetic meridian of a place, or the vertical plane in which the magnetic axis of a freely suspended magnet would set itself.

The *Inclination*, or *Dip*, at any place, is the angle which the direction of the magnetic force makes *in the magnetic meridian* with the horizontal at that place. Thus in Fig. 76, if the plane of the paper represent the magnetic meridian and AI the direction of the magnetic force at A, then the angle HAI represents the inclination or dip at A.

The *intensity* of the earth's field at any place is measured by determining the *horizontal component* of the total magnetic force at that place.

45. Declination. To determine the declination it is necessary to determine the geographical and magnetic meridians, and to measure the angle between them. In this article we shall consider only a very simple method of effecting these measurements. Suppose, for example, we wish to mark on a large table in the laboratory a line showing the positions of the two meridians, and including the angle of declination between them. Let a straight

piece of wire about a foot long be fastened vertically into the table, so that the sun casts its shadow on the table. The direction of this shadow when *it is shortest*, which will be about noon, will approximately give the geographical meridian of the place. Let this direction be marked by a line NS (Fig. 77), drawn on the table.

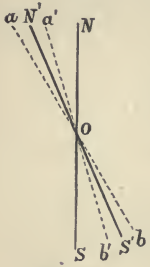


Fig. 77.

To determine the magnetic meridian, let a light bar magnet be suspended over the table by a single silk fibre. To each end of this magnet let a short piece of fine brass wire be attached by shellac or sealing-wax, in such a position that, when the magnet is properly suspended, the pieces of wire are, as accurately as possible, vertical (Fig. 78). Suppose the magnet to be suspended over the table, let a needle or piece of wire be stuck into the table at any point, *a*, through which the line, *ww*, joining the pieces of wire at the ends of the magnets, passes. This point is readily determined by one observer sighting along the lower points of the wire and directing another observer where to place the needle. In the same way another needle can be placed at *b*, so that *a o b* represents the direction of the line joining the wires fixed at the ends of the magnet. If *ww* coincided with the magnetic axis of the magnet, then the line *ab* would represent the trace of the magnetic meridian on the table, and the angle N*oa* would be the required angle of declination. In all probability, however, *ww* is not the magnetic axis of the needle; but whatever the position of this axis, we can eliminate the error due to its non-coincidence with *ww* by turning the needle upside down, so that the top face becomes the bottom face, and then determining the line *a'ob'* in exactly the same way as *a o b* was found. The true direction of the trace of the magnetic meridian will now lie midway between the lines *a o b* and *a'ob'*, and

Fig. 78.

may be drawn by bisecting the angle between them. The line N′S′ represents this direction, and the angle NON′ measures the declination. To prove that N′S′, bisecting the angle between *ab* and *a′b′*, gives the true direction in which the magnetic axis of the magnet points, consider Fig. 79. Taking an extreme case, let *ns* represent the position of the magnetic axis of the magnet. Then if the full outline represent the position of the magnet in the first case, on inverting the magnet it will take up the position indicated by the dotted outline, for in this position the magnetic axis has the same direction as before.* From the figure it is evident that the magnetic axis *ns*, and, therefore, the magnetic meridian, lies midway between the two positions of *ww*.

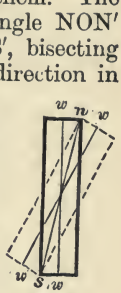


Fig. 79.

The magnitude of the angle of declination will necessarily vary for different positions on the earth's surface. Suppose, in Fig. 80, that we are looking down on the northern hemisphere of the earth. Let G represent the geographical north pole, and M the magnetic north pole; then if the distribution of the earth's magnetism were perfectly regular, the full lines would represent the magnetic meridians, and the dotted lines the geographical meridians, of the northern hemisphere. The angle at any place between the two meridians for that place gives the declination there, and it is evident from the figure that that angle varies for different places. For any place on the line *ab* the angle of declination is zero, for the magnetic and geographical meridians are here

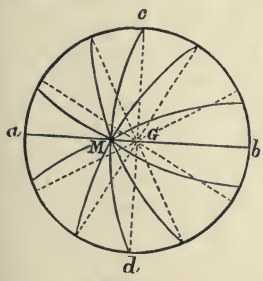


Fig 80.

* The magnetic axis must lie in the magnetic meridian, and therefore, on inversion of the needle it must either be coincident with, or parallel to, its first position. A line fulfilling this condition may be defined as the magnetic axis of a magnet.

coincident; for places on the line *cd*, the angle of declination is a maximum, and for places between these two lines the declination will have intermediate values. Further, in the quadrant *abd*, the declination is to the west of the geographical meridian and in the quadrant *acb* to the east.

Magnetic observation fixes the magnetic north pole in Boothia Felix in latitude 70° 5′ N., and longitude 96° 45′ W. The position of the south magnetic pole has not yet been definitely settled; it was stated by Sir James Ross to be in 75° 30′ S. latitude and 154° E. longitude, but it is possible that there is more than one south polar region. In England the angle of declination is about $17\frac{1}{2}°$ W., that is, the magnetic meridian is about $17\frac{1}{2}°$ West of the geographical meridian.

In Fig. 87 the thick lines diverging from M show roughly the true course of the magnetic meridians; they are called *isogonic* lines, indicating that at all points on any one of them the angle of declination is the same.

46. Inclination, or Dip. If the magnetic meridian be found as in the preceding article, and a magnetic needle free to move in a *vertical plane* (Fig. 81) be placed with its axis in the meridian, it will be found that it does not set horizontally, but dips, in England, at an angle of about 68° with the horizontal. This angle is the angle of Dip, and a magnetic needle mounted so as to indicate it is called a *Dipping Needle.*

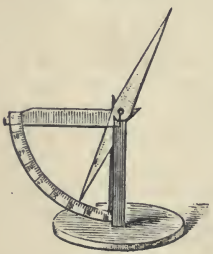


Fig. 81.

To determine the Inclination or Dip at any place it is therefore only necessary to have a magnetic needle mounted so as to move freely in a vertical plane in front of a circular scale on which the angle of Dip may be read off. A proper instrument for measuring the angle of Dip is known as a *Dip Circle.* It consists, as shown in Fig. 82, of three essential parts—the needle mounted so as to move with as little friction as possible in a vertical plane, the vertical circular scale in front of

which the needle moves, and the horizontal circular scale on which the rotation of the frame carrying the vertical scale and needle can be read. The needle is mounted by means of a short horizontal axis of hard polished steel passing, as accurately as possible, through the centre of gravity of the needle. When mounted on its bearings,

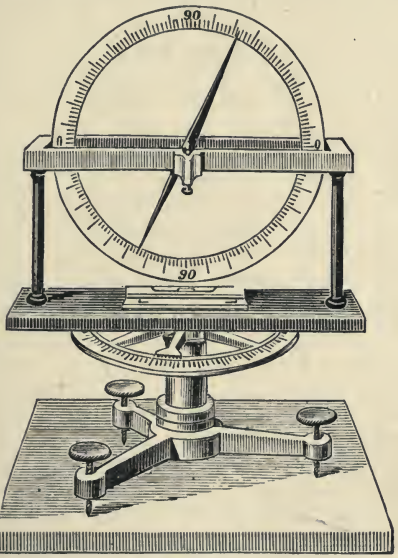


Fig. 82.

this axis should pass through the centre of the vertical scale at right angles to plane of the scale. The scale itself is generally graduated so that both ends of the needle can be read, and since the Dip is measured from the horizontal, the *zero line* is horizontal. Fig. 83 shows generally how the scale is graduated: in the best forms of the instrument, the positions of the ends of the needle are read by small reading microscopes, which carry verniers round the vertical scale. The horizontal scale may be

graduated continuously from 0° to 360°, commencing at any point, or it may conveniently be graduated in the way indicated in Fig. 83—its use will appear in the sequel.

To determine the Dip by a Dip Circle, it is obviously necessary, first, to set the plane of the needle in the magnetic meridian—this might be done approximately by determining the meridian by means of a magnetic needle, but it is more conveniently and accurately done by the indications of the Dip Circle itself. The instrument is carefully levelled by means of the screw feet, and the spirit-level attached to it. The needle and vertical scale are then moved round the horizontal scale until the needle sets exactly vertically.

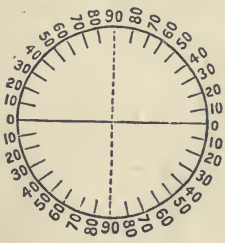


Fig. 83.

When this is the case the plane of the needle is *at right angles* to the magnetic meridian, and hence on turning it through 90°, measured by the help of the vernier attached to the horizontal scale, the needle is set exactly in the magnetic meridian.

To understand this last adjustment let us consider what we know about the magnetic field of the earth. The resultant direction of the magnetic force at any place lies in the magnetic meridian for that place, and makes a definite angle, known as the angle of Dip, with the horizontal in that plane. In Fig. 76 let the plane of the paper, as before, represent the magnetic meridian, and let AI represent the direction of the resultant magnetic force at A. Imagine a magnet, moving in a vertical plane *at right angles* to the plane of the paper, to have its north pole at A—the force acting on the pole will be in the direction AI, *but only the vertical* component of this force will have any effect on the needle, for the horizontal component, AH, being at right angles to the plane of motion of the needle, can have no effect on it, except to produce a slight pressure on its bearings. Hence, when the plane of motion of the needle is at right angles to the magnetic meridian, only the *vertical*

component of the earth's magnetic field is effective, and consequently the needle, being entirely under the influence of this component, sets vertically.

The needle being thus set in the magnetic meridian, it only remains to read off the angle of Dip on the vertical scale, and if the needle be accurately centred and suspended, and the scale accurately levelled and engraved, the direct reading of either end of the needle gives the true Dip. In practice, however, the needle and scale are never in accurate adjustment, and a number of corrections are necessary to eliminate :—

(1) The error due to eccentricity, that is, the error which results if the axis of suspension of the needle should not pass through the centre of the vertical scale.

(2) The error which results if the zero line of the vertical scale is not truly horizontal.

(3) The error due to non-coincidence of the magnetic axis of the needle with its geometrical axis.

(4) The error which results from non-coincidence of the point of suspension of the needle with its centre of gravity.

Error (1) is eliminated by reading both ends of the needle and taking the mean of the two readings. This is evident from Fig. 84, for if O represent the point of suspension of the needle and O′ the centre of the circular scale, then the true Dip on' or os' is evidently the arithmetical mean of on and os, the actual readings of the two ends of the needle.

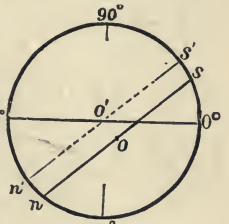


Fig. 84.

Error (2) is eliminated by taking the mean reading for two positions of the needle in the magnetic meridian. When the needle is first set at right angles to the magnetic meridian, it can evidently be put in the meridian by turning through 90°, either to the right or to the left of the observer: let it be turned first to the right, say, and let the position of the needle be read, then let it be turned through 180° in either direction, and it will be again in the magnetic

meridian, but reversed relative to the observer. The mean of the readings of both ends of the needle, taken in these two positions, gives the Dip corrected for the sources of errors (1) and (2). For in Fig. 85, where the right-hand

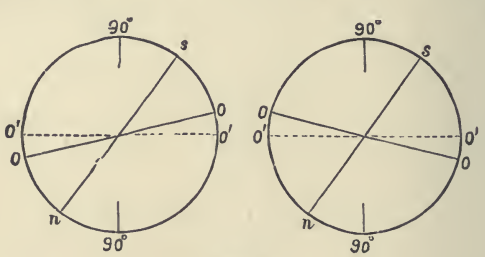


Fig. 85.

drawing represents the reverse (or obverse) view of the left-hand one, if o'o' represent the true horizontal, and oo the actual position of the zero-line, then the true Dip o'*n* or o'*s* is evidently the arithmetical mean of the reading o*n* or o*s* in the left-hand drawing, and o*n* or o*s* in the right-hand drawing.

The source of error indicated in (3) has already been considered in the determination of Declination. It is eliminated in the same way by reversing the needle relative to the scale, that is, by lifting the needle off its bearings, turning it round back for front, and replacing it on the bearings. The four readings indicated above are now repeated, and the mean of the eight readings thus obtained gives the Dip corrected for errors (1), (2), and (3).

To correct for error (4) it is necessary to remagnetise the needle, so as to reverse its poles. The eight readings mentioned above are now repeated, and the mean of the sixteen readings gives the true Dip. The theory of this correction may be seen in a general way thus: suppose the centre of gravity of the needle to be nearer the north pole of needle than the point of suspension, then it is evident that the

observed angle of Dip will be too great, but on reversing the magnetism of the needle the observed angle will, for the same reason, be too small, and hence, by taking the mean of the two readings, the error is approximately liminated.

Like the declination, the angle of Dip is different for different positions on the earth's surface. In the northern hemisphere, the north pole—that is, the north-seeking pole —of the needle always dips downwards, and the magnitude of the angle of Dip varies with change of position on the surface of the hemisphere. At one point the needle sets itself vertically, and the Dip has its maximum value of 90°. This point is the magnetic north pole. Similarly, in the southern hemisphere the south-seeking pole of the needle dips downwards, and at the south pole the needle is vertical. From this it follows that, at a series of points in the neighbourhood of the equator, the needle is horizontal, and dips neither towards the north nor the south pole of the earth—at these points, therefore, the angle of Dip is zero, and the locus of all such points is called the *magnetic equator*, or more properly the *aclinic line*, or line of no dip. Hence, if a dipping needle were carried by any path, say along a magnetic meridian, from the magnetic north pole to the magnetic south pole, the position of the needle would change through 180°. At the magnetic north pole the needle is vertical, with its north-seeking pole dipping down, but as it is carried south the angle of dip gradually decreases from its initial value of 90° at the pole to 0° at the *aclinic line*. On

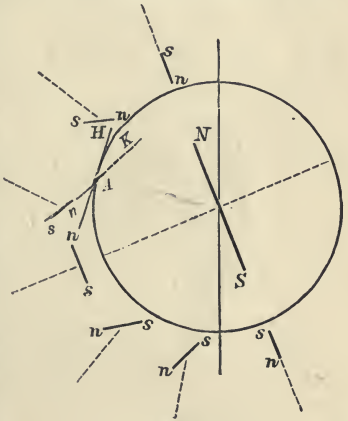


Fig. 86.

crossing this line, the south-seeking pole of the needle begins to dip, and the inclination gradually increases from 0° to 90° in passing from the equator to the south pole. Figure 86 shows these changes diagrammatically: in studying the figure it must be remembered that the Dip at any place is measured by the angle between the horizontal at the place and direction of the magnetic axis of the dipping needle at that place; also that the horizontal at any place is the line drawn tangential to the earth's surface at that place. Thus at A the line AH is the horizontal, and the angle HAK is the angle of Dip. The position of equilibrium of the dipping needle at any place is determined by the principles illustrated in Fig. 73, where NS may represent the earth (as a magnet) and *ns* the dipping needle.

It is evident that a number of places at which the angle of Dip is the same can be found—thus, roughly

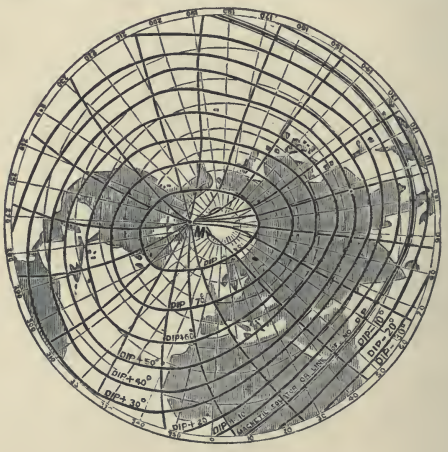


Fig. 87.

speaking, all places at equal distances from either magnetic pole should have the same Dip. Hence, if lines be drawn

through all places at which the Dip is the same, we shall have a series of *isoclinic lines* corresponding on a magnetic map to the lines of latitude on a geographical map—the *aclinic line* is one of these lines and corresponds to the geographical equator. In Fig. 87 the irregular circles concentric with M represent the isoclinic lines for every 10° of Dip—the aclinic line or magnetic equator being the irregular curve of no Dip.

47. The Horizontal Component of the Earth's Magnetic Field. To determine the total intensity of the earth's magnetic field it is usual to measure the *horizontal component* of that force. The reason for this is that in measuring a magnetic field it is necessary to work with pivoted or suspended magnets free to move in the field. Now, owing to action of gravity, it is most convenient to work in a horizontal plane, in which the magnetic field is that due to the horizontal component of the earth's total field.

In Fig. 76, let AI represent the total intensity of the earth's field; then, resolving horizontally and vertically, AH represents the horizontal component, AV the vertical component, and the angle HAI the angle of Dip. These quantities are usually represented by I, H, V, and δ respectively, and from Euc. I. 47 and Art. 43 we have—

$$I^2 = H^2 + V^2.$$

$$I = \frac{H}{\cos \delta}, \text{ or } H = I \cos \delta.$$

$$I = \frac{V}{\sin \delta}, \text{ or } V = I \sin \delta.$$

Also, from figure

$$\frac{IH}{AH} = \tan \delta, \text{ that is, } \frac{V}{H} = \tan \delta.$$

These relations between the magnetic elements are important; from them it is evident that if any two of the quantities I, H, V, and δ are known, then the others can be found. The two usually determined are H and δ: we have already detailed how δ is measured, and we have now

to show how H can be measured. This is a somewhat difficult operation, and we can only indicate generally how it is performed. The determination consists of two steps.

(1) *A Deflection Experiment.* In this experiment a simple piece of apparatus known as a magnetometer is used. It consists of a small needle, *ns* (Fig. 88), pivoted or suspended so as to move freely in a horizontal plane. When left to itself the needle sets itself in the magnetic meridian under the influence of the horizontal component, H, which is the measure of the intensity of the field in which it lies. A magnet, NS, is placed "end on" with its *centre* at a distance *d* to the east or west of the centre of the needle *ns*. As a result the latter is deflected through an angle α, and, by Art. 42, we know that

$$\frac{M}{H} = \frac{d^3}{2} \tan \alpha,$$

where M denotes the magnetic moment of the magnet NS.

When the needle is pivoted, as in the apparatus shown in Fig. 88, the angle α is read off on a circular scale by

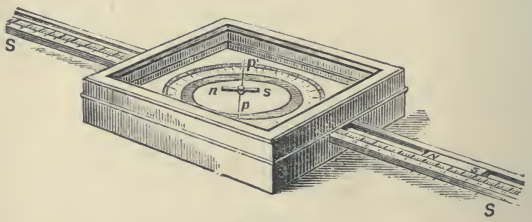


Fig. 88.

means of a light brass pointer *pp'* attached to the needle *ns*, the distance *d* being given by the millimetre scale SS, which is graduated from the pivot of the needle as zero. For more accurate determinations a mirror magnetometer is used; the needle, attached to a small concave mirror, is suspended by a silk fibre, and the angle of deflection is measured by the mirror and scale arrangement described in Art. 27.

For the more exact determination of the angle of deflection, four observations for the same value of *d* are taken in

the way indicated in Fig. 89, the true value of a being

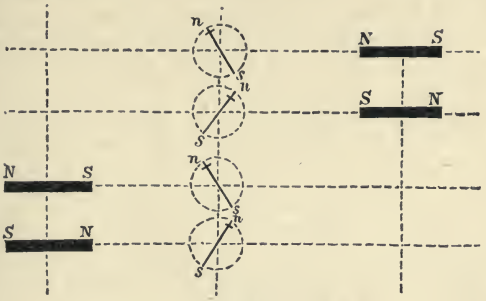


Fig. 89.

the mean of the eight readings obtained by reading both ends of the needle in the four positions of the figure.

The object of the experiment is the determination of $\frac{M}{H}$ from observation of d and a by means of the relation

$$\frac{M}{H} = \frac{d^3}{2} \tan a.$$

(2) *An Oscillation Experiment.* The magnet NS is suspended by a silk fibre inside a box with glass sides, and the position of the box is adjusted until the magnet sets in the magnetic meridian with its axis in the vertical plane passing through two vertical lines ruled on opposite faces of the box. When this adjustment is made the magnet is slightly deflected by bringing the pole of a magnet, held at right angles to it, near one of its poles, and is then allowed to oscillate backwards and forwards in a horizontal plane under the influence of the horizontal component of the earth's magnetic field.

The time of each oscillation of the magnet is then carefully determined—most readily by noting the time (in minutes and seconds) occupied by, say, 50 or 100 oscillations, and dividing this time by 50 or 100 as the case may be. This determination is most easily made by two

observers—one counting the oscillations commencing at 0, and counting an oscillation every time the magnet passes through its initial position of rest; the other noting on a chronometer, or watch with a seconds hand, the exact time of the commencement and finish of the 50 or 100 oscillations observed.

Now, if M denote the magnetic moment of the magnet, the time (t) of its oscillation in a field of intensity H is given by—

$$t = \frac{k}{\sqrt{\mathrm{M\,H}}} \text{ (Art. 41)},$$

where k is a constant involving the moment of inertia of the magnet.* Hence we have—

$$t^2 = \frac{k^2}{\mathrm{M\,H}},$$

or

$$\mathrm{M\,H} = \frac{k^2}{t^2}.$$

Hence by determiming t with a magnet of regular form, for which k can be calculated, we are able to determine the *product* of the two quantities M and H for which the deflection experiment gives the ratio.

Thus we have—

$$\frac{\mathrm{M}}{\mathrm{H}} = \frac{d^3}{2} \tan \alpha, \qquad (1)$$

and

$$\mathrm{M\,H} = \frac{k^2}{t^2} \qquad (2).$$

* For a bar magnet of rectangular section of length l, breadth b, and mass W

$$k^2 = \pi^2\,\mathrm{W}\left(\frac{l^2 + b^2}{12}\right),$$

and for a magnet of cylindrical section of length l, radius r, and mass W

$$k^2 = \pi^2\,\mathrm{W}\left(\frac{l^2}{12} + \frac{r^2}{4}\right).$$

Now, by dividing (2) by (1) we get—

$$H^2 = \frac{2\,k^2}{d^3\,t^2 \tan \alpha},$$

or

$$H = \frac{k}{t}\sqrt{\frac{2}{d^3 \tan \alpha}},$$

and thus H, the horizontal component of the earth's magnetic field, is determined.

If the units involved in k are stated in grams and centimetres, t in seconds, and d in centimetres, then H gives in *dynes* the horizontal component of the force which a unit magnetic pole would experience in the field of the earth.

It should be noticed that the magnetic moment of the magnet NS can also be determined from equations (1) and (2), for, by multiplication, we have—

$$M^2 = \frac{d^3\,k^2 \tan \alpha}{2\,t^2}$$

or

$$M = \frac{k}{t}\sqrt{\frac{d^3}{2} \tan \alpha}.$$

Thus the experimental method which gives H also serves to determine M. The value of H varies with a change of position on the surface of the earth. We have seen that $H = I \cos \delta$; hence H increases as I increases, and decreases as δ increases. In travelling from the equator* to either pole the value of I increases, but that of $\cos \delta$ decreases, and more rapidly than I increases; therefore, on the whole, H decreases in passing from the equator to the poles. Thus, at Quito, near the equator, the value of H is about ·32 C.G.S. units;

* Owing to the irregular distribution of the earth's magnetism, the *aclinic line* is not, as it should be, the line of minimum intensity. If a line were drawn through all points where I is a minimum, we should obtain another curve very close to the aclinic line with a claim to be called the *magnetic equator*. Theoretically, the magnetic equator should be a line of zero dip and minimum intensity, and at all points on it the direction of the magnetic axis of a compass needle should be at right angles to its plane.

in Great Britain it varies from ·18 units in London to ·16 units in Glasgow, and at the north magnetic pole it is zero.

48. Variations of the Magnetic Elements. The magnetic elements vary not only from place to place on the earth's surface, but also from time to time at any given place. These variations may be roughly classified thus:—

(1) *Secular variations.* These are slow periodic changes, taking a great many years to complete their period. Thus, in 1580 the declination at London was 11° East; then it gradually moved westward until, in 1815, it attained an extreme westerly value of 24° 30′ West. Since then it has gradually decreased to its present value, about $17\frac{1}{2}°$ West, and in another 150 years it may be zero, that is, the compass will point true north. The Dip is also subject to secular variations—at London in 1576 it was 71° 50′, and slowly increased until in 1720 it reached a maximum value of 74° 40′. Since that date it has gradually decreased to its present value of about 67° 30′.

(2) Annual variations of the Dip amounting to about a quarter of a degree on either side of the mean annual value.

(3) Diurnal variations of Dip, varying in value from 5′ to 20′ or 30′

In addition to these periodic variations, there are irregular disturbances supposed to be indications of magnetic storms.

Owing to lack of observations, little is known about the variations of H, except in so far as its value is affected by the variations of δ given above.

49. Magnetisation by the Inductive Action of the Earth's Magnetic Field. We have learnt in Art. 37 that when a piece of iron or steel is placed in a magnetic field it becomes magnetised, and that in steel the magnetisation is in some degree permanent, but in iron the residual magnetism varies with the softness of the iron, being practically *nil* for very soft wrought iron.

It follows from this that, since the earth possesses a magnetic field, every piece of iron or steel on it is more or less a magnet; but as the earth's field is, for purposes of magnetisation, extremely feeble, special precautions are necessary in order to experimentally verify the truth of

this fact. Further, a piece of steel being somewhat difficult to magnetise, may not possess magnetic properties unless it has lain for a very long time in one fixed position, or unless the arrangement of its molecules is facilitated by jarring or hammering while in a favourable position in the field. Thus, if a rod of steel or hard iron, for example, an ordinary poker, be placed with its length along the line of Dip, that is, parallel to the lines of force in the earth's field, then, on smartly tapping its upper end with a hammer, it at once becomes a magnet, the lower end being a north pole, so that on presenting this end to the north pole of a compass needle repulsion is distinctly exhibited. Similarly, if the poker were allowed to remain in this position for some weeks or months, it would, under the continued inductive action of the field, gradually acquire magnetic properties without being struck by a hammer.

The poker is placed with its length parallel to lines of force, because in this position it is acted on inductively by the total intensity of the earth's magnetic field—if placed in the magnetic meridian with its length horizontal, its induced magnetism may be said to be due to the inductive action of H, the horizontal component of the earth's field; and if placed vertical, the inductive effect is said to be due to the vertical component V.

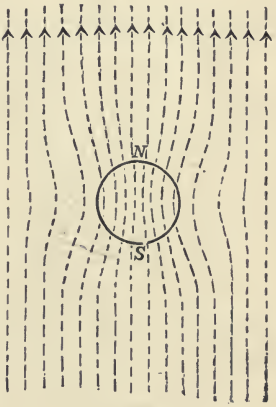


Fig. 90.*

Similarly, a rod of soft iron under the same conditions is acted on in precisely the same way, but the effect is temporary, and too feeble to be readily exhibited.

* The arrows in the figure show the direction of the lines of force; that is, the direction in which a north pole would be urged. Hence the north pole, N, is induced in that direction.

To determine the magnetic effect produced when a piece of magnetic substance is placed in *any* position in a magnetic field, it must be remembered that magnetic induction acts more readily through magnetic substances than through non-magnetic substances. Now the effect on a magnetic substance placed in the field depends upon the distribution of the lines of force through it; it is magnetised by induction, its magnetic axis being parallel to the general direction of the lines of force in it and its poles at the points where the lines enter and leave it. Thus, if an iron or steel rod be placed with its length at right angles to the direction of the lines of force, then, at any point in its length, Fig. 90 shows the polarity and general distribution of the lines of force.

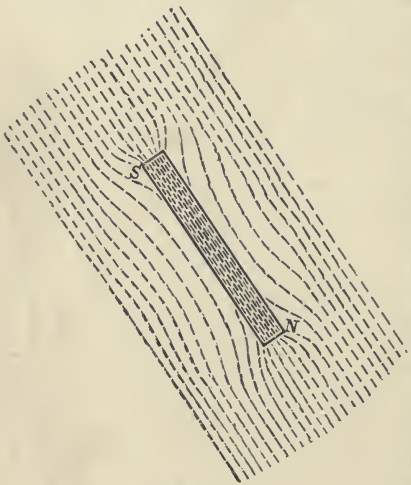


Fig. 91.

Similarly, if placed, as in the case of the poker considered above, with its length parallel to the lines of force, then Fig. 91 indicates the polarity and also the general distribution of lines of force.

The student should verify these and similar results by placing different pieces of iron and steel in various positions in the space between two unlike poles of two strong bar magnets, and obtaining a representation of the field in each case by means of iron filing curves.

CHAPTER IX.

SIMPLE MAGNETIC MEASUREMENTS.

50. The Torsion Balance. The torsion balance described in Art. 8 was applied by Coulomb to the purposes of magnetic measurement. Fig. 92 shows the instrument arranged for magnetic work. The shellac lever *a b* shown in Fig. 5 is replaced by the magnet NS, and the insulating rod *g* by the magnet M.

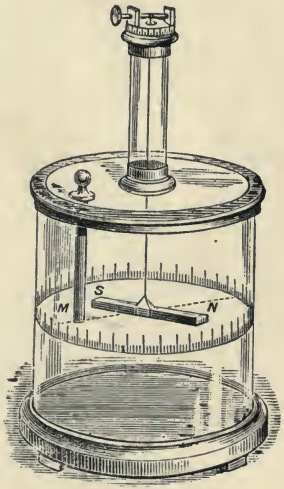


Fig. 92.

The general method of work is exactly similar to that described in Art. 8, but, owing to the directive action of the earth's field on the magnet NS, the following additional adjustments have to be considered.

(1) The suspension wire must be free from torsion when the magnet lies in the magnetic meridian. This may be effected by suspending, in place of the magnet, *a copper* bar of about the same size and weight, and then adjusting the torsion head until this bar lies in the magnetic meridian. When the magnet is now replaced in its stirrup, it at once sets in the meridian without putting any twist on the wire.

(2) When the magnet is deflected out of the meridian by

the repulsion of the lower pole of M, it is urged back by *two* couples, one due to torsion, as explained in Art. 8, and the other due to the horizontal field of the earth. The moment of the former couple is proportional to T, the twist on the wire; the moment of the latter is, according to Art. 41, given by $m\ l\ \mathrm{H} \sin \delta$, where m is the strength of the pole of the magnet, l its length, H the intensity of the horizontal component of the earth's field, and δ the angle of deflection from the meridian. Coulomb, however, for the sake of simplicity, did not apply this relation, but determined by experiment the value of this moment in terms of the torsion moment. This he did by first removing M and adjusting NS in the meridian, so that the suspension wire was free from torsion. He then turned the torsion head so as to deflect NS out of the meridian, and noted the amount of twist that had to be put *on the wire* in order to deflect the magnet 1° out of the meridian. Thus, in one experiment, it was found that the *torsion head* had to be turned through 360° in order to deflect the magnet (in the same direction) 10° out of the meridian, that is, a twist of 350° (360 − 10) had to be put *on the wire* to produce 10° deflection from the meridian. Hence, *assuming the moment due to* H *to be proportional to the angle of deflection*,* this shows that 1° deflection from the magnetic meridian is equivalent to 35° of torsion on the wire.

These adjustments being made, the balance may be used to verify the laws of magnetic attraction and repulsion in precisely the same way as described in connection with the verification of the corresponding laws of electrostatics. For example, in proof of the law of inverse squares, Coulomb made the following experiment. He first found, in the way explained above, that 1° deflection of the magnet out of the magnetic meridian corresponded to 35° of torsion of the wire. The magnet M was then placed in position, and

* Since this moment is expressed by $m\ l\ \mathrm{H} \sin \delta$, it is evidently proportional, not to the angle of deflection, but to the *sine* of that angle. For small angles, however, the sine of an angle is proportional to the angle; so that, unless the deflection exceeds 10 or 20 degrees, Coulomb's method is sufficiently exact.

was found to repel the suspended magnet through 24°. The twist on the wire was thus 24°, and adding to this the 24 × 35° of torsion to which the 24° deflection out of the magnetic meridian is equivalent, the total *torsion equivalent* of the deflection was taken as (24 + 24 × 35) or (36 × 24) = 864°. The torsion head was then turned so as to reduce the deflection from 24° to 12°, and it was found that eight complete revolutions were necessary to effect the desired result. The actual twist on the wire is now equal to 8 × 360 + 12 = 2892°, and adding to this the torsion equivalent of the 12° deflection out of the meridian, we get a total of 2892 + (12 × 35) = 2892 + 420 = 3312°. This proves that, on halving the distance between the two magnetic poles, the force of repulsion between them is increased fourfold, for 3312 is approximately four times 864, and the force of repulsion is roughly proportional to the torsion equivalents of the deflections compared.

As already explained, the torsion balance is a comparatively rough instrument, and is now but little used. It has, however, played an important part in the development of magnetic measurements, and the principles of its construction are still of some practical importance.

51. Comparison of the Intensities of Magnetic Fields. The method of oscillations is the most convenient for comparing magnetic fields. As already explained, the time of oscillation of a freely suspended magnetic needle varies inversely as the square root of the intensity of the field in which it is suspended, that is, the square of the number of oscillations executed by the needle in a given time is directly proportional to the intensity of the field. Hence, if a needle be caused to oscillate at two places on the earth's surface where the horizontal intensities are, say, H_1 and H_2, and the number of oscillations performed in a given time at these places be respectively n_1 and n_2, then we have—

$$\frac{H_1}{H_2} = \left(\frac{n_1}{n_2}\right)^2$$

In this way the intensity of the horizontal component of

the earth's magnetism at any place may be compared with its intensity at any other place.

If we wish to compare the intensity of the field due to a given magnetic system with the horizontal component of the earth's field, it will in general be necessary to eliminate from the former the effect of the latter. Thus, let a bar magnet be adjusted in the neighbourhood of the needle, so that the latter still sets in the magnetic meridian, and let it be required to compare the field *due to the magnet* in the neighbourhood of the needle with the horizontal field of the earth. Let I denote the intensity of the field due to the magnet, and H the horizontal field of the earth, then the total intensity of the field around the needle when the magnet is in position is (I + H); and if n_1 denote the number of oscillations of the magnet in a given time in this field, and n the number in the same time in the earth's field, we have—

$$\frac{\mathrm{I} + \mathrm{H}}{\mathrm{H}} = \frac{n_1^2}{n^2},$$

or

$$\frac{\mathrm{I}}{\mathrm{H}} = \frac{n_1^2 - n^2}{n^2}.$$

Similarly, if any two magnetic fields, from which the field of the earth is to be excluded, are to be compared, the influence of the latter must be eliminated from each. This is done by noting the number of oscillations of the needle, first when under the influence of the earth's field only, and then when under the combined influence of the earth's field and each of the two fields considered, these latter being so arranged that their lines of force are parallel and concurrent with those of the earth. Let n denote the number of oscillations executed by the needle in a given time under the influence of the earth's field only, and n_1 and n_2 the number of oscillations in the compound fields made up respectively of I_1 and H, and I_2 and H, where I_1 and I_2 are the intensities of the fields to be compared. Here then we have—

$$\frac{\mathrm{I}_1}{\mathrm{H}} = \frac{n_1^2 - n^2}{n^2},$$

and

$$\frac{I_2}{H} = \frac{{n_2}^2 - n^2}{n^2},$$

therefore

$$\frac{I_1}{I_2} = \frac{{n_1}^2 - n^2}{{n_2}^2 - n^2}.$$

This method may be applied to determine quantitatively the distribution of free magnetism along a bar magnet. The normal component of the intensity of the magnetic field near any point on a magnet is directly proportional to the amount of free magnetism at that point. Hence, to determine the distribution of the magnetism it is only necessary to oscillate a small needle * opposite suecessive points in the length of the bar, and to compare the magnetic fields, *due to the magnet*, at these points. For this purpose the magnet, as shown in Fig. 93, is fixed vertically to the north of the needle, with its south pole pointing up, so that the needle still sets in the magnetic meridian. The number of oscillations of the needle in a given time, when placed at different distances from the end of the magnet, is now noted; and the number of oscillations executed when under the influence of the earth's field alone being known, the necessary comparisons can be made and the distribution of magnetism along the bar determined. The table given below gives the result of an experiment of this nature, and the student should apply the data supplied to draw the curve of distribution and compare it with that given in Fig. 55.

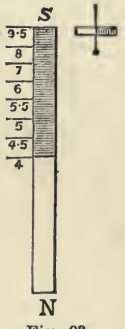


Fig. 93.

* In this case the needle must be weighted, to prevent it being attracted up to the magnet. It is to be noticed that in all these methods account must be taken of the variation of the magnetisation of the needle by inductive action. This last method especially is vitiated by this cause, the magnet acts inductively on the needle, and the amount of this induction is different at different points in the length of the magnet, diminishing from the poles towards the centre—hence the magnetic moment of the test-needle is not constant, but diminishes as it is moved from the poles to the centre of the magnet.

Determination of the Distribution of "Free" Magnetism on the North Half of a Bar Magnet.

Needle makes 4 vibrations per minute when under the influence of the earth's field only.

Distance of needle from centre of magnet.	Number of oscillations of needle.	Relative intensities of fields due to magnet.	Distribution of magnetism from centre towards N. pole.
0 cm.	4	$4^2 - 4^2 =$	0
2 ,,	4·5	$4{\cdot}5^2 - 4^2 =$	4·25
4 ,,	5	$5^2 - 4^2 =$	9
6 ,,	5·5	$5{\cdot}5^2 - 4^2 =$	14·25
8 ,,	6	$6^2 - 4^2 =$	20
10 ,,	7	$7^2 - 4^2 =$	33
12 ,,	8	$8^2 - 4^2 =$	48
14 ,,	9·5	$9{\cdot}5^2 - 4^2 =$	74·25

52. Comparison of Magnetic Moments of Magnets. The comparison of the magnetic moments of two given magnets may be effected by three convenient and reliable methods. The principles involved in these methods have already been explained; we shall therefore proceed at once to their applications.

(1) *The Torsion Balance Method.* Let A and B denote the two magnets whose moments are to be compared. Suspend A in the stirrup of the balance, so that it sets in the magnetic meridian without putting any twist on the wire. Now turn the torsion head so as to deflect the magnet, through an angle δ, out of the meridian. Let T_1 denote the twist on the wire when this adjustment is made.

Repeat these operations with the magnet B, *taking care that the final deflection from the magnetic meridian is exactly the same for each magnet.* Let T_2 denote the final twist on the wire in this case. Then, if M_1 denote the moment of A, and M_2 the moment of B, we evidently have—

$$\frac{M_1 H \sin \delta}{M_2 H \sin \delta} = \frac{T_1}{T_2}$$

for the moment of the couple due to the earth (Art. 41) is in each case proportional to the twist on the wire.

In this relation, however, H sin δ is the same for the two magnets, and therefore—

$$\frac{M_1}{M_2} = \frac{T_1}{T_2},$$

that is, the magnetic moments of the magnets are directly proportional to the respective torsions necessary to deflect them through the same angle out of the magnetic meridian.

The magnetic moment of a magnet, as defined in Art. 41, involves two quantities—the strength of the poles of the magnet and the distance between them. In general this latter quantity is uncertain, but for thin bar magnets it is approximately equal to the length of the magnet. Hence, with thin bar magnets of known absolute or relative length, the strength of their poles can be compared by this method. For example, let magnet B be, say, three times as long as A ; then if l denote the length of A, and m_1 and m_2 the strengths of the poles of A and B respectively, then—

$$M_1 = m_1 l,$$

and

$$M_2 = m_2 \cdot 3l.$$

Therefore

$$\frac{M_1}{M_2} = \frac{m_1 l}{3 m_2 l} = \frac{m_1}{3 m_2}.$$

Hence,

$$\frac{m_1}{3 m_2} = \frac{T_1}{T_2},$$

that is,

$$\frac{m_1}{m_2} = \frac{3 T_1}{T_2}.$$

(2) *The Deflection Method.* To compare the magnetic moments of two magnets by this method, it is only necessary to note the deflection produced by each magnet separately when placed *at the same distance* from a magnetometer needle, the arrangement of apparatus being exactly that described in Art. 47. Then, if M and M_2 denote the

magnetic moments to be compared, and δ_1 and δ_2 the angular deflections produced, we have—

$$\frac{M_1}{H} = \frac{d^3}{2} \tan \delta_1,$$

and

$$\frac{M_2}{H} = \frac{d^3}{2} \tan \delta_2,$$

where d denotes the distance from the centre of the magnet to the magnetometer needle in each case.

Hence we get—

$$\frac{M_1}{M_2} = \frac{\tan \delta_1}{\tan \delta_2},$$

that is, the magnetic moments are directly proportional to the tangents of the angles of deflection. As in the preceding method, if M_1 and M_2 be expressed as $m_1 l_1$ and $m_2 l_2$, then m_1 and m_2 may be compared if l_1 and l_2 or their ratio be known.

(3) *The Oscillation Method.* The comparison of the magnetic moments of *any* two magnets by this method involves considerations beyond the scope of this work. If, however, we wish to compare the moments of two magnets of exactly the same form, size, and weight, this method may be readily applied. In this case, if the magnets are freely suspended and allowed to oscillate under the influence of the earth's field, the square of the number of oscillations performed in a given time will be directly proportional to the magnetic moments of the magnets. Thus, if a magnet of moment M_1 make n_1 oscillations per minute, and another of moment M_2 make n_2 oscillations per minute, then—

$$\frac{M_1}{M_2} = \left(\frac{n_1}{n_2}\right)^2,$$

and since the lengths of the magnets are equal, we at once have—

$$\frac{m_1}{m_2} = \left(\frac{n_1}{n_2}\right)^2,$$

where m_1 and m_2 denote respectively the strengths of the poles of the two magnets.

CALCULATIONS.

53. If two magnetic poles of strengths m and m' are separated in air by a distance d, the force of repulsion exerted between them is given by—

$$f = \frac{m\,m'}{d^2}. \qquad \text{(Art. 39.)}$$

If d be expressed in cms., and m and m' in C.G.S. magnetic units, then f is given in dynes. A north pole is considered of positive sign and a south pole negative, hence a positive value of f indicates a force of repulsion and a negative value a force of attraction. The magnetic force due to any point in a magnetic field at any point is determined in magnitude and direction by the resultant force which a unit north pole would experience if placed at that point. (Art. 40.)

A pole of strength m placed in a field of intensity I experiences a force mI (dynes) in a direction tangential to the line of force passing through the point at which the pole is placed. The magnetic moment of a magnet with poles of strength m and distance l between its poles is given by—

$$\mathrm{M} = m\,l. \qquad \text{(Art. 41.)}$$

If a magnet of moment M is displaced from its position of rest in a uniform magnetic field of intensity I, so that its axis makes an angle α with the direction of the lines of force, then the moment of the couple tending to replace the magnet in its position of rest parallel to the lines of force is given by—

$$\mathrm{M\,I}\sin\alpha. \qquad \text{(Art. 41.)}$$

If the *same* magnet be caused to oscillate in *different* magnetic fields, the intensities of the fields will be directly proportional to the square of the number of oscillations performed by the magnet in each field in a given time. (Art. 41.)

If *different* magnets of the same moment of inertia be caused to oscillate in the same magnetic field, the magnetic moments of the magnets will be directly proportional to the square of the number of oscillations performed by each magnet in a given time. (Art. 41.)

When a magnet of moment M, placed "end on" at a distance d from a magnetic needle freely suspended in a uniform field of intensity I, deflects the needle through an angle α from its position of rest, then the ratio of M to I is given by—

$$\frac{\mathrm{M}}{\mathrm{I}} = \frac{d^3}{2}\tan\alpha.$$

Similarly, if the magnet is placed "broadside on," we have—

$$\frac{\mathrm{M}}{\mathrm{I}} = d^3\tan\alpha.$$

If I denote the total intensity of the earth's magnetic field, H and

V the horizontal and vertical components, and δ the angle of Dip, then we have the following relations—

$$H^2 + V^2 = I^2.$$

$$H = I \cos \delta\,; \quad V = I \sin \delta.$$

$$\frac{V}{H} = \tan \delta. \qquad \text{(Art. 47.)}$$

Examples III.

1. The force of attraction beween two equal dissimilar magnetic poles at a distance of 10 cm. from each other is 4 dynes. Find the strengths of the poles in C.G.S. units.

Here, in the relation—

$$f = \frac{m\,m'}{d^2},$$

we have $m = m'$, $d = 10$ cm., and $f = 4$ dynes.

$$\therefore 4 = \frac{m^2}{10^2}$$

or

$$400 = m^2,$$

or

$$m = 20 \text{ C.G.S. units.}$$

That is, each pole has a strength of 20 C.G.S. units, one being a north pole and the other a south pole.

2. A north pole whose strength is 10 C.G.S. units experiences a force of 20 dynes when placed at a given point in a certain magnetic field; find the intensity of this field at the given point.

Here a north pole of 10 units strength experiences a force of 20 dynes, therefore a north pole of *unit strength* would, at the same point in the field, experience a force of $\frac{20}{10} = 2$ dynes; that is, the intensity of the field at the given point is 2 dynes.

3. A magnet 10 cm. long, with poles of unit strength, is freely suspended in a horizontal uniform magnetic field of intensity 0·18 dyne; find the moment of the couple tending to restore the magnet to its position of rest when it is deflected in a horizontal plane through 30° from that position.

The moment of the couple tending to restore the magnet to its position of rest is given by M I sin α.

Here $M = m\,l = 1 \times 10 = 10$ C.G.S. units.

$I = 0{\cdot}18$ dyne.

$\alpha = 30°$.

$\therefore$ moment of restoring couple—

$$= 10 \times 0{\cdot}18 \times \sin 30°.$$

$$= 1{\cdot}8 \times \frac{1}{2} = 0{\cdot}9 \text{ (dyne cm. or C.G.S units).}$$

4. Two magnets, A and B, are caused to oscillate in the same magnetic field; A performs 15 vibrations per minute, and B 10 vibrations per minute. The magnet A is then caused to oscillate in one magnetic field and B in another; A now performs 5 vibrations per minute and B 20 vibrations per minute. Compare the intensities of the fields in which A and B now oscillate, and compare also the magnetic moments of these magnets.

From the data of the question the second field in which the magnet A oscillates is to the first field in which both A and B oscillate in the ratio—

$$\frac{5^2}{15^2} = \frac{1}{3^2} = \frac{1}{9}.$$

That is, the intensity of the second field is $\frac{1}{9}$th that of the first.

Similarly, the second field, in which B oscillates, is to the first as—

$$\frac{20^2}{10^2} = \frac{2^2}{1} = \frac{4}{1}.$$

That is, the intensity of the second field is 4 times that of the first. Therefore the ratio of the intensities of the second fields in which A and B respectively oscillate is given by—

$$\frac{1}{9} : 4 \text{ or } 1 : 36.$$

That is, the intensity of the second field in which B oscillates is 36 times as great as that in which A oscillates a second time.

From the data given in the beginning of the question we have, assuming the magnets to have the same moment of inertia—

$$\frac{\text{The magnetic moment of A}}{\text{The magnetic moment of B}} = \frac{15^2}{10^2} = \frac{3^2}{2^2} = \frac{9}{4}.$$

5. A magnet suspended by a fine vertical wire hangs in the magnetic meridian when the wire is untwisted. If on turning the upper end of the wire half round the magnet is deflected through 30° from the meridian, show how much the upper end of the wire must be turned in order to deflect the magnet 45° and 60° respectively.

Here the magnet lies in the uniform magnetic field due to the horizontal component (H) of the earth's field, and when deflected the moment of the couple tending to restore it to its position of rest in the magnetic meridian is $M H \sin \delta$, where M denotes the magnetic moment of the magnet and δ the angle of deflection from the meridian. (Art. 41.)

In each case the moment of this couple is balanced by the opposing moment due to the *torsion on the wire*; hence we have—

$$T_{30} : T_{45} : T_{60} :: \sin 30 : \sin 45 : \sin 60.$$

From the data given—

$$T_{30} = 180° - 30° = 150°.$$

Therefore,

$$150 : T_{45} :: \frac{1}{2} : \frac{1}{\sqrt{2}}, \text{ or}$$

$$T_{45} = 150\sqrt{2}.$$

That is, the twist on the wire will be $150\sqrt{2}$ degrees, and the upper end of the wire must be turned through $(150\sqrt{2}+45)$ degrees.
Also,

$$150 : T_{60} :: \frac{1}{2} : \frac{\sqrt{3}}{2}, \text{ or}$$

$$T_{60} = 150\sqrt{3},$$

and the upper end of the wire must be turned through $(150\sqrt{3}+60)$ degrees.

6. Compare the intensities of the horizontal components of the earth's magnetic field at two places A and B, given that at A the dip is 60° and the total intensity ·3 dyne, and at B the dip is 45° and the total intensity ·2 dyne.

Here, since $H = I \cos \delta$, we have at A—

$$H = \cdot 3 \cos 60^\circ = \cdot 3 \times \frac{1}{2} = \cdot 15 \text{ dyne};$$

and at B—

$$H = \cdot 2 \cos 45^\circ = \cdot 2 \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{10} = \cdot 1414 \text{ dyne.}$$

7. A declination needle makes 50 oscillations in one minute at a point on the earth's surface where the dip is 60°, and 57 oscillations in a minute where the dip is 45°. Compare the magnetic intensities at the two places.

8. Show how to compare the magnetic force exerted by the earth on a suspended needle with that exerted by an artificial magnet at a given distance. A *long* bar magnet is placed with its length in the magnetic meridian, and its north pole 20 cm. south of the south pole of a freely suspended needle. If the needle oscillates 25 times per minute when the magnet is not in position, and 100 times per minute when under the influence of the magnet, find the strength of the magnet pole, assuming $H = 0{\cdot}18$ dyne and neglecting the action of the south pole of the magnet.

9. A magnetic needle, the magnetic moment of which remains constant, is suspended so as to move freely in a horizontal plane. When deflected from the magnetic meridian at three different places on the earth's surface, it is observed to oscillate 7·5, 8·3, and 10·4 times respectively in one minute. Compare the intensities of the earth's horizontal magnetic force at the three places.

10. What is meant by the three magnetic elements at any point on the earth's surface? Let the horizontal force be 3·8, and the vertical force 8·5. Find the total magnetic force.

11. Explain how to determine the ratio of the magnetic moment of a magnet, to the earth's horizontal magnetic force

PART III.

CURRENT ELECTRICITY.

CHAPTER X.

INTRODUCTORY.

54. Voltaic Cells and their Action. From Part I., the student has already learnt that electricity flows along conductors—thus, if a positively charged body be connected to the earth by means of a long copper wire, the positive electricity at once flows along the wire, and the body is discharged. In this case the flow lasts for a mere instant of time, but if it were possible to supply electricity to the body as quickly as it loses it, then we should have a continuous *current* of electricity along the wire. It is found possible by chemical means to maintain for some time a constant supply of electricity, and the arrangement employed for this purpose is known as a *voltaic cell.* It was discovered by Volta (hence the name *voltaic*) that if a strip of copper and a strip of zinc be placed in a vessel containing dilute sulphuric acid, and connected by a wire, then a current of electricity is maintained through the wire. This arrangement (Fig. 94) is the simplest form of voltaic cell, and we have now to consider the explanation of its action.

As to the two metals, zinc and copper, the essential

chemical difference between them is that zinc is more easily oxidised than the copper, that is, zinc has a greater attraction for oxygen than copper has. Again, consider the liquid in which the metals are placed,—it is practically acidulated water,—and therefore its molecules are principally water molecules containing two atoms of hydrogen and one of oxygen. Now, it is supposed that the atoms of any one molecule are not always the same, but are continually changing, that is, there is a constant interchange of atoms among the molecules, and thus there are instants at which a given oxygen atom in passing from one molecule to another is free, or *dissociated* from the molecules of the liquid. Hence, considering the immense number of molecules in the liquid, it is evident that there must *always* be a large number of dissociated atoms present.

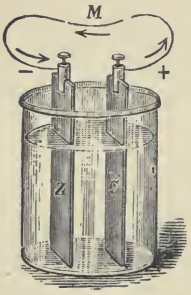


Fig. 94.

Now both zinc and copper attract oxygen atoms, and when placed in the liquid these metals attract all dissociated oxygen atoms coming within the range * of their attraction. In this way both the copper and the zinc attract the oxygen atoms in the liquid, but the attraction of the zinc is greater than that of the copper. Now there is a good deal of evidence in support of the hypothesis that all atoms are charged, some positively and some negatively; oxygen is supposed to be *electro-negative*, that is, its atoms are supposed to be negatively charged, and hydrogen is the type of the *electro-positive* elements.

Hence, as the metals combine with the attracted oxygen atoms, they become negatively charged, and soon repel the free oxygen atoms electrically, as strongly as they attract them chemically. Equilibrium is thus quickly attained, and both metals are negatively charged; but the attraction of the zinc for oxygen being much greater than that of the

* The range of this molecular attraction is very small—perhaps about the ten-millionth part of a millimetre.

copper, the charge on the zinc is greater than that on the copper.

So far, then, the effect of immersing the metals in the acid is to charge both negatively, but the zinc to a greater degree than the copper, which is thus at a higher (positive) potential than the zinc. Hence, if the two metals be now joined by a piece of wire, positive electricity at once flows from the copper to the zinc, or negative electricity flows from the zinc to the copper, and the equilibrium in the cell is at once destroyed. The zinc plate becomes *less* strongly charged with negative electricity, and therefore again attracts adjacent oxygen atoms, while the copper plate becomes *more* strongly charged with negative electricity, and therefore repels instead of attracting the free oxygen atoms near it. In this way a continuous current of *negative* electricity is maintained from the zinc through the wire to the copper, and back to the zinc through the liquid, as a sort of convection current of negatively charged oxygen atoms. Similarly, by considering zinc to *repel* hydrogen *less* than copper does, we may consider the current, as a flow of *positive* electricity from the copper through the wire to the zinc, and back to the copper through the liquid, as a convection current of *positively* charged hydrogen atoms. Since the flow of oxygen atoms through the liquid in one direction necessarily implies a flow of hydrogen atoms in the opposite direction, this explanation of the action of a voltaic cell points to the fact that a current possibly consists in a transfer of positive and negative electricity in opposite directions. It is, however, usual to consider only the "positive current," and the current is said to travel in the direction in which positive electricity is supposed to be carried. Thus, in the simple cell shown above, the current, starting from the copper plate, passes round its *circuit* through the wire to the zinc plate, and then through the liquid back to the copper plate. The copper plate is called the *positive pole* of the cell, and the zinc plate the *negative pole*; in the wire or *external circuit* of the cell, the current passes from the copper to the zinc, or from the positive to the negative pole, while in the liquid or *internal circuit* it

passes from zinc to the copper, or from the negative to the positive pole.

The metals here used as the elements of the cell are copper and zinc, but any two substances which attract oxygen to markedly different extents may be employed—thus zinc and platinum, zinc and carbon, copper and carbon, zinc and silver, iron and platinum, and many other pairs of substances, act more or less efficiently as the plates of a voltaic cell. In fact, if the metals be arranged in order of their chemical affinity for oxygen, and any two of the series be chosen as the elements of a cell, the efficiency of the cell will be greater the further apart these metals are in the series. Thus, the following is a list indicating decreasing affinity for oxygen, and from it we see that a cell having plates of zinc and copper is less efficient than one having, say, zinc and carbon as its elements—further, if a cell be made up with any two substances on the list, the higher on the list, having the stronger affinity for oxygen, must always be the negative pole of the cell.

1. Magnesium.
2. Zinc.
3. Lead.
4. Tin.
5. Iron.
6. Copper.
7. Silver.
8. Gold.
9. Platinum.
10. Carbon.

The chemical action* of the cell should here be considered. The zinc on combination with the oxygen becomes zinc oxide, and as this is formed it is acted on by the sulphuric acid present, and zinc sulphate is formed. In this way the zinc and the acid are used up and zinc sulphate formed, the energy of the electric current produced being the equivalent of the chemical work done in the cell. The copper is not chemically acted on to any extent.

55. Polarisation. In theory this simple cell is quite satisfactory and should work well, but in practice it is found

* The student who is ignorant of Chemistry should, before proceeding further, read up the elementary facts of the science from some simple text-book such as Roscoe's "Primer of Chemistry."

that after working for a few minutes its action is greatly weakened and may ultimately stop.

Experimental investigation into the cause of this defect has shown that it is due to the fact that a layer of hydrogen bubbles collects on the copper plate. This layer of hydrogen spoils the action of the cell in two ways: first, it interposes a thin insulating layer in the path of the current, and thus an additional resistance is opposed to the passage of the current round its circuit; second, the hydrogen bubbles attached to the copper plate attract the oxygen atoms in the liquid even more strongly than the zinc plate does, and thus the layer of gas, acting in combination with the zinc plate, opposes the action of the zinc and copper plates. In fact the action of the cell is now of a double nature—the zinc and copper plates, owing to their different attractions for oxygen, cause a current to travel in the external circuit from the copper to the zinc; in a precisely similar way, the hydrogen layer and the zinc plate, by virtue of their different attractions for oxygen, tend to produce a current in the external circuit from the zinc to the hydrogen, that is, in the opposite direction to that due to the zinc and copper plates. In this way the total current produced by the cell is greatly diminished, and may be stopped altogether if the hydrogen be allowed to accumulate to such an extent as to interpose a completely insulating layer in the circuit.

This defect in the action of the cell is known as **polarisation**, and the various modifications of Volta's simple cell have been devised with the intention of removing this defect as completely as possible.

56. Various Forms of Cells. The most obvious method of preventing polarisation is to remove the layer of hydrogen bubbles mechanically. Thus, if the copper plate be repeatedly brushed, the cell continues to act satisfactorily, and does not become polarised. This, however, is a troublesome method, and Smee was the first to suggest an improvement. He constructed a cell having a platinised silver plate instead of a copper plate. The rough surface formed by the finely divided platinum which was deposited on the plate allowed the hydrogen bubbles to escape freely from

its numerous sharp points, and thus polarisation was in a great measure prevented without the trouble of brushing the plate.

These mechanical methods of preventing polarisation were, however, not very satisfactory, and it was next proposed to adopt a chemical method of preventing the accumulation of the hydrogen on the positive plate.

Poggendorff's bichromate cell is constructed on this principle. It is made up with zinc and carbon elements, but the fluid—a solution of potassium bichromate mixed with strong sulphuric acid—is one which, on account of its powerful oxidising properties, rapidly oxidises the hydrogen, and thus prevents polarisation. Fig. 95 shows the usual form of the cell—the two outer plates shown are of carbon (gas coke), and *together* form the positive plate of the cell. The middle plate is of zinc, and can be raised out of the liquid, by means of a sliding rod to which it is attached, when the cell is not in use. The plates are connected to the *binding screws*, *a* and *b*; the screw *a* is connected with both carbon plates, forming the positive pole of the cell, and *b*, connected with the zinc through the brass sliding rod, forms the negative pole. Hence, when a piece of wire connects *a* and *b*, a current at once passes from *a* to *b*, but no current passes unless this connection be made. The defect of this cell is that, owing to the action of the hydrogen and zinc on the liquid, the latter quickly changes in character and gradually becomes less and less efficient, so that for a short time the cell gives a good current, which quickly falls off, though it recovers considerably if the cell be put out of action for some hours.

Fig. 95.

A more effectual method of preventing polarisation is afforded by the use of *two-fluid* cells. The cells described above are all made up with one fluid, but in the two-fluid cells two fluids are used, one to act on the zinc, the other to deal with the hydrogen tending to accumulate on the

positive plate. The two fluids are separated by a porous partition, which allows diffusion to go on, but prevents direct mixture of the liquids. *Bunsen's cell* is a good example of this class. The plates are zinc and carbon arranged in two cells—the zinc in an outer one containing dilute sulphuric acid, and the carbon in an inner *porous* cell containing *strong nitric acid.* Fig. 96 shows the arrangement and general appearance of the cell; but as it is absolutely essential for the student to see, handle, and work with these cells, we shall not waste space on a detailed description. The chemical action in the cell is readily understood. The zinc acted on by the *dilute* acid gives zinc sulphate and sets free hydrogen; thus—

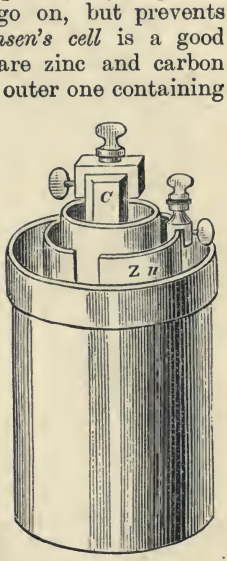


Fig. 96.

$$\underset{\text{Zinc}}{Zn} + \underset{\substack{\text{(Sulphuric}\\\text{Acid)}}}{H_2SO_4} = \underset{\substack{\text{(Zinc}\\\text{Sulphate)}}}{ZnSO_4} + \underset{\text{(Hydrogen)}}{H_2}.$$

Possibly, as explained above, this reaction takes place in two steps—the first involving the oxidation of the zinc and the liberation of hydrogen, and the second the formation of zinc sulphate; thus—

$$\underset{\text{(Water)}}{H_2O} + Zn = \underset{\text{(Zinc Oxide)}}{ZnO} + H_2. \qquad (1).$$

$$ZnO + H_2SO_4 = ZnSO_4 + H_2O. \qquad (2).$$

Whatever be the exact nature of the reaction, free hydrogen is certainly liberated, and travels by alternate combination and dissociation towards the positive pole. In this cell, however, before reaching the carbon plate, it passes through the wall of the porous cell and meets the nitric acid. Now nitric acid being a strongly oxidising agent, the hydrogen is rapidly oxidised, causing reduction of the acid. The lower oxides of nitrogen resulting from this reduction are mostly dissolved in the acid, and thus prevented from escaping into the air.

Grove's cell (Fig. 97) is essentially the same as Bunsen's, the only difference being that a sheet of platinum foil is used instead of the plate of carbon.

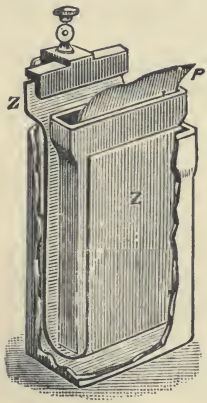


Fig. 97.

Daniell's cell is one of the most familiar of two-fluid cells, and differs from the Bunsen and Grove cells in the fact that the hydrogen is not got rid of by oxidation, but by a substitution reaction. It is made up with copper and zinc plates, and the two fluids employed are dilute sulphuric acid (or a semi-saturated solution of zinc sulphate) and a saturated solution of copper sulphate. In the form that the cell usually takes (Fig. 98) the outer cell is of copper and forms the positive plate of the cell. This cell contains the copper sulphate, and is usually fitted with a ledge, on which a quantity of crystals are placed, which serve to keep the solution saturated. The zinc plate or rod is fitted by means of a wooden stopper into the inner porous cell, which contains the acid or solution of zinc sulphate. The action of the cell is as follows: first in the inner cell the zinc and dilute acid act on one another so as to give zinc sulphate and free hydrogen—

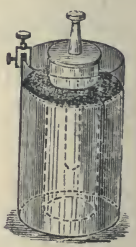


Fig. 98.

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

This free hydrogen passes from molecule to molecule through the acid towards the copper, until it meets the porous cell wall. Here it soon encounters a molecule of copper sulphate ($CuSO_4$) and at once displaces the copper; thus—

$$\underset{}{H_2} + \underset{\text{(Copper Sulphate)}}{CuSO_4} = \underset{}{H_2SO_4} + \underset{\text{(Copper)}}{Cu}.$$

The displaced copper atom then travels from molecule to molecule, until it reaches the copper plate on which it is

deposited. Hence, instead of a layer of hydrogen forming on the plate, a thin film of pure copper is deposited on it, and as a consequence no polarisation results, for the copper film is an excellent conductor, and being of the same material as the plate on which it is deposited, there is no tendency to set up an opposing current.

The *Leclanché cell* (Fig. 99) is a convenient cell now greatly used; its elements are zinc and carbon. The zinc, in the form of a rod, is placed in the outer cell in a solution of ammonium chloride, and the carbon in the inner porous cell is packed round with black oxide of manganese and pieces of carbon. The solution of ammonium chloride acts on the zinc, producing a double chloride of zinc and ammonium, and liberating ammonia gas and hydrogen. The hydrogen, on penetrating into the inner cell, is slowly oxidised by the manganese dioxide. This oxidising action goes on very slowly, so that if the cell is worked for some time it becomes partially polarised, but gradually recovers if left to itself.

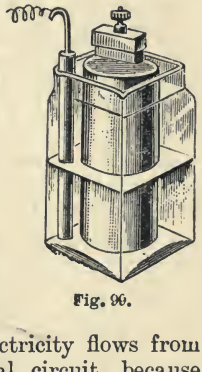


Fig. 99.

57. Electromotive Force. In the description given above it has been stated that on joining the copper and zinc of Volta's simple cell by a conducting wire, a current of positive electricity flows from the copper to the zinc in the external circuit, because the copper is at a higher positive potential than the zinc. That is, the flow is due to the *difference of potential* of the plates of the cell, and this difference of potential, to which the current is due, is the measure of the **electromotive force** of the cell. Generally defined, electromotive force * is whatever causes motion of electricity. Difference of potential is an electromotive force, but it is not the only one. Just as the difference of potential energy,

* The term *force* is used from analogy to mechanical force, which causes motion of *matter*, but it does not imply that electromotive force is a force in the dynamical sense of the term.

due to difference of level in a liquid, produces flow of the liquid, so difference of potential causes flow of electricity; but a flow of liquid can be produced by other means than by the aid of difference of level, for example, by the use of a force pump; so with electricity a current may be produced without being primarily due to difference of potential. Now, a voltaic cell of any kind is practically an electricity pump, the seat of its electromotive force lying in the unexplained attraction of zinc (or other electropositive metal) for oxygen; and when the circuit of the cell is complete this electromotive force drives a current of electricity round it, but if the circuit is incomplete, then the cell works until a difference of potential is set up at its poles which is equivalent to the electromotive force to which it is due. Hence, when the circuit of a cell is *open*, that is, when its poles are not connected by a conductor, the difference of potential between the poles measures the electromotive force of the cell; but if the circuit is closed, then the electromotive force is spent in driving the current round the circuit and cannot therefore set up a difference of potential equal to itself between the poles of the cell. The action of the cell will probably be more easily understood by considering the following hydrostatic analogy. Let A B C D (Fig. 100)

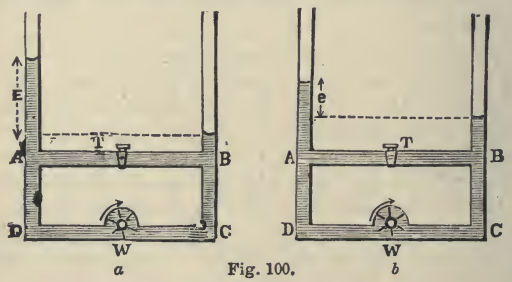


Fig. 100.

represent a circuit of pipes filled with water, and having a stopcock at T, and a small paddle-wheel at W. By rotating the paddle wheel the water can, when T is open, be driven round the circuit in the direction CDAB; but if T is closed,

then no current can be produced, but the water is driven into the tube DA, until the pressure due to the difference of level in the tubes DA and CB is sufficient to balance the force exerted by the driving wheel at W. The left-hand drawing in Fig. 100 indicates this case, and the difference of level, E, is a measure of the driving force of the paddle wheel. When, however, the cock T is opened and a current established in the circuit, the levels in the tubes DA and CB change in the way shown in the right-hand drawing of the figure, and *the difference of level* in the vertical tubes is now considerably reduced. The work done by the paddle wheel is now spent in driving the current of water round the circuit, and the difference of the levels at A and B (e) is a measure of that portion of the driving force of the wheel which is spent in driving the current through the tube AB. Now this hydrostatic arrangement roughly illustrates the action of a voltaic cell—the pressure exerted by the paddle wheel at W corresponds to the electromotive force of the cell; and if AB represent the external portion of the circuit, the difference of levels at A and B represents the difference of potential at the poles of the cell. Hence, we see that when the circuit is open and no current flows, the potential difference at the poles is a measure of the electromotive force of the cell; but when the circuit is closed, it is only a measure of that portion of the electromotive force which is spent in driving the current through the external portion of the circuit. Further, the work done in driving the paddle represents the chemical work done in the cell, and the energy of the current of water which is equivalent to the work done on the wheel corresponds to the energy of the electrical current resulting from the chemical work done in the cell. If the tubes A B C D be filled with small shot, the analogy to an electric circuit will be more complete, for the shot may be taken to represent the molecules of the conductors composing the circuit, and the resistance which they offer to the current of water corresponds to the **electrical resistance** which even the best conductors offer to the passage of electricity through their substance. Moreover, just as in one case the energy of the stream of water is spent in doing

work against the friction of the shot, so, in the case of an electric circuit, the energy of the current is expended in doing work against molecular friction, and in both cases transformation of energy takes place and the energy of the current is dissipated as heat in the circuit.

Considering the question of electrical resistance introduced above, it is evident that from its assumed nature the resistance of a given conductor will vary with the substance of the conductor, and also with its dimensions. Thus, similar conductors of different material will have different resistances, and for a given material the resistance will vary with the length of substance which the current has to traverse, and with the area of cross section of the path along which it travels. The greater the length traversed the greater the resistance offered to the current, but the greater the cross section of the conductor the smaller the resistance, for if the hydrostatic analogy holds, the resistance will be less for a wide than a narrow path. Now, an essential part of every circuit is the cell itself, and from what has been said it is evident that the resistance of the cell is that offered by the layer of liquid or liquids between the two plates, and must therefore depend upon these plates and their distance apart—the larger the plates and the

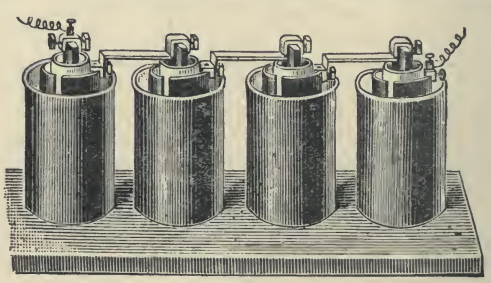


Fig. 101.

nearer they are together the smaller will be the resistance. The arrangement of the plates in Figs. 95-97 will now be better understood—the doubling of the carbon plate in

Fig. 95 and of the zinc plates in Figs. 96, 97 not only adds to the compactness of the cell, but also by increasing the size of the plates diminishes the resistance in it without unduly increasing its size.

58. Batteries of Cells. In practice cells are seldom used singly, but usually in *batteries* of two or three, or more, suitably connected. Fig. 101 shows a battery of four Bunsen cells arranged *in series*; that is, commencing with the end cell on the left hand, the negative pole is connected to the positive pole of the next cell, and the negative pole of that cell is in turn connected to the positive of the third, and so on.

The *poles of the battery* are the free poles of the *end*

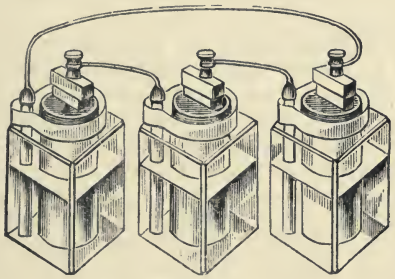


Fig. 102.

cells—Fig. 102 shows a similar arrangement of Leclanché cells with its circuit closed by a wire joining its poles.

In this series arrangement the electromotive force and the resistance *of the battery* are proportional to the number of cells employed. For example, if the battery be made up of five similar cells, then the electromotive force of the battery is five times that of one cell, and its resistance is also five times the resistance of a single cell. If the cells are not similar, then the electromotive force of the arrangement is the sum of the electromotive forces of the constituent cells, and, similarly, the resistance is the sum of the individual resistances of the cells.

Another arrangement frequently adopted is that shown in Fig. 103. The four cells shown are said to be arranged "*in parallel*" or "*for quantity*"—all the positive poles are connected together and all the negative poles separately, and the four cells are thus practically converted into one cell four times the size of a single cell. Hence, the electromotive force of the combined cells is the same as that of one, but the resistance of the arrangement is only one-fourth that of a single cell. Since in this arrangement all similar poles are connected together and practically made one, *any* two unlike poles may be taken as the poles of the battery.

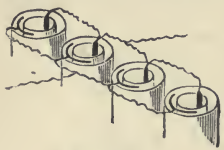


Fig. 103.

59. Local Action. There is one point of some practical importance in the use of cells which should here be noticed. It is well known that ordinary commercial zinc is readily dissolved by dilute sulphuric acid with formation of zinc sulphate, whereas *pure* zinc is only very slightly attacked. Now the zinc plates used in cells are always made of ordinary commercial zinc, and would therefore be quickly used up if some precaution were not adopted to prevent their dissolving in the acid when the cell is not giving a current. It is impracticable to use pure zinc plates, but it is found that by amalgamating * the ordinary plates with mercury, they are protected from the action of the acid, except when the cell is giving a current. The chemical action that goes on in a battery with unamalgamated zincs, when the circuit is open and no current flows, is called *local action*, and its energy is spent in maintaining a multitude of small currents. These circulate between portions of the impure zinc plates, which differ from each other, either chemically or physically, to such an extent as to act as the plates of minute cells. The energy resulting

* Zinc plates are readily amalgamated by rubbing them alternately with dilute sulphuric acid and mercury. A pad of wash-leather mounted on a handle may be conveniently adopted for this purpose. Very little mercury should be used.

from the solution of the zinc in the acid is thus wasted in local work, and it is to prevent this waste of energy that we resort to amalgamation.

60. Voltaic Pile and Crown of Cups. In investigating the action of voltaic cells, Volta constructed an interesting apparatus known as the *Voltaic pile* or the *dry pile*. It usually takes some such form as that shown in Fig. 104, and is made by arranging alternately in a pile zinc and copper discs, separated by discs of felt or flannel soaked in brine; each couple acts as a small cell, and if arranged so that the zinc of one couple is in contact with the copper of the next, the pile acts as a battery made up of a large number of separate cells. Such a pile may have a very considerable electromotive force, equal to the sum of the individual electromotive forces of the component cells, but its resistance will also be large, so that it cannot give a strong current. Another arrangement devised by Volta was his *Crown of Cups*,—this was merely a battery made up of a large number of the simple zinc and copper cells described above.

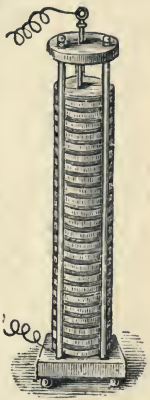


Fig. 104.

61. Essentials of a Good Cell. Having now described the voltaic cell in some detail, it may be instructive to consider what are the essentials of a good cell, and to notice how far the cells described fulfil the necessary conditions. A good voltaic cell should meet the following requirements :—

(1) Its electromotive force should be high and constant.

(2) Its resistance should be small.

(3) It should be free from polarisation.

(4) It should give a constant current for a considerable time, and should therefore have a good supply of suitable working materials, which should not be rapidly exhaustible.

(5) No chemical action should go on in it except when the current is passing.

(6) It should be convenient and economical in use.

None of the cells described above fulfil all these conditions,

but each of them fulfils some of them; and in order to choose the proper cell for use under given conditions, it is necessary to know their several characteristics. Bunsen and Grove cells fulfil conditions (1) and (2), and consequently give a strong current, but unless they are of large size the current will not remain constant for more than a few hours. They emit a small quantity of corrosive and suffocating fumes, and should therefore not be used where the fumes may give rise to annoyance, or attack the metal work of instruments or fittings. Daniell's cell has a constant but not very high electromotive force (rather more than half that of a Bunsen or Grove), and its resistance is rather high, but when in good order it will give a constant current of medium strength for a very long time. It is convenient to use, and emits no fumes whatever. The electromotive force of a Leclanché cell is somewhat higher than that of a Daniell, and its resistance is also higher, but its current is by no means constant. As already explained, it becomes polarised when in continued use, but as it recovers gradually and unlike the other cells, may always be kept made up ready for use, it is one of the most convenient cells when a fairly good current is wanted for only a few minutes at a time, for example, in ringing an electric bell.

Volta's simple cell is of theoretical interest only, and Poggendorff's bichromate cell is not greatly used, because, although its electromotive force is high and its resistance low, its current, though strong at first, quickly diminishes almost to zero. Care also has to be taken always to remove the zinc out of the liquid when the cell is not in use, but Fuller's two-fluid form of cell is now much used in the Post Office.

62. Volta's Experiment. In his study of the action of a voltaic cell Volta performed the following experiment, which is of historic interest:—

A bar formed by soldering together a short rod of zinc and a short rod of copper end to end was held, say, by the copper end (Fig. 105), and the zinc end was put in contact with the lower plate of a condensing electroscope.

On touching the upper plate of the electroscope, removing the bar in contact with the lower plate, and then raising

the upper plate, the leaves of the electroscope were found to diverge with *positive* electricity. On repeating the experiment with the bar reversed, so that the copper end is in contact with the lower plate of the electroscope, it was found on removing the upper plate that the leaves diverged with negative electricity.

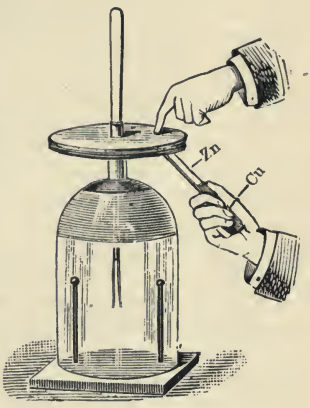


Fig. 105.

63. Contact Theory and Explanation. These results indicate that electrical separation takes place somewhere, and since the zinc is apparently charged positively, and the copper negatively, Volta framed the theory that the separation was due to the *contact* of the zinc and copper; and on this and similar experiments subsequent experimenters have based the *contact theory* of voltaic electricity, which states that the seat of the electromotive force of a voltaic cell lies in the contact of the dissimilar metals employed as its elements. Modern investigation, however, goes far to show that this theory must be abandoned—contact of dissimilar metals or other substances does give rise to electrical separation, but not to *the* separation we have to deal with in a voltaic cell. For metals the difference of potential established by contact may be exceedingly minute compared with the electromotive force of a cell, and has been held to have little or nothing to do with voltaic currents, in the ordinary meaning of the term. The most probable explanation of the action of a voltaic cell is given in the preceding article, and an explanation of the experiment just described can be given in an exactly similar way. Consider a piece of zinc (Fig. 106) surrounded by air—here, just as when immersed in dilute acid, it attracts the neighbouring oxygen atoms, but in the liquid these atoms,

while dissociated, are free to obey this attraction, whereas in air, *where there is no dissociation*, they are not free to do so. Hence, the oxygen of the air *tends* to combine with the zinc, but is not free to do so; the negative electricity of the zinc however is, as it were, under pressure by the wall of *negatively* charged oxygen atoms surrounding the zinc and endeavouring to obey the attraction to which they are subjected. The zinc thus practically becomes negatively charged, and its potential is lower than that of the surrounding air, but, as the charge is due to molecular attraction, the slope of potential up from the zinc does not extend indefinitely into the surrounding medium, but is confined to the film of attracted molecules surrounding the zinc. Considering again the case of a piece of copper surrounded by air, all that has been said applies exactly; only, since the attraction of copper for oxygen is less than that of zinc, the "pressure" on the negative electricity in the copper is less than in the zinc, that is, the slope of potential from the air to the copper is less than from the air to the zinc. Now if the pieces of copper and zinc be put in contact, as in Fig. 107, the negative electricity, being under greater pressure in the zinc than in the copper, at once flows to the copper, leaving the zinc positively charged and charging the copper negatively, and thus equalising the potentials of the *metals*, but leaving a difference of potential between the outer surfaces of the films of air surrounding the metals. Thus, in Fig. 108, let *aa* represent the potential of the air surrounding the metals before they are put in contact, and

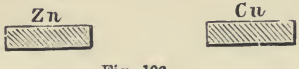


Fig. 106.

Fig. 107.

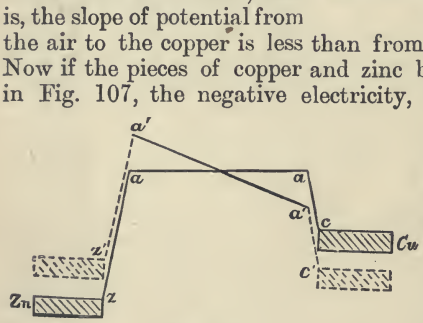


Fig. 108.

let az and ac represent the slopes of potential from the air to the zinc and copper respectively. On joining the metals, however, the zinc receives a positive charge (loses a negative charge) and the copper a negative charge, and in consequence their potentials respectively rise and fall by the same amounts * to the dotted positions z' and c', thus attaining the same potential; and as the slopes of potential $a'\ z'$ and $a'\ c'$ remain nearly equal to az and ac, a slope of potential $a'\ a'$ is set up in the intervening medium. In the case of a voltaic cell the intervening medium is a conductor and not an insulator; and therefore instead of a slope of potential being produced in it a current is set up through it.

This experiment of Volta's is thus merely an illustration of the action of a voltaic cell in which the medium which surrounds the plates is an insulator, and not a liquid which conducts by virtue of the dissociation which goes on in it. A liquid of this latter kind is usually called an *electrolyte*.

* Assuming the pieces of metal to be of equal capacities.

CHAPTER XI.

GENERAL EFFECTS OF CURRENTS.

64. Introductory. In the preceding articles we have spoken of a current of electricity flowing round the circuit of a voltaic cell, but we have not stated how this current manifests itself, nor how its existence can be detected. In the cell itself, it is true, chemical action indicates the passage of a current; but as it is necessary to be able to detect and even to measure the current in any conductor, we must consider whether the passage of a current of electricity through that conductor has any effects which we may take advantage of for this purpose.

There are two broad classes of conductors to be considered. The first consists of those conductors which conduct electricity like they conduct heat, with little or no permanent effect on their physical or chemical state. The second comprises *electrolytic conductors* or *electrolytes*, in which the passage of current is accompanied by chemical action.

Of the first class, metallic conductors afford typical examples, and, as we shall have to deal mostly with conductors of this kind, we shall here consider a few experiments illustrating the effects attendant on the passage of a current through metal wires.

65. Heating Effect. Let the poles of a Bunsen cell be joined by a short piece of iron wire about half a millimetre in diameter. When the connection is made good, and the current flows through the wire, it will be found, on holding it between the fingers, that it gradually becomes hot, and before long may become too hot to touch. This

proves that one of the effects of passing the current through the wire is to heat it; and by varying the experiment a little, we may learn something more of this **heating effect** of the currents. Let the piece of wire joining the poles of the cell be replaced by a much thinner piece of the same length and composed of the same material. It will now be found that the heating effect is much more evident, for the wire quickly becomes red hot. This seems to point to the fact that the heating effect varies with the resistance which the conductor offers to the passage of the current, for the only difference between the two wires is one of diameter; and as the thin wire offers a much narrower path to the current than the thick one, its electrical resistance, as explained above, must necessarily be greater. Again, if thin wires of the same dimensions but of different material be employed, it will be found that the heating effect is different for each. For example, let short, thin wires of copper, iron, brass, and platinum be tried: it will be found that, with the exception of the copper wire, all the wires quickly become white hot. The copper wire may slowly attain a dull red heat, and of the other three the platinum heats most quickly, the iron next, and the brass at nearly the same rate. In each of these experiments approximately the same current is used, but the same results may be exhibited in a more satisfactory way as follows. Take short pieces of thick copper wire, thin copper wire, platinum wire, iron wire, and brass wire, and fasten them end to end, so as to form one compound conductor. This is most onveniently done by the help of small binding screws of the form shown in the figure. If the poles of a battery of three or four Bunsen cells be joined by this conductor, then precisely the same current passes through each wire composing it, but the heating effect in each will be very different. The platinum, iron, and brass wires will, at slightly different rates, become white hot, the thin copper wire will be nearly red hot, but the thick copper wire will be only slightly heated, and the thick brass binding screws

Fig. 109.

may not be perceptibly heated at all. From these experiments we may deduce the following general results :—

(*a*) The passage of a current through a conductor heats it.

(*b*) For conductors of the same material the amount of heat developed in a conductor by a given current varies with the resistance of the conductor. The greater the resistance, the greater the amount of heat developed, but further experiment is necessary to determine the quantitative nature of the relation.

(*c*) The amounts of heat developed by a given current in similar conductors of different material is different. This points to the fact that electrical resistance varies with the material of the conductor, and that the amount of heat developed in it will be small or great, according as it is composed of good or bad conducting material. Looked at in this way, this result is practically the same as the preceding one—here the difference of resistance is due to difference of material, in (*b*) it is due to difference in dimensions. As a corollary to this result, it may be noticed that copper appears to be a good conducting material, while brass, iron, and platinum, though readily conducting the current, offer considerable resistance to its passage.

(*d*) When a conductor is made up of several different parts of different conducting powers, the heat developed in any portion of it depends upon the resistance offered *by that portion* to the passage of the current.

If the experiments described above be repeated with, say, Daniell or Leclanché cells to supply the current, the same general results will be obtained, but the heating effects noticed will be very slight. In these cases the current used will be much weaker than in the first case, and we may therefore infer that—

(*e*) The heat developed in a given conductor varies with the current used. The stronger the current, the greater the amount of heat developed, but the quantitative nature of this relation has yet to be found.

It is probable that the passage of a current through a conductor affects most of the physical properties of its material, but the heating effect is the only one we need

consider at present. Its importance will be realised from the following considerations. Imagine a Bunsen cell working, with its poles joined by a piece of wire. So long as the cell is working, chemical energy is being used up in the cell, and a question arises as to what becomes of this energy. The answer is, that it produces an electrical current which develops heat in the circuit, the amount of heat energy evolved being equivalent to the chemical energy used up in the cell, and thus furnishing a measure of the energy of the current. In this case, the circuit is made up of the piece of wire and the cell itself: part of the heat is developed in the wire and the remainder in the cell, for every conductor which offers resistance to the passage of the current must become heated when the current passes through it, the amount of heat developed in it being equivalent to the work done by the cell in driving the current through it.

66. Magnetic Effects. The passage of a current through a conductor is attended by another effect which is even more noteworthy than the development of heat. It is found by experiment that a conductor which is carrying a current has a magnetic field surrounding it, and consequently currents are capable of influencing magnets, and may be employed for the magnetisation of magnetic substances.

In order to exhibit the magnetic field of a current, let a hole be bored in a piece of glass or cardboard, and let a wire carrying a strong current be fitted through it, as in Fig. 110, with its length at right angles to the horizontal surface of the glass. Then, on sprinkling iron filings over the glass and gently tapping, the filings at once arrange themselves in curves of force, taking the form of concentric circles round the wire as their common centre. This arrangement is shown in Fig. 111,

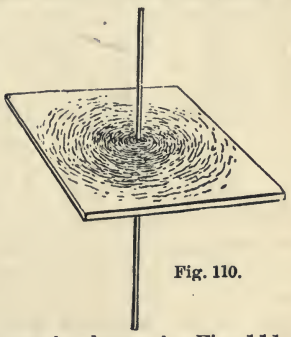


Fig. 110.

where the current is supposed to come up through the centre of the field, in a direction at right angles to the plane of the paper. The direction of the magnetic force at any point in the field due to the current is thus seen to be at right angles to the plane determined by the direction of the wire and the point considered.

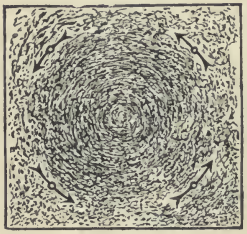


Fig. 111

Thus, if w (Fig. 112) denote the section of a wire running vertically through the paper, the direction of the magnetic force at A due to a current down through the wire is along T T′, the tangent to the curve of force A A′ which passes through the point A. That is, the direction of the force is at right angles to the vertical plane represented by the trace AwA′ passing through the wire and the point A. If the current in the wire flows down into the paper, the experiment shows that the magnetic force at A is along AT, but if the current flows upwards, out of the paper, it is along AT′. Hence, if a magnetic needle be placed, as in the figure, with north pole at A, it is in the first case directed to the left and in the second to the right.

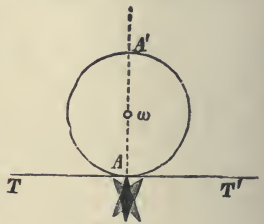


Fig. 112.

This effect is more strikingly shown by an experiment due to Œrstedt. A wire carrying a current is held, as in Fig. 113, over a magnetic needle with its length parallel to the axis of the needle. The poles of the needle at once experience the magnetic force due to the current, and the needle, acted on by a horizontal couple, is deflected out of the magnetic meridian until it reaches a position of equilibrium, where the moment of the couple due to the magnetic field of the current is balanced by the moment of the couple due to the magnetic field of

the earth. If the direction of the current be from north to south, then the north pole of the needle is urged eastwards, but if the current flow from south to north, the

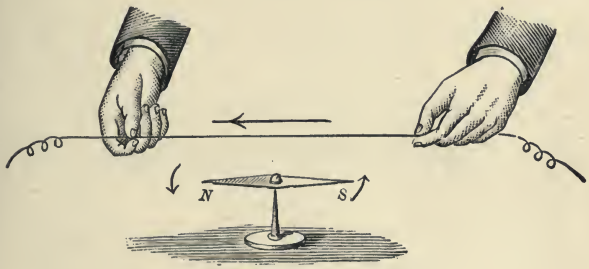


Fig. 113.

north pole is urged towards the west. If the current pass below the needle, then a consideration of the conditions shows that these effects are reversed. As it is somewhat puzzling to deduce or remember these results, the following rule due to Ampère, and commonly called *Ampère's rule*, will be found very useful: *To determine the direction in which the* **north pole** *of a needle will be urged by the magnetic action of a current, imagine a man swimming* **with** *the current with his face turned so as to look at the north pole considered, then that pole will in all cases be deflected towards his* **left** *hand.*

Another convenient form of this rule is this. If one look along a conductor, *in the direction in which the current flows*, then the direction in which a *north* pole would be urged round the circular lines of force in the field is the same as that of the hands of a watch.

To consider the magnetising action of a current, let a wire insulated by a gutta percha covering be coiled round a bar of iron or steel in the way shown in Fig. 114, and let a current be passed through the wire. Immediately the current is established the bar becomes a magnet while the current passes. If made of steel the bar remains

permanently magnetised after the current is stopped, but if made of iron the magnetisation is more or less temporary according to the quality of the iron, so that with a soft

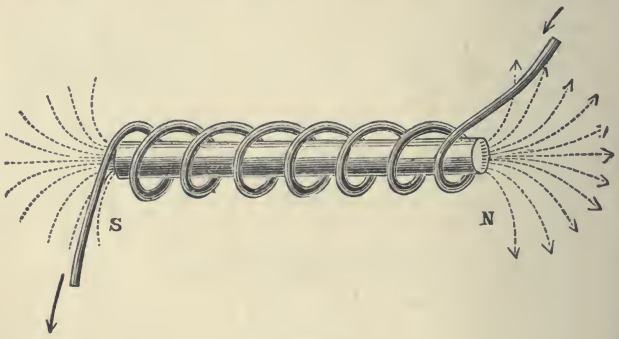


Fig. 114.

iron core the magnetisation comes and goes with the current.

This method is the most satisfactory for the magnetisation of steel bars as permanent magnets, and with a strong current much more powerful magnets can be obtained by this method than by any of the methods described in Art. 38.

With a given current a much more powerful magnet can be obtained while the current passes by using a soft iron core instead of a steel core; hence, when very strong magnets are required, soft iron bars wound with coils of wire suitable for carrying strong currents are used. These magnets are called *electromagnets*, and are greatly used in practical electrical work. Small electromagnets are also much used in automatic appliances, where rapid magnetisation and demagnetisation are produced by making and breaking the circuit round the coils.

The polarity of the bar in Fig. 114 is readily determined by the application of Ampère's rule. The molecules of the bar, on experiencing the magnetic field due to the current, tend to set themselves parallel to the lines of force inside the coil, and their north poles will all lie to the left of a

person swimming round the coil and looking inwards at the bar, that is, the north pole of the bar will be to his left and the south pole to his right. This result is more easily remembered in the following equivalent form. When one looks, end on, at the south pole of the bar, the current in the coil will be found to be flowing round the bar in the same direction as the hands of a watch.

In what has been said above we have spoken of the direction of "the lines of force inside the coil," and it may be well to consider here the general nature of the magnetic field due to a coil of wire carrying a current. At any point in the wire the lines of force run round it in concentric circles, but from the arrangement of the coil, it is evident that in the interior the general direction of the resultant magnetic force is parallel to the axis of the coil; similarly, in the region outside the coil the line of action of the resultant force is also approximately parallel to the axis, but in a direction opposed to that in the interior. In the spaces between the turns of the coil, however, the magnetic field due to one turn is more or less neutralised by the field due to the adjacent turns on either side. Thus, in Fig. 115, which represents a section of a few turns of the coil, the general direction of the resultant force in the interior is along *ab*, and in the region outside the coil along *cd*, but at *e* and *f*, between the turns, the field due to one turn is neutralised by that due to the adjacent turn. Hence, if a wire be wound in a coil on a sheet of cardboard in the way shown in Fig. 116, the magnetic field produced when a current is passed through the coil may be exhibited by sprinkling iron filings on the card, and in accordance with what has been said above the lines of force will have the

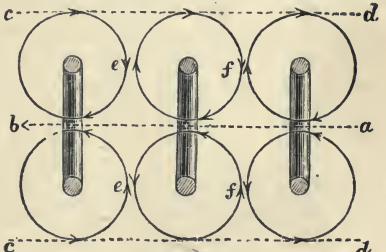


Fig. 115.

distribution shown in the figure. If the magnetic field shown here be compared with that of Fig. 64, it will be found that the two fields are practically identical, and therefore, as far as magnetic effect is concerned, a conducting helix carrying a current is practically equivalent to a magnet. The polarity of the helix may be determined by Ampère's rule; remembering that the north-seeking pole will be that from which the lines of force * diverge, it is only necessary to know the direction of the current in the coil in order to determine the poles. Thus in Fig. 114, if the arrows indicate the direction of the current, the lines of force will by Ampère's rule have the direction indicated, and the polarity of the needle will be as shown in the figure. From this it will be seen that the rule for determining the polarity of a bar magnetised by a current flowing round it in a coil applies also to the polarity of the coil itself.

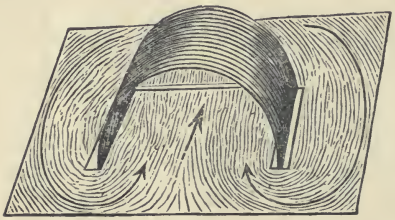


Fig. 116.

These results are strikingly illustrated by an arrangement known as the *solenoid*. This is merely a coil of wire arranged as shown in Fig. 117, so as to be capable of free suspension in a horizontal plane by pivoting the terminal of the coil in the small cups *m m* (Fig. 117). These cups can be filled with mercury, and a current passed through the coil by connecting these cups by means of the wires, *w w*, to the poles of a battery. Immediately the current

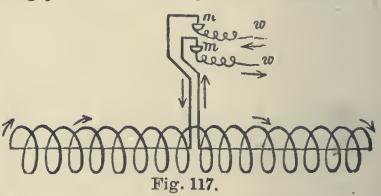


Fig. 117.

* The direction of a line of magnetic force being that in which a north pole would be urged. Art. 40.

passes, the coil acquires magnetic properties, and being free to move, at once sets itself in the magnetic meridian. Its poles are attracted and repelled by the poles of an ordinary magnet, and in every respect the coil behaves as a freely suspended bar magnet would. The floating battery of Fig. 118 also illustrates these effects in a simple and striking manner. It consists of zinc and copper plates fixed through a piece of cork and carrying a short coil of copper wire. On floating the arrangement in dilute acid, a current passes through the coil and endows it with magnetic properties. If the pole of a magnet be presented to one of the poles of the coil, the resulting attraction or repulsion is at once made evident by the motion of the coil.

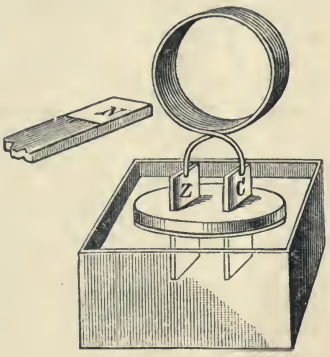


Fig. 118.

67. Chemical Effects: Electrolysis. Another important effect of the passage of a current through a conductor has still to be noticed. This is the *chemical effect* which is produced when a current is passed through an electrolyte. As already noticed in the theory of the voltaic cell, the electrical current is the result of chemical action going on in the cell; conversely, if an electrical current be passed through a conductor capable of electrolytic conduction, then the passage of this current is attended with chemical action. Thus, if the poles of a Bunsen's cell be connected to two pieces of platinum foil immersed in water acidulated with sulphuric acid, the passage of the current through the liquid decomposes it into oxygen and hydrogen. These gases are liberated separately at the surfaces of the pieces of platinum foil—the oxygen coming off from that connected with the positive pole of the cell, and the hydrogen from the other. Remembering that the dissociated atoms of oxygen and hydrogen

in the liquid are supposed to be charged respectively with negative and positive electricity, this result is just what should be expected; for, the foil connected with the positive pole of the cell being charged positively relative to that connected to the negative pole, at once attracts up to its surface the dissociated oxygen atoms in its neighbourhood, and the dissociated hydrogen atoms seek the other piece. Thus the gases are liberated separately at the two platinum terminals—oxygen at the one and hydrogen at the other—and the current is maintained through the liquid by the stream of dissociated atoms passing from one piece to the other. The chemical action in this case is very similar to that which goes on in a voltaic cell, but it is essential to distinguish clearly between the two cases. In the voltaic cell, the stream of dissociated atoms is set up by a difference between the chemical attractions of the two cell plates for oxygen; but in the case just considered the platinum plates used are exactly similar and have no chemical attraction for oxygen at all, so that it is only on electrifying them to different extents, by putting them in contact with the poles of a cell, that the necessary difference of attraction for the dissociated atoms is established, and a current thereby set up. Further, since in the voltaic cell the motion of the dissociated oxygen atoms is due to the greater attraction of one of the plates, these atoms on reaching that plate combine with it, and the chemical action which maintains the current goes on. In the case here considered, however, neither the oxygen nor hydrogen atoms combine with the plates to which they are attracted, but their charges, which they give up before liberation, neutralise the charges on the plates at the same rate as these charges are supplied by the cell to which the plates are connected. In this way a current is maintained round the circuit, and the chemical decomposition produced is the result, and not the cause, of its existence.

The chemical decomposition resulting from the passage of a current through an electrolyte is called *electrolysis*. The terminals at which the current enters and leaves the liquid are known as *electrodes*—the positive electrode, or *anode*,

being that connected with the positive pole of the cell, and by which the current enters the liquid; and the negative electrode, or *cathode*, that connected to the negative pole of the cell, and by which the current leaves the liquid. The products of electrolysis which appear at these electrodes are sometimes called *ions*. An apparatus in which the electrolysis of a given electrolyte may be suitably effected, and by which one or both of the products of electrolysis may be retained and measured, is called a *voltameter*. Fig. 119 shows one form of voltameter for the electrolysis of water.

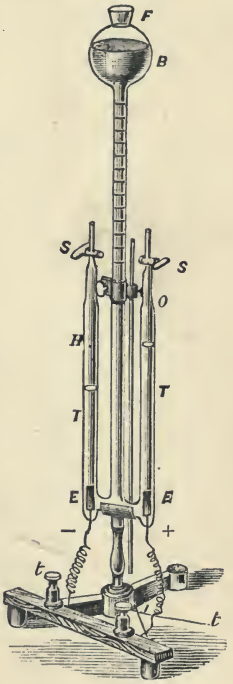


Fig. 119.

The taps at SS being open, acidulated water is poured into the apparatus by the funnel F until the vertical tubes TT are filled. The taps are then closed, and on joining the terminals *tt* to the poles of a battery, a current flows between the platinum-foil electrodes EE, which are attached to platinum wires fused into the bottom of the vertical tubes TT. The gas liberated at each electrode rises in the tube in which the electrode is fixed, and collects in the upper part of the tube, displacing the liquid up into the bulb B. By this means oxygen is collected in one tube, and hydrogen in the other, and it will then be seen that the volume of the hydrogen is approximately twice that of the oxygen. The identity of the gases may be verified by opening the taps SS, and testing them in the ordinary way, by means of a lighted taper and a glowing splinter of wood—the hydrogen burns with a pale blue flame, and the oxygen ignites the glowing splinter.

Many other substances are decomposed by the passage of a current through them. For example, hydrochloric acid is at once decomposed into hydrogen and chlorine, the former being liberated at the negative pole and the latter at the positive pole. A solution of common salt, or sodium chloride (NaCl), is first decomposed into sodium (Na) and chlorine (Cl), thus :—

$$NaCl = Na + Cl.$$

The chlorine is liberated at the anode, or positive electrode; but the sodium does not appear at the cathode, for immediately it is set free it reacts on the water present in the solution, forming sodium hydrate, and liberating hydrogen thus :—

$$Na_2 + 2H_2O = 2NaOH + H_2.$$

The hydrogen liberated by this *secondary* reaction then appears at the cathode as one of the products of the electrolysis, while the presence of the alkaline sodium hydrate may be exhibited by means of litmus solution. For, if a small quantity of reddened litmus be added to the solution before the electrolysis begins, it will be found that, as the decomposition proceeds, the colour of the solution gradually becomes blue, the blue colour first appearing in the neighbourhood of the cathode, but quickly extending throughout the liquid.

Similarly, if a solution of sodium sulphate (Na_2SO_4) be subjected to electrolysis, the first products of decomposition are sodium at the cathode and SO_4 at the anode; but neither of these substances are allowed to appear—the sodium, as before, attacking the water and liberating hydrogen at the cathode, while the SO_4 also attacks the water with formation of sulphuric acid (H_2SO_4) and liberation of oxygen :—

$$SO_4 + H_2O = H_2SO_4 + O.$$

Thus the final results of electrolysis will be hydrogen and sodium hydrate at the cathode, and oxygen and sulphuric acid at the anode. The gases are liberated, and the hydrate

and acid slowly diffuse from the electrodes through the liquid. If the electrolysis be performed in a U-tube (Fig. 120), the solution coloured with litmus, the liquid in the cathode limb of the tube gradually becomes blue, owing to the formation of the sodium hydrate, while the liquid in the other limb slowly becomes red, owing to the presence of the free sulphuric acid.

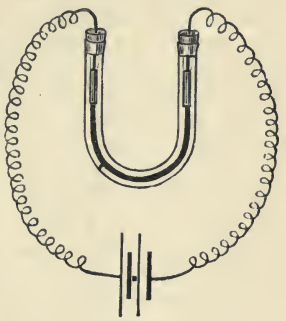


Fig. 120.

In the same way a solution of caustic soda or caustic potash may be decomposed by electrolysis, yielding oxygen at one electrode and hydrogen at the other. Potassium iodide is also readily decomposed, yielding free iodine at the anode and hydrogen at the cathode, and this reaction furnishes a very delicate test for the existence of weak currents. The free iodine liberated at the anode turns starch-paste blue, and if the terminals of a circuit be placed on starch iodide paper, the point at which the positive terminal rests is at once marked by a blue dot.

Solutions of copper sulphate and silver nitrate are electrolytes of some practical importance. Let two pieces of platinum foil, dipping into a beaker containing a solution of copper sulphate ($CuSO_4$), be connected to the poles of a battery. When the current passes the liquid is decomposed into Cu and SO_4. The copper, like all other metals, appears at the cathode *on which it is deposited as a thin film*, but the SO_4, as described above, reacts with the water, producing sulphuric acid and oxygen. If copper plates be used instead of platinum foil, copper is deposited on one plate and the SO_4 attacks the other plate, forming $CuSO_4$; and thus copper is, as it were, carried through the liquid with the current from one plate to the other, and the average concentration of the solution remains unchanged. Similarly, if a solution of silver nitrate ($AgNO_3$) be electrolysed by a current passing between two silver plates, silver, being a

metal, is deposited on the cathode, and the NO_3 set free at the anode dissolves it with re-formation of $AgNO_3$, and thus, while the quantity of silver in the solution remains unchanged, silver is carried with the current from one plate to the other.

These experiments illustrate how electro-plating is effected. For example, suppose it is required to coat a brass or copper spoon with silver, the result might be effected in the following simple way. Take a silver coin, or a strip of silver and the spoon, and arrange them as electrodes for the electrolysis of a solution of silver nitrate, the spoon being made the cathode. On passing the current a thin layer of silver is deposited on the spoon, and when this layer is sufficiently thick the spoon may be removed, and its surface burnished so as to present the usual polished appearance of an electro-plated spoon. In practice, however, the process is not so simple; special arrangements have to be made to get a coherent layer of silver that will take a good polish, and the liquid generally subjected to electrolysis is not silver nitrate, but a double cyanide of silver and potassium.

Most electrolytes, like those described above, are liquids, and this is quite what we should expect, for electrolysis can only take place where dissociation exists, and it is difficult to imagine dissociation occurring in anything but a fluid wherein molecular motion is possible. It is not, however, always necessary to make a solution of a salt in order to effect its electrolysis, for many salts, especially chlorides, when fused conduct electrolytically. Thus, when fused, silver, magnesium, and aluminium chlorides are readily decomposed by a current, the pure metal appearing at the cathode. This, in fact, is one important method of obtaining a number of metals in a pure state, and the metal potassium was first discovered by Sir Humphry Davy, by subjecting the solid hydrate to electrolysis. The hydrate was allowed to deliquesce slightly, so as to become like a paste; then, on passing a strong current through it, it quickly liquefied, and small globules of potassium appeared round the wire where the current left the hydrate. The metal here appeared in a free state, because there was not

sufficient water present to combine with all of it as it formed.

68 Less Important Effects. In addition to the important effects described above, currents produce certain obscure physical and physiological effects. Thus the phenomena of *electric osmose* and distillation and electro-capillary action are physical effects about which comparatively little is known. The physiological effects are intimately connected with the nervous organisation of the body, but a consideration of this subject belongs more to physiology than to electricity.

69. Galvanoscopes. We have seen above that when a current passes near a magnet it tends to deflect it out of the magnetic meridian, and that, under known conditions, the sense of the deflection of the magnet indicates the direction of the current. Any arrangement in which advantage is taken of this effect to indicate the existence of a current is called a galvanoscope. Thus, the simple arrangement shown in Fig. 121 acts as an efficient current indicator or galvanoscope.

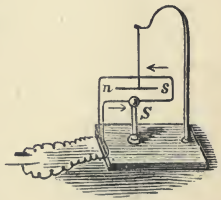


Fig. 121.

The magnet *ns* is freely suspended inside the rectangular loop of wire supported by the stand S, and indicates by its deflection the existence and direction of a current in the wire. It will be noticed that the current passing *under* the needle in one direction and back *over* it in the *opposite* direction tends, in both positions, to deflect the needle in the same direction. The arrangement may be made more effective by coiling the wire a large number of times round the needle, and thus increasing the force with which the current tends to deflect the needle.

With a given current a thin coil of twenty turns is twenty times as effective as a coil of one turn, for there are then practically twenty currents flowing round the needle, and each one has an equal effect in producing deflection. It must however be carefully noticed that this result is true

only when we compare the action of a given current in one coil with *an equal* current in the other coil. Fig. 122 shows a galvanoscope arranged with a coil of several turns.

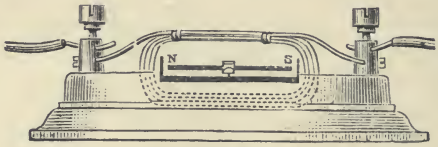


Fig. 122.

As explained above the magnetic field due to the earth opposes the deflection of a magnet by the magnetic action of a current; hence it is evident that if the current be very weak it may not be able to produce any appreciable deflection of the needle of a galvanoscope. To remedy this defect, a combination of two magnetic needles known as an *astatic pair* is employed. Fig. 123 shows the arrangement: two magnets of equal strength and size are attached to a short piece of aluminium or brass wire, and fixed *in reversed positions* with their magnetic axes parallel. If the magnets are exactly equal and accurately fixed, the combination, when freely suspended, will set in any position, for the couple tending to set one magnet into the magnetic meridian is exactly counterbalanced by the opposite couple acting on the other. In practice, however, it is impossible to secure the exact equilibrium of an astatic pair; for it is difficult to obtain magnets of exactly equal strength, and it is more difficult to fix them accurately parallel to one another. Even an imperfectly astatic needle is, however, practically free from the restraining action of the earth's field; and if the coil be arranged so that the current passing through it tends to deflect both needles in the same direction, a very sensitive galvanoscope

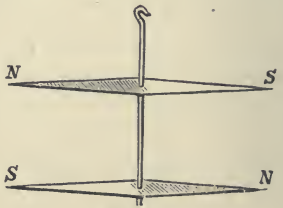


Fig. 123.

may be constructed. Fig. 124 shows how the coil may be

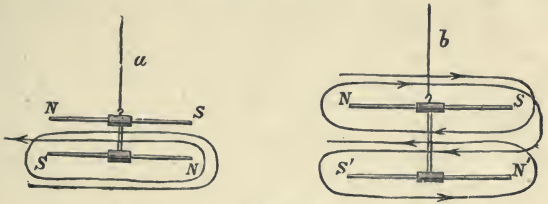


Fig. 124.

arranged—it may be wound round the lower magnet only, as shown at *a*, or round both magnets, as at *b*; in both cases the student will see, by applying Ampère's law, that the action of the current in the coil tends to deflect *both* magnets in the same direction. The couple opposing the deflection of the needle will be due to the slight difference between the opposed couples acting on the component magnets, and the torsion on the suspension fibre or wire. Fig. 125 shows an elaborate form of galvanoscope, with an astatic needle, known as *Nobili's Astatic Galvanometer*, though it is in principle a current indicator, and not a current measurer

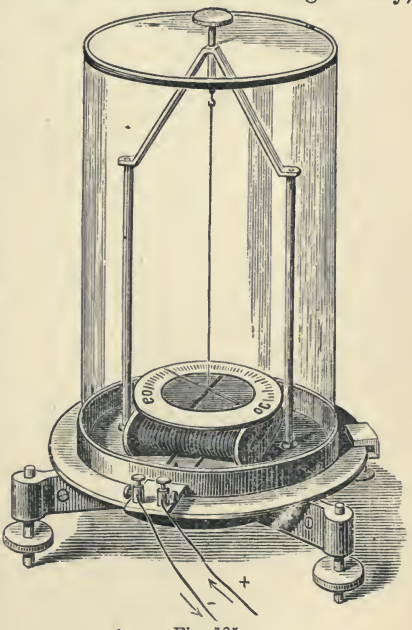


Fig. 125.

as its name implies. The coil is wound round the lower needle, and the upper needle moves over a circular scale graduated in degrees. When in use, the torsion head should be turned until the needle sets with its length at right angles to the magnetic meridian; the frame of the coil should be set accurately parallel to the needle, and the pointer should then indicate the zero of the scale.

A galvanoscope of this kind, in which the deflection produced may be read off on a scale, may be *calibrated* for use as a galvanometer. That is, by comparing its indications with those of a measuring instrument, a table of current strength corresponding to the divisions of the scale may be drawn up, and the current strength for any given deflection may then be determined by reference to this table.

CHAPTER XII.

MAGNETIC EFFECTS OF CURRENTS.

70. Magnetic Field of a Current. In Art. 66 we have discussed the general character of the magnetic field due to a current. We must now consider what determines the *intensity* of the magnetic field at any point in the neighbourhood of a conductor carrying a current.

Let AB (Fig. 126) represent a wire carrying a current; then, considering the field due to a very small element, *ab*, it can be shown that the intensity of the field at *c* due to the current in that element is :—

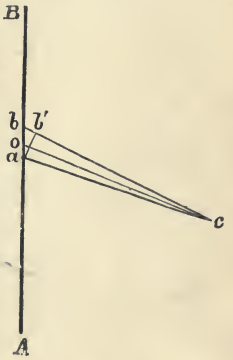


Fig. 126.

1. *Directly* proportional to the strength of the current in AB.

2. *Directly* proportional to the distance *ab′*, that is, to the apparent length of the element *ab* as seen from *c*. (The line *ab′* is drawn at right angles to the line *co*, which joins *c* to *o*, the middle point of *ab*.)

3. *Inversely* proportional to the square of the distance of the element *ab* from *c*, that is, to co^2.

Hence, if i denote the intensity of the field at *c* due to the element *ab*, and C the strength of the current in AB, we have—

$$i \propto \frac{\mathrm{C} \,.\, ab'}{(oc)^2}.$$

The direction of i is perpendicular to the plane through *c* and AB. To determine, by application of this result, the

intensity, at any point of the field, due to a current in a conductor of given form and position, it is evidently necessary to sum up the effects due to each element of the conductor considered separately—the result of the summation giving the total intensity, at the point considered, of the field due to the current in the conductor taken as a whole. The general solution of this question is beyond the scope of our work, but there is one case of great importance which must here be considered. Consider the conductor ACB to be looped into a circle as shown in Fig. 127, and let us determine the intensity of the magnetic field at *c*, the centre of the circle, due to the current in the circular conductor AB.

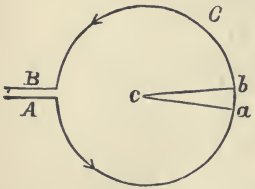


Fig. 127.

Considering the action of the element *ab*, the intensity of the field due to this element is evidently proportional to $\frac{C \times ab}{r^2}$, where C denotes the strength of the current in AB, and r the radius of the circle. Now, since each element of the conductor is similarly placed relatively to *c*, the intensity of the field at *c* due to the current in the conductor ACB taken as a whole is proportional to $\frac{C \times ACB}{r^2}$, that is, since AB $= 2\pi r$, to $\frac{C \times 2\pi r}{r^2}$, or to $\frac{2\pi C}{r}$. Hence, if I denote the total intensity of the field at C, then—

$$I \propto \frac{2\pi C}{r}.$$

The line of action of the force will be at right angles to the plane of the coil, and the direction along this line may be determined by Ampère's rule when the direction of the current is known.

Thus, for the case of Fig. 127, the direction of the force, that is, the direction in which a north pole would be urged, is upwards, towards the reader, along the normal to the plane of the paper at *c*.

If the circular coil AB consist of n turns (sufficiently thin to be regarded as coincident) instead of one, then, since the current passes n times round the coil, the intensity of the field will be n times as great as for one turn, that is—

$$I \propto \frac{2\pi n C}{r}.$$

where r is as before the radius of the coil, and C the current in it. Fig. 128 shows roughly the lines of force for the magnetic field inside a coil of wire carrying a current, and it should be noticed that, within a small space near the centre of the coil, the field is practically uniform (Art. 40), that is, the lines of force are approximately parallel equidistant straight lines.

Fig. 128.

These results may be readily verified experimentally by looping a long thin wire carrying a constant current successively into a number of coils varying in size and number of turns, and comparing the magnetic fields produced at the centre of each coil. This may be done by noting in each case the number of oscillations performed by a small needle placed at the centre of the coil. Care must be taken to arrange the plane of the coils at right angles to the magnetic meridian, so that the field due to the current may be parallel to that due to earth; then, for example, if the needle make 20 vibrations per minute when vibrating in the earth's field *only*, and 25, 30, and 40 respectively when vibrating at the centre of three different coils, then the intensities of the three fields *due to the current in the coils* are as $(25^2-20^2) : (30^2-20^2) : (40^2-20^2)$, that is, as 225 : 500 : 1200, or as 9 : 20 : 48. Results obtained in this way will be found to verify the statement that $I \propto \frac{2\pi n C}{r}$, and it should here be noticed that by proving the field for n turns of a current C to be n times the field for one turn,

we practically prove that $I \propto C$, for n turns of a current C are practically equivalent to one turn of a current nC; and if by increasing the current n-fold the magnetic field is also increased n-fold, then $I \propto C$.

A slight extension of the results obtained above shows that the intensity of the field at any point *on the axis* of a thin coil of n turns, at a distance x from the centre of the coil, is proportional to—

$$2\pi n C \frac{r^2}{(r^2 + x^2)^{\frac{3}{2}}},$$

where C denotes the current in the coil.

For, by the law given above, the intensity of the magnetic field at any point on the axis of the coil, due to a small element of the coil, is proportional to—

$$\frac{l C}{(r^2 + x^2)^{\frac{1}{2}}},$$

where l is the length of the small element, r the radius of the coil, and x the distance of the point on the axis from the centre of the coil. The direction of this force is at right angles to the plane passing through the given point on the axis and the element of the coil, and its component along the axis of the coil is therefore proportional to—

$$\frac{r l C}{(r^2 + x^2)^{\frac{3}{2}}}.$$

The total resultant force along the axis is got by summing up the effects of all the elements of the coil, that is, by substituting for l the total length of the coil; hence the resultant intensity along the axis is proportional to—

$$2\pi n C \frac{r^2}{(r^2 + x^2)^{\frac{3}{2}}}.$$

Evidently, when $x = 0$ this expansion reduces to $\frac{2\pi n C}{r}$, the result obtained above for the intensity at the centre of the coil.

A further extension of this result shows that the intensity of the magnetic field in the interior of a coil similar

to that shown in Fig. 114, but closely wound, is approximately proportional to—

$$4\pi n\,C,$$

where n denotes the number of turns *per unit length* of the coil, and C the current flowing in it. The magnetic field in the interior of such a coil is practically uniform except near the ends.

71. Electromagnetic Unit of Current Strength. It is evident from the preceding article that we may *compare* the strengths of current by comparing the intensities of the fields which they produce at the centre of a given coil. For if the same coil be used for different currents, then, in the relation $I \propto \frac{2\pi n C}{r}$, n and r are constants and C is the only variable, and we have $I \propto C$ or $C \propto I$.

In order to measure a given current absolutely, however, it is necessary to decide upon a unit of current strength. This is most conveniently done by adopting a unit such that the relation $I \propto \frac{2\pi n C}{r}$ may be written as an equality, thus—

$$I = \frac{2\pi n C}{r}.$$

That is, *unit current is that current which, flowing in a single coil of unit radius, produces a magnetic field of* 2π *units of intensity at the centre of the coil.* For, from the relation given above—

$$C = \frac{I r}{2\pi n},$$

and if $I = 2\pi$ units, r be of unit length, and $n = 1$, then—

$$C = \frac{2\pi \times 1}{2\pi \times 1} = 1,$$

that is, the current is of unit strength.

This definition is perhaps more neatly expressed by saying that *unit current is that current which, flowing in an arc of unit length and unit radius, produces a magnetic field of unit intensity at the centre of the arc.* If it be remembered that a single complete coil of unit radius contains 2π arcs of

unit length, it will be seen that the two definitions are identical. If the unit of length be the centimetre, and the unit of force the dyne, then the unit of current defined above is known as the *absolute electromagnetic unit* of current strength, and is equal to 10 amperes.*

72. Galvanometers. Speaking generally, galvanometers may be divided into two classes.

1. Instruments adapted for exact measurement of current. Of this class the *tangent* and *sine galvanometers* are the best types.

2. Instruments constructed so as to secure extreme sensitiveness. Galvanometers of this class are not suitable for the measurement of current, but they are the most useful instruments in the general work of electrical measurement, where it is often more essential to have a sensitive instrument capable of indicating, and if necessary measuring, the feeblest current, rather than one capable of exact measurement of current generally.

The general principle of the action of all galvanometers depends upon the effect of the current in a coil of wire on a magnetic needle freely suspended at the centre of the coil. When no current passes the needle sets in the magnetic meridian, and the plane of the coil should also be in the magnetic meridian, that is, the needle should set with its length in the plane of the coil. Then, on passing a current through the coil, the direction of the magnetic field, due to the current, being at right angles to the plane of the coil, tends to deflect the needle out of the magnetic meridian, and sets it in such a position that the moment of the couple due to the field of the current is balanced by the moment of the opposing couple due to the field of the earth. The magnitude of this deflection is directly connected with the strength of the current, and by noting its value it is possible with a properly constructed instrument to obtain a measure of the current.

From this it is evident that the essential parts of any galvanometer are (*a*) a coil of wire, (*b*) a magnetic needle freely suspended at the centre of the coil, and (*c*) a scale on which the deflection of the needle may be read. We shall

now describe the forms of galvanometer most generally used; these are the *Tangent* and *Sine galvanometers* and *Thomson's mirror or reflecting galvanometer.*

73. The Tangent Galvanometer. A simple form of tangent galvanometer is shown in Fig. 129. The needle, which may be suspended or pivoted at the exact centre of the coil, is very short, and carries a light wire pointer moving over a circular scale. The needle and scale are enclosed in the shallow cylindrical box B by a glass cover, and the arrangements for accurately reading the deflection of the needle are exactly those described in Art. 47 in connection with the apparatus of Fig. 88, which may be taken to represent the needle and scale of a tangent galvanometer.

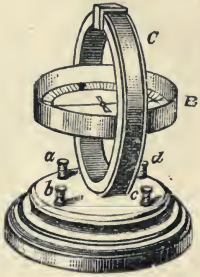


Fig. 129.

The coil of carefully insulated copper wire is wound round the frame C, and the ends of the wire are brought out under the base of the instrument and connected to screw terminals fixed in the base-board. Most instruments, however, are usually fitted with several coils of different lengths wound on the same frame, but the ends of each coil are connected to the terminal screws in such a way that any one coil may be used separately, or any two or more combined into one by choosing the proper screws as terminals. Thus, in Fig. 129, if the screws a and b are taken as terminals the current passes only through the coil joining them; but if the screws a and c are taken, then the current passes through the coil joining a and b, and also through that joining b and c. Similarly, if a and d be taken, the current passes through the coils a to b, b to c, and c to d. Three coils are sufficient for ordinary purposes—one consisting of a single turn of thick copper wire, a second of a number of turns of wire of medium thickness, and a third composed of a large number of turns of thin wire. To adjust the galvanometer for use it is carefully levelled, and the plane of the coil adjusted in the magnetic meridian. The needle

then lies in the plane of the coil, and the ends of the pointer should point to the zeros of the scale. To eliminate errors due to inaccurate adjustment of the needle and pointer, both ends of the pointer should be read, and the deflection taken first on one side of the zero, and then, by reversing the current in the coil, on the other side. The mean of the four readings thus obtained gives the true value of the deflection. The coil to be used in any given case must be chosen in accordance with principles to be discussed in the next chapter, but it will be readily understood that to prevent injury to the instrument all very strong currents should be sent through the single coil of stout wire.

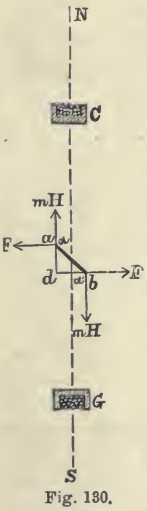


Fig. 130.

We have now to consider the theory of the instrument in order to establish a relation between the strength of a current and the magnitude of the deflection produced by it.

Let Fig. 130 represent a horizontal section of coil in the plane of the needle. The plane of the coil is supposed to lie in the magnetic meridian NS, at right angles to the plane of the paper, the sections at CC representing the cut ends of the coil, and *ab* the needle in its deflected position. For the sake of clearness the length of the needle is here shown much longer than it should be relative to the diameter of the coil: it should be comparatively short, so that in all positions it may lie within the small space in which the magnetic field due to the current may be considered of uniform intensity, equal to the intensity at the centre of the coil. That is, referring to Fig. 128, the needle should be small enough to lie within the small space *a b c d* at the centre of the coil where the lines of force are approximately parallel and equidistant.

The intensity of the magnetic field within this space, due to a current C in the coil, is approximately given by—

$$I = \frac{2\pi n C}{r},$$

where r denotes the radius of the coil, and n the number of turns in the coil.

Hence, if m denote the strength of either pole of the needle, the forces constituting the couple tending to deflect it out of the magnetic meridian are each given by—

$$F = mI = \frac{2\pi mnC}{r},$$

and act at right angles to the plane of the coil.

When the needle is in its position of equilibrium, the moment of the couple due to the earth's field balances the moment due to this couple. Hence, for equilibrium we have—

$$F \,.\, ad = mH \,.\, bd,$$

that is,

$$\frac{2\pi mnC}{r} \,.\, ad = mH \,.\, bd,$$

or

$$\frac{2\pi nC}{r} \,.\, ad = H \,.\, bd,$$

and therefore

$$C = \frac{r}{2\pi n} \,.\, H \,.\, \frac{bd}{ad}.$$

But $\frac{bd}{ad} = \tan bad$, and as the angle bad is equal to α, the angle of deflection of the needle, we have—

$$\frac{bd}{ad} = \tan \alpha,$$

and

$$C = \frac{r}{2\pi n} \,.\, H \,.\, \tan \alpha.$$

The quantity $\frac{r}{2\pi n}.H$ is known as the *working constant* or *reduction factor* of the galvanometer,* and denoting it by k, we have—

$$C = k \tan \alpha,$$

* The constant of the galvanometer *coil* is $\frac{2\pi n}{r}$

where k is a constant. That is, *the strength of the current varies as the tangent of the angle of deflection.* For example, if one current (C_1) produce a deflection of 30°, and another (C_2) a deflection of 60°, then—

$$\frac{C_1}{C_2} = \frac{\tan 30°}{\tan 60°} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3},$$

that is, C_2 is three times as great as C_1. We are thus able to *compare* currents by simply noting the deflections of the needle and taking the value of the tangents of the angles; but if we wish to measure a current absolutely, then the value of the constant k must be determined, either by calculation from the relation $\frac{rH}{2\pi n}$ or by experiment, as explained in Art. 81.

The Sine Galvanometer, one form of which is shown in Fig. 131, is an instrument exactly similar in principle

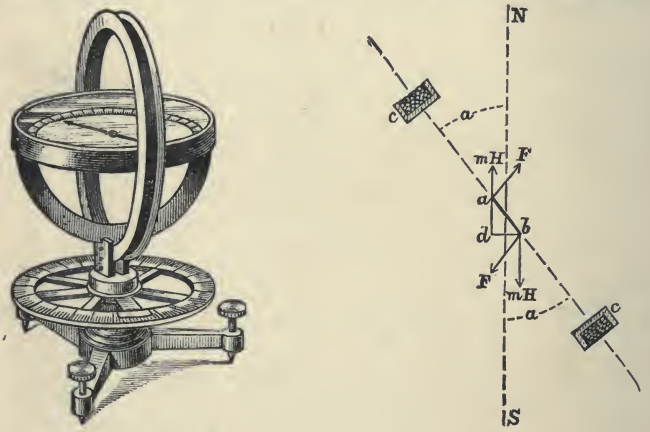


Fig. 131.

and construction to the tangent galvanometer. It differs from it only in the fact that the coil and needle box can

be rotated round a central vertical axis, and a horizontal circular scale is provided on which the amount of this rotation can be accurately read.

For use the instrument is adjusted in the same way as the tangent galvanometer, but when the needle is deflected the coil is rotated after it until the needle is overtaken by it and *in its deflected position* lies in the plane of the coil. The diagram of Fig. 131 shows the conditions of equilibrium of the needle, and as before we must have—

$$F \cdot a\,b = m\,H \cdot b\,d,$$

that is,

$$\frac{2\pi m n C}{r} \cdot a\,b = m\,H \cdot b\,d,$$

or

$$\frac{2\pi n C}{r} \cdot a\,b = H \cdot b\,d,$$

and therefore

$$C = \frac{r}{2\pi n} \cdot H \cdot \frac{b\,d}{a\,b},$$

that is, since $\frac{b\,d}{a\,b} = \sin\, b\,a\,d = \sin a$, where a denotes the deflection of the needle or the rotation of the coil, we have—

$$C = \frac{r}{2\pi n} \cdot H \cdot \sin a,$$

or

$$C = k \cdot \sin a,$$

where k denotes the reduction factor of the instrument. That is, the current is proportional to the *sine* of the angle of deflection of the needle; for example, if one current, C_1, produce a deflection of 30°, and another, C_2, a deflection of 60°, then—

$$\frac{C_1}{C_2} = \frac{\sin 30°}{\sin 60°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

It should be noticed that with the sine galvanometer the

needle, when in its position of equilibrium, lies in the plane of the coil ; and therefore the forces of the couple F, F, always act at right angles to the length of the needle, and therefore the arm of the couple is always given by the length of the needle *ab*.

This is true whatever be the length of the needle, and hence, for a needle of given length, the moment of the couple varies only with strength of the current. For *comparative measurements*, therefore, the instrument may have a long needle, but for absolute measurements the needle must be short, in order that the intensity of the field in which it moves may be uniform and easily determined.

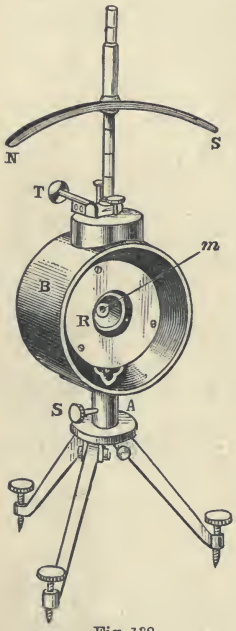


Fig. 132.

74. The Mirror Galvanometer. A good form of Thomson's *mirror galvanometer* is shown in Fig. 132. The coil is wound on a comparatively small circular reel, R, enclosed in B, a cylindrical brass box with a glass front. The needle of the instrument is a short, carefully magnetised strip of steel, attached by shellac or cement to the back of a small circular concave mirror, *m*, suspended by a single silk fibre at the centre of the coil. The deflection of the magnet is measured by the mirror and scale method explained in Art. 27, and Fig. 133 shows the galvanometer and scale arranged for use. The large magnet NS, supported above the coil, has several important uses.

If this magnet be removed, the needle of the galvanometer sets in the magnetic meridian, and the magnetic field in which it lies is that due to the horizontal component of the earth's magnetism. If now the magnet be replaced

with its length in the magnetic meridian, and its *south* pole pointing northwards, the magnetic field which it pro-

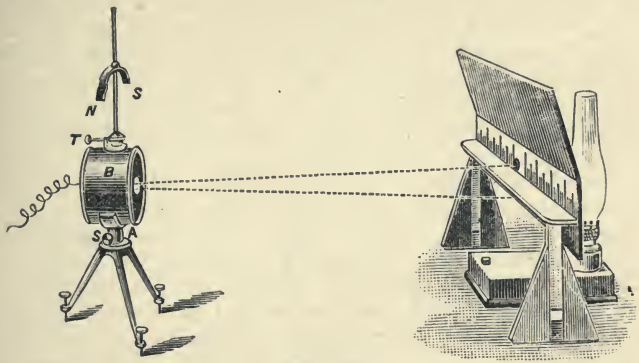


Fig. 133.

duces at the centre of the coil will be added to that due to the earth. The needle will now be more difficult to deflect, and, consequently, the galvanometer will be less sensitive. The nearer the magnet is to the coil, the greater will this effect be; hence, by lowering the magnet on its support, the sensitiveness may be very considerably diminished. If, however, the magnet be replaced with its *north* pole pointing northwards, the field it produces at the centre of the coil tends to neutralise that due to the earth; and hence, by lowering the magnet, the field in the coil may be decreased until, at a certain point, the field of the magnet exactly balances that of the earth. For this position of the magnet the needle is unstable, and remains at rest in any position; and if the magnet be lowered still more, the direction of the field is reversed, and the needle tends to turn round through 180°. Hence, in this case, the galvanometer cannot be used with the controlling magnet lower than the position of instability of the needle, but with the magnet slightly above this position the instrument is extremely sensitive. Its sensitiveness may be slightly decreased by raising the magnet, or, in a greater degree, by

removing it, or, more effectually, as explained above, by reversing it and lowering it on its support.

So far we have considered the galvanometer placed with the plane of its coil in the magnetic meridian, but this is not an essential adjustment—by turning the controlling magnet on its support the needle may be made to set in almost any required plane, and the coils are then set parallel to it by rotating the body of the instrument round a central axis at A.

In adjusting the galvanometer for use it is first levelled so that the mirror m hangs truly at the centre of the coil. The position of the magnet NS is then adjusted so as to secure the required sensitiveness, and to set the needle in a plane convenient for work. The coil is then set parallel to the plane of the needle, and fixed in position by the screw at S. The scale must now be arranged with its length parallel to the plane of the coil, and the line joining the centre of the mirror to the zero of the scale should be approximately at right angles to this.

On adjusting the lamp and scale to the proper height, the spot of light reflected on to the scale will now come to rest somewhere near the zero of the scale, and the dark line across the spot may be adjusted exactly to the zero mark by slightly rotating the magnet NS by means of the tangent screw T. The galvanometer will now be ready for use, and Fig. 133 shows how it is connected in circuit, but the student can realise the entire process of adjustment only by going through it for himself.

75. The Magnetisation of Iron. It has been stated in Art. 70 that the magnetic field produced in the interior of a long coil wound regularly and uniformly with n turns per unit of length is uniform and of magnitude $4\pi n$C, where C denotes the current in the coil. Hence it is evident that the magnetic field in the interior of the coil can be varied by varying the current in the coil. If, then, we take a long regularly wound coil, and place in it a long thin iron wire, we can by varying the current in the coil subject the wire to the inductive action of magnetic fields of varying intensity, and in this way investigate the

magnetic behaviour of iron in magnetising fields of different strengths.

In an investigation of this kind the usual method of work is to determine by experiment the Intensity of Magnetisation, I, or the Magnetic Induction, B (Art. 43), corresponding to known values of H, the Magnetic Field. The relation between I and H or between B and H can then be exhibited by means of a curve drawn by taking values of H for abscissæ, and the corresponding value of I or B as ordinates. To determine H and I or B the most satisfactory arrangement is that in which the conditions of magnetisation are such that the demagnetising force is negligible : H will then be given by the intensity of the field in the interior of the coil, and I can be determined from the magnetic moment of the magnetised wire and the dimensions of the wire; B, if required, can be got from the relation $B = H + 4\pi I$.

To realise these conditions practically, an iron wire, whose length is four or five hundred times its diameter, is placed in a long carefully wound coil of length about equal to that of the wire. The coil is then placed in a vertical position, with one end close to the needle of a mirror magnetometer, at a point behind the needle on a line drawn through its centre at right angles to its length. A very small current is then passed round the coil, and by means of an adjustable resistance in the circuit the strength of the current is gradually increased. As the current is increased the deflection of the light on the magnetometer scale also increases—quickly at first, but afterwards more and more slowly, showing that the magnetic effect on the wire appears to tend to a maximum. The deflection of the magnetometer needle is evidently due to two things—the magnetic effect of the coil itself (Art. 66), and the magnetic effect of the thin core of iron wire. These two effects can readily be separated; that due to the coil is proportional to the current flowing in the coil, so that if the deflection produced by the *coil alone* for one or two known values of the current be obtained by experiment, then the deflection corresponding to any given

current can be determined: the effect due to the iron wire core for any given value of the current can then be readily obtained by subtracting the deflection due to the coil alone from that given by the coil and the wire. The deflection of the magnetometer due to the action of the wire core alone will be proportional to the magnetic moment of the wire, and therefore proportional to the intensity of magnetisation of the iron, assuming that the wire is uniformly magnetised, an assumption which under the

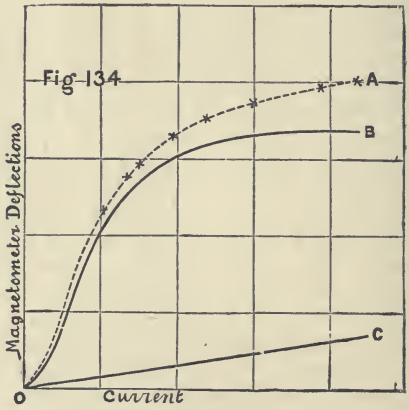


Fig. 134.

conditions considered is practically true. The intensity of the magnetising field being given by $4\pi n$ C is evidently proportional to C, and C can readily be determined by means of a tangent galvanometer or other current measurer placed in the circuit. Hence if we take for abscissæ the values of C, and for ordinates the corresponding values of the magnetometer deflections, we obtain a curve similar to the dotted curve OA (Fig. 134), showing the relation between the magnetic moment of coil and wire and the intensity of the magnetic field in the coil. If we now

similarly plot a curve OC, showing the deflections due to the coil alone for different values of C, then by subtracting the ordinates of the curve OC from the corresponding ordinates of OA we get the curve OB, showing the relation between the magnetic moment of the iron wire and the intensity of the magnetising field. An examination of this curve shows that the magnetic moment, and therefore the intensity of magnetisation, increases with increase in

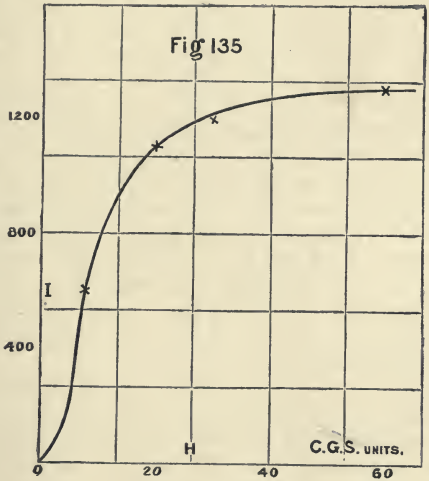


Fig. 135.

the value of H: the increase is at first somewhat slow, then more rapid, and finally slower and slower, until it becomes almost imperceptible. In the upper part of the curve, where a large increase in the intensity of the magnetising field produces a very small increase in the intensity of magnetisation, the iron has almost attained magnetic *saturation*; but experiment seems to show that absolute saturation cannot be attained—there is always

a slight increase in I when H is increased, and I only attains its maximum value when H is infinite.

Instead of plotting a curve from the values of the current for abscissæ, and the deflections as ordinates, the absolute values of I and H can be deduced from these and other data of the experiment, and a curve obtained by taking values of H as abscissæ, and the corresponding values of I as ordinates. The curve so obtained will be exactly similar in general form to that discussed above, but it has the additional advantage of showing the actual values of I and H; and since the magnetic susceptibility, κ, is given by the ratio I/H, the value of this constant for any point on the curve can be obtained by taking the ratio of the values of I and H at the given point. Similarly, if a curve be drawn showing the relation between B and H, then the *permeability*, B/H, for any point on the curve can be determined, or the value of this constant can be obtained from the I and H curve by means of the relation $\mu = 1 + 4\pi\kappa$. Fig. 135 shows a curve for I and H in C. G. S. units. Looking at this curve, it is evident the values of κ and μ vary with the magnetic state of the iron, and calculation for a number of points on the curve will show that the values of these constants vary with the intensity of magnetisation of the iron.

The determination of I and H in absolute C. G. S. units from the data of an experiment such as that described above is comparatively simple. Omitting unimportant details and corrections, the following is a brief sketch of the method of reducing the data. In considering the magnetic action of the iron wire on the needle of the magnetometer, only the effect of the upper pole need be considered—the other pole will be at a considerable distance from the needle, and its position relative to the needle is such that its effect in causing deflection may be neglected. Let the strength of the upper pole be denoted by m; then if its distance from the magnetometer needle be d, the intensity of the magnetic field due to this pole at the needle is given by m/d^2. Then if H denote the uniform field in which the needle is initially at rest, it is evident

that for equilibrium of the deflected needle we must have, as in Art. 73,*—

$$\frac{m}{d^2} = \mathrm{H} \tan \alpha,$$

where α is the deflection of the needle.

That is—

$$m = d^2 \mathrm{H} \tan \alpha.$$

Now I is given by $\frac{m}{a}$ where a denotes the area of cross section of the wire; therefore we have—

$$\mathrm{I} = \frac{m}{a} = \frac{d^2 \mathrm{H} \tan \alpha}{a}.$$

To determine I it is therefore necessary to know d, a, $\tan \alpha$, and H. The first two can be measured directly; the third, $\tan \alpha$, is readily given by the deflection on the magnetometer scale and the distance of the scale from the needle; for if s denote the former and L the latter distance, then, remembering that the angular deflection of the reflected ray is twice that of the needle, we get $\tan \alpha = \frac{\mathrm{S}}{2\mathrm{L}}$, provided the deflection is small enough for $\tan \frac{1}{2}\alpha$ to be taken as $\frac{1}{2} \tan \alpha$. Finally, the value of H can be determined by causing the needle of the magnetometer to oscillate first in the field which controls it during the experiment, and then by removing all control magnets, and taking its period of oscillation in the field of the earth only. By comparing the periods of oscillation as in Art. 51, the value of the control field can easily be obtained in terms of the horizontal component of the earth's field at the place of experiment.

The value of H, the intensity of the magnetising field, is more readily obtained; it is given at once by $4\pi n \mathrm{C}$, and is evidently known when n and C are known. The value of C, which must be expressed in absolute C. G. S. units,

* From p. 230 it is evident that since $m\,\mathrm{I} \,.\, ad = m\,\mathrm{H} \,.\, bd$, we have $\mathrm{I} = \mathrm{H}\,\frac{bd}{ad}$, or $\mathrm{I} = \mathrm{H} \tan \alpha$, and in this case I is given by m/d^2.

is given by the current measurer in the circuit, and n can be accurately obtained directly by counting the number of turns in a considerable length of the coil. When I is once obtained in this way, B can be easily calculated from the relation $B = H + 4\pi I$; B can *also be* obtained by direct experiment as described in Art. 43.

It should be noticed that in the method of experiment described above the iron wire is in a vertical position, and is therefore subject to the inductive action of V, the vertical component of the earth's field: for this reason the

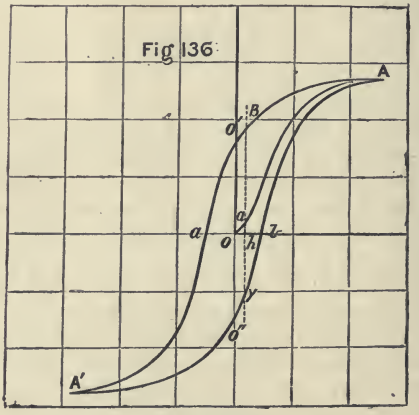


Fig. 136.

strength of the magnetising field is given, not by $4\pi nC$, but by $4\pi nC \pm V$. The value of V is comparatively small—about ·43 C. G. S. units—but its inductive effect is quite appreciable.

In connection with this subject the important phenomenon of *hysteresis* must not be passed over. The curve shown in Fig. 136 is obtained, as explained above, by gradually increasing the current in the coil, and so subjecting the wire to the inductive action of a gradually increasing field. If this process is carried on until the wire practically

attains magnetic saturation, and the current is then gradually *decreased*, it will be found that the stages in the magnetic condition of the wire during *increase* of the magnetising field are not repeated in the reverse order during the decrease of the field. The magnetic intensity for any given value of H during decrease is greater than during increase, so that when the magnetising field is zero the wire is not demagnetised, but possesses a considerable amount of residual magnetism. Fig. 136 represents graphically what we have just described: OA is the curve, showing the increase of I with H as H is increased, AO′ shows how I decreases with H as H is decreased, and OO′ indicates the residual intensity of magnetisation when H is zero. The curve AO′ can, however, be continued. If the current after being reduced to zero is reversed and then gradually increased, the value of H changes sign, and under the influence of this reversed magnetic field the wire first loses its residual magnetism, and then becomes magnetised in the sense opposite to that first induced. This can be continued until the wire again approaches saturation with reversed polarity. The curve O′*a*A′ represents this process: from O′ to *a* the effect of the reversed field is merely to reduce the residual magnetism to zero, and at *a* we have the wire in a neutral condition but the magnetising field is not zero; it has a negative value given by O*a*. From *a* to A′ the intensity of magnetisation, with sign reversed, gradually increases and tends towards a maximum at A′, just as in the portion OA of the curve.

If the current be now decreased, the intensity of magnetisation during the decrease of H will not pass through its old values in the reverse order, but the values during decrease will be numerically greater than during increase. This is indicated in the figure by the portion A′O″ of the curve corresponding to the portion AO′. At O″ the current has been reduced to zero; the residual intensity of magnetisation however is not zero, but represented by OO″. If the current be now reversed and gradually increased, the magnetic state will change in the way indicated by the portion O″*b*A of the curve. This portion corresponds to

O′aA′. From O″ to b the effect of the reversed field is to reduce the residual magnetism to zero; at b, with a magnetic field indicated by Ob, the intensity of magnetisation is zero; and from b to A the value of I increases with that of H. At A the wire has returned to one of its initial states on the curve OA. In passing through the states indicated by the complete curve AO′aA′O″bA, the iron has passed through a cycle of states, and by repeating the operations involved in the cycle the iron may be made to pass through a similar cycle any number of times.

It will be seen from a study of this curve that the magnetic state of the iron does not entirely depend on the intensity of the magnetising field. For example, take the value of H indicated by Oh (Fig. 136). It is evident that there are *three* values of I corresponding to this value of H. These are indicated by $h\alpha$, $h\beta$, and $h\gamma$; and it is evident that the difference in these values is connected with the previous history of the iron: thus the value $h\alpha$ is for iron which was in a neutral condition before being subjected to the action of a gradually increasing field; $h\beta$ refers to iron which has been first magnetised up to saturation, and then subjected to the action of a gradually decreasing field; $h\gamma$ is for iron which after being reduced to the neutral condition at a is then magnetised in the reverse sense up to saturation, then subjected to a gradually decreasing field, and finally to a reversed field gradually increased from zero to the value indicated by Oh.

It is evident from this that the magnetic state of the iron depends not only on the intensity of the magnetising field, but also on its magnetic history since last leaving the neutral condition. It will be noticed that the behaviour of the iron is such as to suggest that the original magnetic condition of the iron, whatever it may be, tends to persist—that is, change in the magnetisation of the iron "lags" behind the changes in the magnetising force acting on it. This phenomenon has been called by Professor Ewing *Hysteresis*, from the Greek ὑστέρειν, to lag behind.

The existence of the *hysteresis* in iron is the cause of a considerable loss of energy when, as in the case of iron in

dynamo and motor armatures, cores of transformers, etc., the iron has to go through cycles of magnetic changes with great rapidity. Each time a cycle is gone through a quantity of energy proportional to the area of the closed curve in Fig. 136 is dissipated, and causes heating of the iron.

The whole subject of magnetic induction is one of the greatest interest, and students wishing to gain further knowledge of the subject cannot do better than read Professor Ewing's book on "Magnetic Induction in Iron and other Metals," where the whole subject is fully discussed.

CHAPTER XIII.

OHM'S LAW.

76. Electromotive Force. Electromotive force has been defined generally in Art. 57 as that which causes motion of electricity. The absolute electromagnetic unit of electromotive force is defined in Chapter XV., and is approximately equivalent $\frac{1}{3 \times 10^{10}}$ of an electrostatic unit of potential. The unit of electromotive force in practical use is however equal to one hundred million (10^8) absolute electromagnetic units, and is called a **volt**. The electromotive force, or, as it is usually written, the E. M. F. of a Daniell's cell, is about 1·07 volts, and may therefore be taken without very great error as a rough realisation of the practical unit. The electromotive forces of different cells may be *compared* by comparing the differences of potential set up at their poles when no current is allowed to pass. Thus, if one pole of a cell be connected to one pair of quadrants of a quadrant electrometer, and the other pole and the other pair of quadrants earthed, then the two pairs of quadrants being unconnected no current passes, but a difference of potential equal to the E. M. F. of the cell is set up between them, and the needle of the electrometer is deflected through an angle proportional to it. By noting in this way the several deflections produced by the cells to be compared, we may by comparing the deflections determine the comparative electromotive forces of the cells; and if the E. M. F. of any one cell be known absolutely, this method may be applied to determine the absolute E. M. F. of any given cell. For example, if the E. M. F. of a Daniell's cell be taken as 1 volt, and it produces a deflection of 100 divisions on the electrometer

scale, then the E. M. F. of a cell giving 190 divisions deflection is evidently 1·9 volts.

The following table gives the approximate E. M. F. of the cells described in Art. 56.

Cell.	E. M. F. in volts.
Volta's simple cell	1 volt (about)
Smee's cell	0·7 „ („)
Poggendorff bichromate cell ...	1·8 to 2·3 volts
Daniell's cell	1·1 volt (about)
Grove's cell	1·9 „ („)
Bunsen's cell	1·9 „ („)
Leclanché cell	1·4 to 1·6 volts
Latimer Clark cell (standard) ...	1·433 volt at 62° F.

The Latimer Clark cell is a standard cell of very constant E. M. F. Mercury and zinc are the elements; the mercury is at the bottom of the cell, and on it rests a paste made by mixing mercurous sulphate with zinc sulphate. The zinc element takes the form of a rod or strip of zinc immersed in the paste, but not going deep enough to touch the mercury. Contact is made with the mercury by means of a platinum wire passing through the bottom of the cell and forming the positive pole of the cell. Fig. 137 shows a convenient form of the cell; it is used only as a standard of E. M. F., and is soon spoiled if allowed to give a current.

Fig. 137.

When a number of similar cells are arranged *in series* so as to form a battery, the E. M. F. of the battery is found by summing up the E. M. Forces of the component cells. Thus, the E. M. F. of a battery of n similar cells is n times the E. M. F. of one cell. If, however, the cells are arranged *in parallel*, then they practically constitute one large cell, and the E. M. F. of the arrangement is the same as that of one cell. Similarly, if mn cells be arranged in n rows, each row containing m cells arranged in parallel and the rows connected in series, the E. M. F. of the

arrangement is n times the E. M. F. of one cell, for we have practically n large cells in series, each cell being m times as large as a single cell.

77. Current. The absolute electromagnetic unit of current has already been defined in Art. 71, but the practical unit, called the **ampere**, is only one-tenth of this absolute unit. The quantity of electricity which a current of one ampere conveys in one second past any section of the conductor is called the **coulomb**, and is equivalent to 3×10^9 * electrostatic units of quantity as defined in Art. 6.

The quantity of electricity conveyed by a current C in time t is given by the relation $Q = C\,t$. If C is expressed in amperes and t in seconds, then Q is given in coulombs.

The beginner should realise that when a current travels round a continuous undivided circuit the strength of the current is the same at all points in the circuit. But in the case of a *divided circuit*, such as that shown in Fig. 138, when the current reaches the point of division, a, it splits up along the several branches meeting at a, and as explained in the next article, the relative strengths of these branch currents depend upon the relative resistances of the branches.

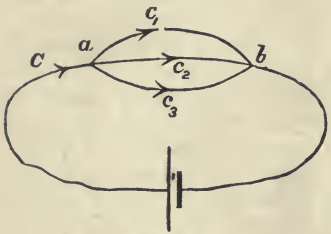


Fig. 138.

If C denotes the *total* current flowing in the circuit, and c_1, c_2, and c_3 the branch currents, then we evidently must have—

$$C = c_1 + c_2 + c_3;$$

that is, $C - c_1 - c_2 - c_3 = 0$, or the *algebraic* sum of the currents meeting at points a and b is zero. This result is an example of *Kirshhoff's first law*, which states that in any network of wires carrying currents the algebraic sum of the currents meeting at any point is zero.

* The electromagnetic unit of quantity is 3×10^{10} electrostatic units, and the coulomb, being $\frac{1}{10}$th the electromagnetic unit, is 3×10^9 electrostatic units.

78. Resistance. The resistance of a conductor may be defined as the ratio of the electromotive force producing a current in the conductor to the strength of the current so produced.

Thus, if an electromotive force, E, applied to a given conductor, produce in it a current, C, then the resistance of that conductor is given by the ratio $\frac{E}{C}$. That is, if R denote the resistance of the conductor, we have—

$$R = \frac{E}{C}.$$

If E and C be expressed in absolute electromagnetic units, then R is also expressed in electromagnetic units, the absolute electromagnetic unit of resistance being the resistance of a conductor in which unit E. M. F. produces unit current. The practical unit of resistance is that resistance in which an E. M. F. of one volt produces a current of one ampere. It is called the **ohm**, and is equal to one thousand million (10^9) absolute units. It may conveniently be taken as the resistance of a column of mercury 106·3 cm. long, and having a uniform section of one square millimetre, the temperature of the mercury being 0° C.

In preparing practical standards of resistance, it is extremely difficult to arrange columns of mercury in a form convenient for general use; hence the resistances used in practical measurements are generally sets of coils of insulated wire wound on bobbins, and so arranged that they may be conveniently used in any desired combination. Such an arrangement of coils is called a *resistance box*, the bobbins carrying the coils being arranged inside a box. The ends of each coil are attached to thick pieces of brass fitted in rows on the lid of the box. These

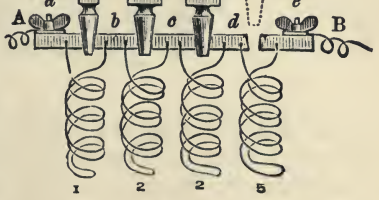


Fig. 139.

brass pieces are insulated from each other, but may be joined together by brass plugs, fitting into the spaces between them. Fig. 139 shows diagrammatically how the coils of a resistance box are arranged, and how they may be combined so as to place any desired resistance in the circuit. The current enters at A and leaves at B. Coils of 1, 2, 2, and 5 ohms' resistance are connected, as shown, to the brass pieces *a b c d e*, which may be connected together by the brass plugs *k l m n*. These plugs fit tightly into circular gaps between the brasses, and it will be seen that when any one is removed the corresponding coil is thrown into the circuit. For example, when the plug *n* is removed, the coil of 5 ohms' resistance is placed in the circuit; if the plugs *k* and *n* are removed, then a resistance of 6 ohms is inserted; and it is evident that with these four coils it is possible to insert a resistance of any integral number of ohms between 0 and 10. When all the plugs are in, the only resistance between A and B is that offered by the intervening brasses and plugs, and these being massive and made of good conducting material, their resistance is practically *nil*, provided that the plugs fit well and the surfaces of contact are clean. Fig. 140 shows the general appear-

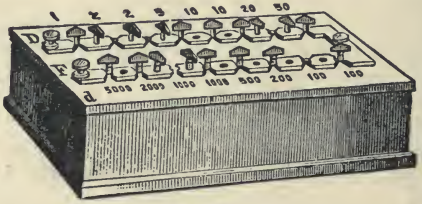


Fig. 140.

ance of a resistance box capable of supplying a resistance of any exact number of ohms between 0 and 10,000. Resistance boxes are also constructed so as to give fractional parts of an ohm. For example, a box containing four coils of $\frac{1}{10}$, $\frac{2}{10}$, $\frac{2}{10}$, $\frac{5}{10}$ ohm, in addition to the usual coils, would give any required resistance to $\frac{1}{10}$th of an ohm.

It will be noticed that the coils in a resistance box are arranged, with respect to magnitude, in the same way as a set of weights, and the resistance of any given conductor may be measured in a way exactly analogous to that employed in determining the weight of any body.

Let the given resistance be arranged in circuit with a cell or battery of constant action—say a battery of four Daniell's cells—and a delicate galvanoscope. When the current is steadily established, let the deflection of the galvanoscope needle be carefully noted. Now remove the unknown resistance, and substitute a resistance box in its place. Let the resistance in the box be adjusted until the galvanoscope deflection is exactly the same as at first then the resistance in the box must be equal to the resistance of the given conductor.

This simple *substitution* method of measuring resistance is, however, rather inconvenient in practice and is not often resorted to.

Experiment shows that the resistance of a conductor of uniform section, such as a wire or rod, varies directly with its length and inversely with its area of cross section. That is, the longer the conductor is the greater is its resistance, and the thicker it is the less its resistance. Hence, if l denote the length of a conductor of uniform cross section of area a, then—

$$\mathrm{R} \propto \frac{l}{a},$$

where R denotes the resistance of the conductor. That is,

$$\mathrm{R} = \mathrm{S}\,\frac{l}{a},$$

where S is a constant, depending in magnitude upon the material of the conductor—the greater the conductivity of any material, the smaller the value of S for that material. The constant S for any substance is thus a *specific* constant for that substance, indicating its inherent power of resisting the passage of electricity through it. This constant for any substance is known as the *specific resistance* of that

substance. If we consider a conductor of unit length and unit area of cross section, then—

$$R = S\frac{1}{1} = S;$$

that is, the *specific resistance* of any substance may be defined as the resistance of a conductor of that substance of unit length and unit area of cross section—for example, the resistance between two opposite faces of a unit cube of the substance.

The *specific conductivity* of a substance is sometimes defined as the reciprocal of the specific resistance of the material. Accepting this definition, the *conductivity* of any conductor may be defined as the reciprocal of its resistance. Thus the conductivity, k, of a conductor of resistance R is given by

$$k = \frac{1}{R}.$$

If two conductors, a and b (Fig. 141), of resistances r_1 and r_2, are arranged *in series* in the circuit, the combined

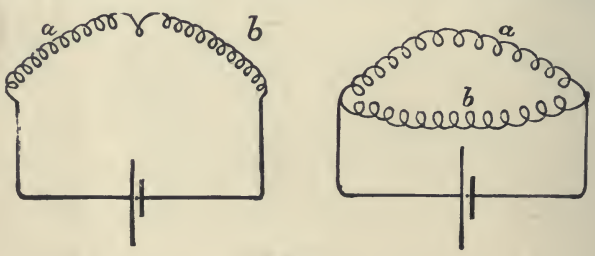


Fig. 141.

resistance of the two is simply $r_1 + r_2$. If, however, they are *in parallel* or *in multiple arc*, as shown in figure, then it is evident that the combined resistance of the two is *less* than either of them, for the current has now two paths open instead of only one. The *conductivity* of the two conductors arranged as here shown is evidently equal to the

sum of the conductivities * of the individual conductors a and b. That is, if k denote the conductivity of the combined conductors, then $k = k_1 + k_2$, where k_1 and k_2 denote respectively the conductivities of the conductors a and b. Hence we have—

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2},$$

where R denotes the resistance of the combined conductors; that is,

$$R = \frac{r_1 r_2}{r_1 + r_2}.$$

Similarly, if any number of conductors of resistances r_1, r_2, r_3 . . . r_n be arranged in parallel, their combined resistance, R, is given by

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots + \frac{1}{r_n}.$$

Considering the relation

$$R = \frac{r_1 r_2}{r_1 + r_2},$$

it is evident that if $r_1 = r_2 = r$, say, then

$$R = \frac{r}{2}.$$

That is, the combined resistance of two conductors of *equal* resistance arranged in parallel is half that of either. This result is evident at once from first principles, for, if the current has two exactly similar paths through which it may divide, then the resistance of the double path will be just one-half that of either of the single paths by itself. Similarly, the combined resistance of n *equal* resistances arranged in parallel is $\frac{1}{n}$-th that of any one.

When a current flows through a *divided circuit* such as

* When the conductors are arranged in series they increase the *resistance* of the circuit, but when arranged in parallel they increase the *conductivity* of the circuit.

that formed by the conductors a and b in Fig. 141, then the current divides along the separate branches: the portion of the total current in any one branch *is proportional to the conductivity of that branch.*

Thus, if c_1 and c_2 denote the currents in a and b respectively, then—

$$\frac{c_1}{c_2} = \frac{k_1}{k_2} = \frac{\frac{1}{r_1}}{\frac{1}{r_2}} = \frac{r_2}{r_1}.$$

That is, the currents in the branches are *inversely* proportional to their resistances. For example, if the resistance of the conductor a be 2 ohms, and that of b 3 ohms, then $\frac{2}{5}$th of the total current flows through b and $\frac{3}{5}$th through a, the *greater* current flowing in the conductor of *lesser* resistance.

An important application of this principle of divided circuits is found in the use of *shunts*. It sometimes happens that the current to be passed through a galvanometer turns out to be too strong for the instrument. When this occurs, instead of changing the galvanometer for another, it is more convenient to join the terminals of the instrument by a wire of known resistance, thus allowing a known portion of the current to be shunted through the wire. The remainder of the current passes through the galvanometer, and may be regulated to any desired strength by varying the resistance of the wire across the terminals. This wire is called a *shunt*, and when a galvanometer has a shunt across its terminals, it is said to be shunted. In Fig. 142, which represents a simple circuit containing a cell and galvanometer, S represents

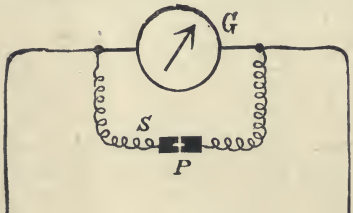


Fig. 142.

P

the shunt and G the galvanometer. The gap at P represents a plug *key* similar to that shown in the figure, and by means of this key the shunt circuit may be opened or closed without disconnecting it from the galvanometer terminals. If G denote the resistance of the galvanometer, S the resistance of the shunt, and C the main current in the circuit, then the current *through the galvanometer* is equal to $\frac{S\,C}{G+S}$, and if S and G are known, the measure of this current given by the galvanometer evidently serves to measure C. When the resistance of the shunt is $\frac{1}{9}$th the resistance of the galvanometer ($S = G/9$), then the current through the galvanometer is $\frac{1}{10}$th the main current. Similarly, if $S = G/99$ or $G/999$, the current in the galvanometer will be $\frac{1}{100}$th or $\frac{1}{1000}$th of the main current. Shunts of this kind are generally supplied with the galvanometer with which they are to be used, and are called the $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th shunts, or the $\frac{1}{10}$th, $\frac{1}{100}$th, or $\frac{1}{1000}$th shunts, according as resistance or current is implied. In general, $S = \frac{G}{n-1}$ where S is the shunt resistance required in order that $\frac{1}{n}$th of the main current may flow round a galvanometer of resistance G. Properly adjusted shunts of this kind are made up in boxes, like small resistance boxes, with convenient arrangements for using any one of the three. For rough general purposes, however, a piece of wire joining the terminals of the instrument to be shunted acts as an efficient shunt, the portion of the current diverted through it being greater the less the resistance of the wire.

The reason given above for the use of shunts is not the only one; they are used generally in practical methods of measurement for a great many different purposes.

The resistance between the terminals of a galvanometer of resistance G, when shunted by a shunt of resistance S, is given by $\frac{S\,G}{S+G}$, and is always less than G; hence, when a galvanometer is shunted, the resistance in the circuit is

diminished, and consequently the main current is increased. When it is desired to keep the main current constant, the shunt boxes used should contain *compensating resistances*, connected to each shunt in such a way that when any shunt is used a compensating resistance equal to $G - (S\,G/S + G)$ is put in the circuit.

Experiment shows that the resistance of a conductor varies with its temperature, the resistance being greater the higher the temperature. If R_0 denote the resistance of a given conductor at 0° C., then the variation with temperature is such that for the resistance at $t°$ we may write—

$$R_t = R_0 (1 + at),$$

where a denotes the coefficient of increase of resistance with temperature.

Owing to this variation of resistance with temperature, it is evidently necessary when stating the specific resistance of any substance to specify the temperature for which the resistance is given.

79. Ohm's Law. In its simplest form, Ohm's law states that the current produced in any portion of a homogeneous conductor lying between two equipotential cross sections at different potentials is directly proportional to the difference of the potentials of these two sections. That is, if C denote the current produced by a difference of potential or electromotive force E, then—

$$C \propto E.$$

In this form, however, the full significance of the law is not at once apparent. The *law* involved is perhaps best explained by reference to the definition of resistance given in the preceding article. The resistance of a conductor is there defined as the ratio of the E. M. F. applied to the conductor to the current produced in it; that is,

$$R = \frac{E}{C}.$$

Now, Ohm's law states that, assuming resistance to be defined in this way, the resistance of a given conductor is constant for all values of E and C, for, if C is proportional to E, then the ratio of E to C must be constant. That is, a

given conductor presents the same resistance to all currents, and is independent of the E. M. F. producing the current. Hence, combining our definition of resistance with Ohm's law, we may write—

$$C = \frac{E}{R},$$

where C denotes the current produced by an electromotive force E in a conductor of resistance R, and R is a constant for a given conductor. The relation, $C = E/R$, which states that the current produced in a conductor of resistance R by an E. M. Force E is given by the ratio E/R, is usually taken as the full expression of Ohm's law.

If E be expressed in volts and R in ohms, then C is given in amperes; that is,

$$C\ (\text{amperes}) = \frac{E\ (\text{volts})}{R\ (\text{ohms})}.$$

This relation may be written in the form

$$E = C\,R;$$

that is, if a current C flow through a conductor of resistance R, then the E. M. F. driving this current *through this conductor* is given by the product C R—or, if a current C is maintained in a circuit of resistance R, the E. M. F. in the circuit is C R. Thus, let a cell of electromotive force E and resistance r have its terminals joined by a wire of resistance R, then the total resistance of the circuit is $(R + r)$, and if C denote the current in the circuit, we have—

$$E = C\,(R + r).$$

Similarly, if e denote the difference of potential of the terminals of the cell, then—

$$e = C\,R,$$

for this difference of potential is a measure of that portion of the E. M. F. of the cell spent in driving the current C through the wire joining the terminals. The application of these results supplies a simple and accurate method for the measurement of the internal resistance of a cell or battery. Let the terminals of the cell be joined to the terminals of a quadrant electrometer, and the deflection corresponding to the E. M. F. of the cell noted. Then, with-

out disconnecting the cell from the electrometer, let its terminals be joined by a wire of *known* resistance R; the presence of the electrometer will in no way interfere with the current, but the deflection indicating the difference of potential at the terminals will decrease from the value corresponding to E to a lower value proportional to *e*. Hence, if D and *d* denote these deflections respectively, we have—

$$\frac{E}{e} = \frac{D}{d};$$

that is,

$$\frac{C(R + r)}{CR} = \frac{D}{d} \text{ or } \frac{R + r}{R} = \frac{D}{d},$$

and therefore

$$r = \frac{R(D - d)}{d}.$$

We have here applied the relation $E = CR$ to a *restricted* portion of a circuit. This is a slight extension of Ohm's law, first made by Kirchhoff, and hence often called *Kirchhoff's second law*. For its simple application, however, the restricted portion considered should be a simple conductor *containing no source of E. M. F.* When the relation $E = CR$ or $C = \frac{E}{R}$ is applied to a complete circuit, then

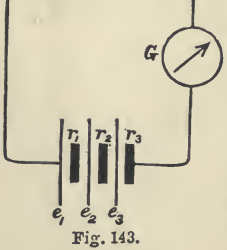


Fig. 143.

R denotes the *total* resistance of the circuit, E the *total* effective E. M. F. in the circuit, and C the current in the circuit. Thus, if three cells of electromotive forces e_1, e_2, e_3 and resistances r_1, r_2, r_3 be arranged, as in Fig. 143, in circuit with a galvanometer of resistance G and a wire of resistance W, then the total effective E. M. F. in the circuit is $e_1 + e_2 + e_3$, and the total resistance $r_1 + r_2 + r_3 + G + W + x$, where x denotes the resistance of the connecting wires of the circuit. Hence, to determine

the current in the circuit, we have—

$$C = \frac{E}{R} = \frac{e_1 + e_2 + e_3}{r_1 + r_2 + r_3 + G + W + x}$$

In general x is so small that it need not be considered. If, in the above arrangement, one of the cells, say that of electromotive force e_3, be reversed, then, assuming $e_1 + e_2$ greater than e_3, the total effective E. M. F. in the circuit would be $e_1 + e_2 - e_3$, and the current would be given by—

$$C = \frac{e_1 + e_2 - e_3}{r_1 + r_2 + r_3 + G + W + x}.$$

It should be noticed that the reversal of the cell in the circuit does not alter the total resistance, that is, we still write $r_1 + r_2 + r_3$.

Similarly, if a battery of n cells *arranged in series*, each of electromotive force E and resistance r, give a current through a circuit in which the total *external* resistance is denoted by R, then the strength of the current is given by—

$$C = \frac{n E}{R + n r}.$$

If, however, the same n cells be arranged *in parallel*, the E. M. F. of the battery is that of one cell, E; but its resistance is $\frac{1}{n}$th the resistance of one cell, for the cross section of the path of the current in the cell is now n times as great as before. Hence the current produced in a circuit of external resistance R is given by—

$$C = \frac{E}{R + \frac{r}{n}},$$

that is,

$$C = \frac{n E}{n R + r}.$$

By comparing these two expressions we can readily compare the efficiency of the two arrangements. It will

be seen that the two values for C differ only in the fact that for the *series* arrangement the resistance of the cell, r, is multiplied by n, while in the parallel arrangement the external resistance R is multiplied by n. Now the value of C will be greatest when the denominator of the equivalent expression is least; hence, the series arrangement gives the better current when r is less than R, and the parallel arrangement is better when R is less than r. The resistance of a cell is generally small; hence we may say generally that parallel grouping should be employed when the external resistance is very small, and the series arrangement when it is comparatively large. When r = R it is immaterial which arrangement is adopted, but the greater R is compared with r the greater the advantage of the series over the parallel arrangement, and the smaller R is compared with r the greater the advantage of the parallel grouping. For example, with ten cells, each having a resistance of 2 ohms and an E. M. F. of 2 volts, let us compare the current given with the two arrangements (a) through an external resistance of 1 ohm, (b) through an external resistance of 1000 ohms. Here, for the case (a) we have—

(i) By series arrangement—

$$C = \frac{10 \times 2}{10 \times 2 + 1} = \frac{20}{21} \text{ ampere.}$$

(ii) By parallel grouping—

$$C = \frac{2}{\frac{2}{10} + 1} = 1\tfrac{2}{3} \text{ amperes.}$$

And for the case (b) we have—

(i) By series—

$$C = \frac{10 \times 2}{10 \times 2 + 1000} = \frac{20}{1020} = \frac{1}{51} \text{ ampere}$$

(ii) In parallel—

$$C = \frac{2}{\frac{2}{10} + 1000} = \frac{20}{10002} = \frac{1}{500} \text{ ampere (nearly).}$$

That is, with the small external resistance the parallel arrangement gives the better current, but with the large

external resistance the current given by the series arrangement is almost ten times as great as that given by the parallel grouping. The series arrangement of case (*b*) shows such a decided advantage because the external resistance of 1000 ohms is very much greater than the 2 ohms internal resistance of the cell.

If we have mn cells, each of electromotive force E and resistance r, arranged in n rows in series, each row containing m cells grouped in parallel, then the current given by this grouping through an external resistance R is given by—

$$C = \frac{n\,E}{n \cdot \frac{r}{m} + R}$$

for the arrangement practically consists of n large cells each of electromotive force E and resistance $\frac{r}{m}$. It can be shown that with a given number of cells the most advantageous grouping is that which gives the internal resistance of the arrangement equal to the external resistance. Thus, in the case just considered, to obtain the best current we must have—

$$n \cdot \frac{r}{m} = R,$$

and combining this condition with the known value of mn, the values of n and m can be determined for any given case. For example, to determine the best * method of grouping 24 cells each of 4 ohms resistance to give a current through the external resistance of 6 ohms, we have—

$$\frac{4\,n}{m} = 6 \qquad (1)$$

and

$$m\,n = 24. \qquad (2)$$

* The *best* method here means the method giving the *strongest* current. It is not the most economical method.

From (1) we get—

$$n = \frac{3m}{2},$$

and substituting this value, in (2) we have—

$$m^2 = 16, \text{ or } m = \pm 4.$$

That is, the cells should be arranged in 6 rows connected in series, each row containing 4 cells in parallel.

The experimental verification of Ohm's law is based upon the following considerations. Let AB (Fig. 144) represent a conductor carrying a *constant* current C. Then, considering a portion, *ab*, of the conductor, the E. M. F.

Fig. 144.

driving the current C through this portion, that is, the difference of potential, e, between the points a and b is given by—

$$e = Cr$$

where r denotes the resistance of *ab*.

But, *since C is supposed to be constant*, we may write—

$$e = (\text{constant}) \times r,$$

or

$$e \propto r$$

that is, the difference of potential between any two points in a conductor carrying a *constant* current is proportional to the resistance of the portion of the conductor included between these points. Now this deduction is very readily put to the test of experiment—the difference of potential between any two points may be measured by a quadrant electrometer as already explained, and with a resistance box in the circuit a number of points with different known resistances lying between them are readily available.* For

* Strictly speaking the resistance between different points on a uniform wire should be taken, for Ohm's law is assumed in the adjustment of the coils of a resistance box, and the experiment here described in reality verifies only the accuracy of the adjustment.

example, if Fig. 139 represent the portion of the circuit including the resistance box, then, all the plugs being out, let the quadrant electrometer be properly connected to the brasses *a* and *b*. A deflection proportional to the difference of potential between *a* and *b* is then obtained, and this difference of potential corresponds to a resistance of *one* ohm between *a* and *b*. Let this deflection be noted, and the electrometer terminals next connected to the brasses *a* and *c*. A deflection proportional to the difference of potential between the points *a* and *c* is now obtained, and this difference of potential corresponds to a resistance of *three* ohms between the points *a* and *c*.

Now, if Ohm's law be true, this second deflection should be exactly three times as great as the first, for the law states that under the conditions of this experiment the difference of potential between any two points is proportional to the resistance between those points. Here then, since the resistance between the points *a* and *c* is three times greater than that between *a* and *b*, the difference of potential, and therefore the electrometer deflection, should also be three times greater. Similarly, if the electrometer be connected, say, to the points *d* and *e*, the deflection should be five times greater than the first deflection for the points *a*, *b*,—and so on. Careful experiments of this nature entirely support Ohm's law, and show that it is, as far as can be detected, rigorously true.

In conducting this experiment, care must be taken to secure an absolutely constant current in the circuit—this may be done by using a battery of, say, a dozen Daniell cells working through a circuit of very *large* resistance. Further, the experimenter must be careful not to alter the resistance of the circuit during the experiment. Connection is readily made with the brass pieces of the resistance box if they are drilled with small holes, to take a small plug carrying a binding screw.

So far we have been accustomed to use the quadrant electrometer whenever it is required to measure the difference

of potential between any two points, but in practice a high resistance reflecting galvanometer is much more convenient to use, and, provided its resistance is high enough, it gives practically the same results as an electrometer.

Suppose, for example, that the terminals of a galvanometer are connected to the points a and b in Fig. 144. The current through the galvanometer will be proportional to the difference of potential existing between the points a and b *after the galvanometer has been joined on.* Now if the galvanometer is of low resistance, the total resistance between a and b will be greatly diminished by connecting the galvanometer to these points, and therefore, by Ohm's law, the difference of potential between a and b is greatly diminished by joining on the instrument intended to measure it. If, however, the resistance of the galvanometer is very large compared with the resistance of ab, then the total resistance between a and b will be very little changed when the galvanometer connections are made, and therefore the deflection produced will be proportional to the difference of potential between a and b when these points are united only by the conductor ab. Considered in this relation, an electrometer acts as a galvanometer of infinite resistance, and therefore the resistance and difference of potential between a and b are unchanged when its terminals are connected to these points.

***80. Comparison of Electromotive Forces.** The quantitative comparison of electromotive forces of voltaic cells or batteries is an important part of electric measurements. We shall here notice a few only of the more important methods. Theoretically, the simplest method of comparison is by means of the quadrant electrometer—one terminal of the cell or battery is connected to one pair of quadrants and the other terminal and pair of quadrants earthed; the deflection obtained is proportional to the difference of potential between the terminals—that is, to the electromotive force of the cell, since the external circuit is not closed. A comparison of deflections obtained in this way with different cells gives a comparative measure of their electromotive forces, and if the E. M. F. of one cell as a standard

(for example Latimer Clark's cell) be known in volts, the E. M. Forces of the other cells can also be expressed in volts. A good form of quadrant electrometer is, however, somewhat troublesome to put in working order, so that in applying this method it is simpler in practice to employ a galvanometer of very high resistance instead of the electrometer, or a condenser and ballistic galvanometer combined in the way described below.

Wiedemann's Sum and Difference method is an easy, convenient method of comparing the E. M. Forces of any two cells not of too nearly equal E. M. F. The cells to be compared are arranged first *in series*, so as to send a current in the *same* direction round a circuit made up of the cells, a tangent galvanometer and a resistance box, and second *in opposition*, so as to send currents in opposite directions round the *same* circuit. The resistance of the circuit is adjusted by means of the resistance box, so that suitable deflections are obtained on the galvanometer. Then if R denote this resistance, which must be same in both cases, and E_1 and E_2 the E. M. Forces to be compared, we have, for the first arrangement,

$$C_1 = \frac{E_1 + E_2}{R} = k \tan \delta_1,$$

where δ_1 is the deflection of the galvanometer needle, and for the second arrangement

$$C_2 = \frac{E_1 - E_2}{R} = k \tan \delta_2,$$

where δ_2 is the galvanometer deflection,

Whence we get

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{\tan \delta_1}{\tan \delta_2},$$

or,

$$\frac{E_1}{E_2} = \frac{\tan \delta_1 + \tan \delta_2}{\tan \delta_1 - \tan \delta_2}$$

A convenient arrangement of apparatus is indicated in Fig. 145.

The mercury cups *a*, *b*, *c*, *d*, can be connected by two short bent pieces of copper wire; and when *a c* and *b d*

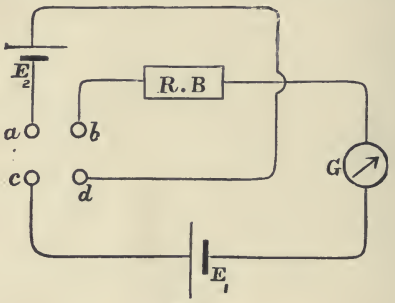


Fig. 145.

are connected the cells are in series, but when *a b* and *c d* are connected, they are in opposition.

Another important method of comparing E. M. Forces is that known as *Poggendorff's Compensation Method.* The

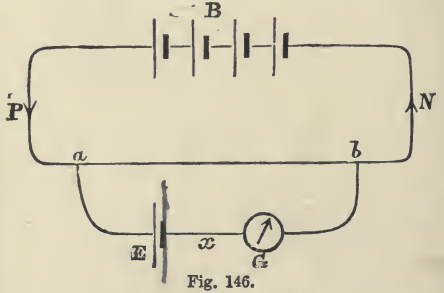


Fig. 146.

principle of this method consists in balancing the E. M. F. of the cells to be compared against the difference of potential between two points in the external circuit of a battery of considerably higher E. M. F. than that of either of the cells. Let B (Fig. 146), represent the battery and PN its external

circuit, the resistance of which is sufficiently great for the difference of potential between the terminals of the battery to be greater than the highest of the E. M. Forces to be compared. The difference of potential between any two points *a*, *b*, on the external circuit will depend upon the resistance between these points, being equal to C R where C denotes the current in the circuit and R the resistance of the portion, *a b*, of the circuit. If the points *a*, *b*, be connected by a wire, *a x b*, a current, due to the difference of potential between *a* and *b*, will flow through it in the direction *a x b*; but if a cell, E, be included in the circuit, so as to tend to give a current in the opposite direction, it is possible, by properly choosing the position of the point of contact *b* to obtain a balance between the E. M. F. of the cell E and the difference of potential between the points *a*, *b*. When this balance is obtained there will be no current in the branch *a x b*, and there will be no deflection of the galvanometer G placed in the circuit. Hence, if the point of contact *b* be moved along the main circuit until the galvanometer G shows no deflection, then we have the E. M. F. of the cell equal to the difference of potential between *a*, *b*, the points of contact—that is, $E = C R$. Thus, if two cells of E. M. Forces, E_1 and E_2, are to be compared, the above adjustment is made with each separately, and we have

$$E_1 = C R_1 \text{ and } E_2 = C R_2,$$

where R_1 and R_2 are the resistances in each case between the points *a*, *b*, when balance is attained. Hence we get

$$\frac{E_1}{E_2} = \frac{C R_1}{C R_2} = \frac{R_1}{R_2}.$$

If the external circuit PN consists of a long uniform wire, the resistance between any two points on it is proportional to the length of wire between the points. Hence in this case the E. M. Forces of the cells will be directly proportional to the length of wire necessary to be included between the points *a*, *b*, to obtain balance in each case. An external circuit of this kind is sometimes called a *Potentio-*

meter, on account of its application to the measurement of potential in this way.

It should be noted that this method is applicable to the absolute measurement of E. M. F., for we have $E = CR$, and if C is measured in some way, say by a tangent galvanometer or voltameter, and R is known, then E can be determined absolutely. This is practically the method adopted by Lord Rayleigh in determining the E. M. F. of the Latimer Clark Cell.

***81. Comparison of Resistances.** In this article we shall consider only the simpler methods based on the *Wheatstone's Bridge* arrangement.

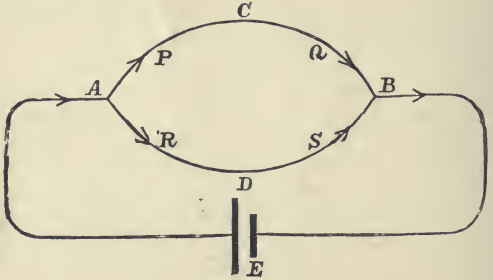


Fig. 147.

In the circuit shown in Fig. 147, the cell E sends a current round a divided circuit; and the difference of potential between the extremities of each branch of the divided circuit is the same; being, in each case, the difference of potential between the points A and B on the main circuit. Hence, in going from A to B along either of the branches A C B or A D B, the same fall of potential takes place, and points can therefore be found in each branch at which the potential is the same as at corresponding points in the other branch. Let C and D be two such corresponding points, then the potential at C is the same as at D—that is, the fall of potential from A to C is the same as that from A to D. Now, the fall of potential between any two points *on the same conductor* is proportional to the resistance

between these points, hence it will be evident that the fall from A to C will be same as that from A to D when the resistance from A to C is the same fraction of the resistance of A C B as the resistance from A to D is of A D B. For example, if the difference of potential between A and B is 3 volts and the resistance of A C B and A D B respectively 15 ohms and 9 ohms, then the fall of potential along A C B is one volt for every five ohms resistance, and along A D B one volt for every three ohms resistance; and if the resistance of A C is 10 ohms (two-thirds of A C B) then the resistance of A D must be 6 ohms (two-thirds of A D B) in order that the points C and D may be at the same potential. The common potential of these points will in this case be two volts lower than that at A and one volt higher than that at B. It is evident that on the branches A C B and A D B any number of pairs of corresponding points, such as C, D, can be found, and in each case the following proportion must hold between the resistances involved: let the resistance of the *arms* A C, C B, A D and D B be denoted by P, Q, R and S respectively, then we must have

$$P : Q :: R : S,$$

that is

$$PS = QR.$$

Now, it is evident that if three of the four resistances P, Q, R, S be known, the fourth can be determined from this relation; or, if *the ratio* of two of them, say P: Q, be known, and another, say R, be known, then the fourth, S, can be determined; provided the four resistances P, Q, R and S are connected and adjusted by experiment in the way indicated above.

The loop A C B D, made up of the four *arms* A C, C B, A D, D B, is known as the *Wheatstone bridge* or *Wheatstone net* arrangement; it may be applied to the measurement of resistance in the following way. Let two equal resistances, say of about 10 ohms each,* constitute the two arms

* Their resistance need not be known if they are known to be equal.

A C and A D, and let the arm C B be a resistance box, and the arm D B the resistance to be measured. Then, if the terminals of a sensitive reflecting galvanometer G (Fig. 148) are connected to the points C, D, it will be possible to adjust for equality of potential of the points C and D; for, if these points are at the same potential, then no current will flow through the galvanometer—that is, there will be no deflection of the galvanometer needle. This adjustment may be made by adjusting the resistance in the resistance box which constitutes the arm C B, for since the resistance of A C is equal to that of A D the adjustment will be complete

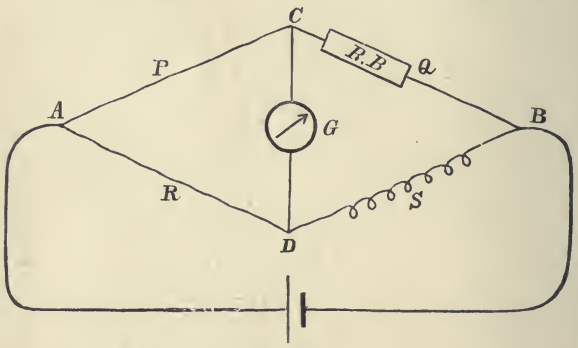


Fig. 148.

when the resistance in the box is equal to the unknown resistance D B.

The details of the construction and practical use of the different forms of Wheatstone's bridge used in the measurement of resistance are best learnt in the laboratory, and for this reason we shall not give any further description of the arrangement.

This Wheatstone's bridge method of comparing resistance is directly adapted to the measurement of simple resistances, but it may be modified so as to be applicable in some other cases. For example, *Thomson's* method of measuring the resistance of a galvanometer by its own deflection is a

modification of the method just described. The galvanometer is placed in the arm B D (Fig. 149), and the points C, D, are connected through a key K, which admits of the connection between C and D being made and broken at will. The resistance in the resistance box in C B is then adjusted until the deflection of the galvanometer is the same, whether the key K makes contact between C and D or not. This adjustment evidently depends on the same principle as the one just described: if the points C and D are at the same potential, it is evidently immaterial

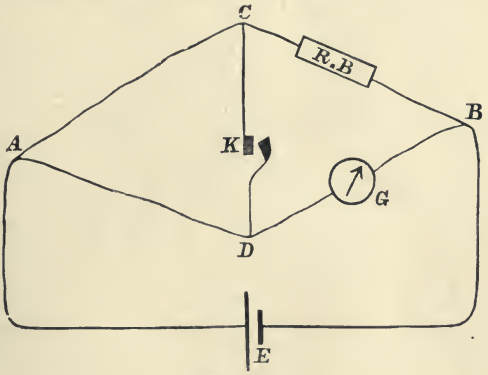


Fig. 149.

whether the key makes contact or not, for in either case no current will flow along C D and consequently the current through the galvanometer will not be changed. If, however, C and D are not at the same potential, then when the key is depressed and makes contact between C and D, a portion of the current passing through the galvanometer is shunted along C D and the deflection of the galvanometer is changed. Hence, when the resistance in C D is so adjusted that the depression of the key has no effect on the galvanometer deflection, the resistance of the galvanometer can be determined from the relation $P S = Q R$; or if, as assumed above, $P = R$, then $Q = S$—that is, the resistance

out in the resistance box when the adjustment is complete gives the resistance of the galvanometer.

Another application of this method is *Mance's* method of measuring the internal resistance of a cell or battery. It evidently would not do to place the cell in the arm B D and attempt to measure its resistance as if it were a simple resistance; for being a source of E. M. F., the current produced by it in the circuit would disturb the adjustment. In Mance's method the cell whose resistance is to be measured is the only one in the circuit, and it is placed in the arm B D, and the place of the cell E is taken by a key K

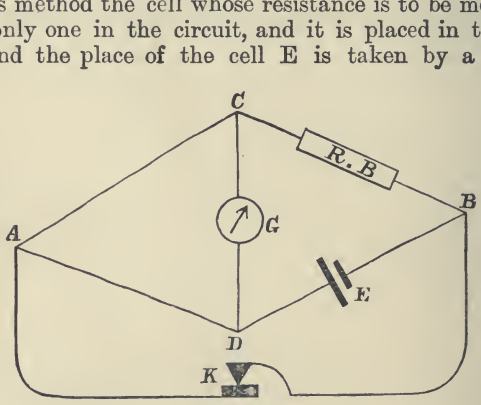


Fig. 150.

(Fig. 150): that is, just as in Thomson's method the *galvanometer* whose resistance is to be measured is placed in the arm B D and the usual place of the *galvanometer* taken by a key, so in this method, the *battery* whose resistance is to be measured is placed in the arm B D and the usual place of the *battery* is taken by a key. The adjustment is exactly as in Thomson's method,—the resistance in the arm C B is adjusted until the deflection of the galvanometer is not affected by the working of the key, and the resistance of the battery determined by calculation from the relation P S = Q R. The principle of this adjustment is slightly different to that in the two cases just described; here the adjustment does not depend upon equality of potential of

the points C, D, whether the key K is open or closed, but upon the constancy of the difference of potential between these points whether the circuit through K be made or not. The conditions of this adjustment are however the same as before, for when K is closed the resistance between the points A, B, is lessened and the difference of potential between these points changes. Hence, when the key is closed, a sudden change takes place in the difference of potential between the points A, B, and this change will, or will not, make itself felt between the points C, D, according as the relation $P S = Q R$ holds or not.

For clearness in describing the above methods we have always supposed the resistance box to be in the arm B C, the unknown resistance in B D, and so on, but this arrangement is evidently not essential—any one of the resistances may be placed in any one of the arms without affecting the general principle of adjustment.

82. The Ballistic Galvanometer. A ballistic galvanometer is an ordinary low-resistance reflecting galvanometer with a needle of large moment of inertia. It is used for measuring the *quantity* of electricity passed through it not as a continuous current but as a sudden discharge, and the needle has a large moment of inertia, so that it will be slow in beginning to move under the impulse of the sudden discharge, and will therefore not have moved appreciably from its position of rest during the time the discharge takes to pass through the galvanometer.

It can be shown that in a galvanometer of this kind the sine of the half the angle of deflection is proportional to the quantity of electricity discharged through it, or where the angle of deflection is small, as it always is in actual work with a reflecting galvanometer, the quantity of electricity discharged is directly proportional to the deflection of the spot on the galvanometer scale.

The capacity of two condensers may be readily compared by means of a ballistic galvanometer. Let the capacities of the two condensers be C_1 and C_2, and let them be, separately, first charged by a battery of constant E. M. F., E, and then discharged through the galvanometer. The

quantities of electricity discharged are respectively $E C_1$ and $E C_2$, and if d_1 and d_2 are the corresponding galvanometer deflections, then,

$$\frac{E C_1}{E C_2} = \frac{C_1}{C_2} = \frac{d_1}{d_2},$$

—that is, the capacities are in the same ratio as the deflections.

Similarly, the E. M. Forces of two cells or batteries are readily compared by this method. Let a condenser of capacity C be charged first by the cell of E. M. F. E_1 and discharged through the galvanometer giving a deflection d_1 and afterwards charged by the other cell of E. M. F. E_2, and again discharged through the galvanometer producing a deflection d_2. Then, as before, we get

$$\frac{E_1 C}{E_2 C} = \frac{E_1}{E_2} = \frac{d_1}{d_2},$$

—that is, the E. M. Forces are directly proportional to the deflections.

This is a simple, accurate and ready method of comparing electromotive forces. By means of a key known as a discharging key the above operations of charging and discharging are readily and easily performed, but the student will more readily and satisfactorily learn the practical detail of the above methods in a laboratory than from a text-book.

TABLE OF ELECTRICAL UNITS.

	Electrostatic C.G.S. units in 1 electromagnetic C.G.S. unit.	Electromagnetic C.G.S. units in 1 (electromagnetic) "practical" unit.	Electrostatic C.G.S. units in 1 electromagnetic "practical" unit.	Name of electromagnetic "practical" unit.
Charge or quantity	v	$\frac{1}{10}$	$\frac{v}{10}$	Coulomb
Potential	$\frac{1}{v}$	10^8	$\frac{10^8}{v}$	Volt
Capacity	v^2	10^{-9}	$10^{-9}\,v^2$	Farad
		10^{-15}	$10^{-15}\,v^2$	Microfarad
Current	$[v]$	$\frac{1}{10}$	$\left[\frac{v}{10}\right]$	Ampere
Resistance	$\left[\frac{1}{v^2}\right]$	10^9	$\left[\frac{10^9}{v^2}\right]$	Ohm

v is the velocity of light or any other electrical disturbance of the ether in C.G.S. units, and its numerical value is about 3×10^{10}.

The electrostatic units in square brackets are not in use.

There are no names for the C.G.S. units of either system. They are called simply "C.G.S. electrostatic unit of potential," and so on.

10^9 EMu = 1 ohm

1 ESu = 3×10^{10} EMu

10^9 EMu = 1 ohm = 30 ESu

1 ohm = 10^9 EMu = 30 ESu

CHAPTER XIV.

CHEMICAL EFFECTS OF CURRENTS.

83. Chemical Equivalence. When dilute sulphuric acid acts on zinc, the chemical reaction which takes place may be represented by the equation—

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

Here it will be noticed that one atom of zinc displaces two atoms of hydrogen, and hence we say that one atom of zinc is chemically equivalent to two atoms of hydrogen. The atomic weight of zinc is 65, and that of hydrogen is 1; therefore we may say that 65 grams, or pounds, or tons, of zinc are chemically equivalent to 2 grams, etc., of hydrogen.

Similarly, from the equation—

$$AgNO_3 + HCl = AgCl + HNO_3,$$

representing the reaction between silver nitrate and hydrochloric acid, we see that one atom of silver is equivalent to one atom of hydrogen. Combining this result with the previous one, we see that one atom of zinc is equivalent to two atoms of hydrogen, and also to two atoms of silver; that is, 65 grams of zinc are equivalent to 2 grams of hydrogen or to 216 grams of silver. In this way, if the atomic weights and valency of the elements are known, we are at once able to state the mass of any element chemically equivalent to unit mass of hydrogen. The number ex-

pressing this mass is called the *chemical equivalent* of the element, and is evidently obtained by dividing the atomic weight of the element by its valency. Thus, zinc being a dyad, its chemical equivalent is $\frac{65}{2} = 32{\cdot}5$; but silver being a monad, its chemical equivalent is $\frac{108}{1} = 108$.

84. Laws of Electrolytic Chemical Action. The simple laws of chemical action are two in number:—

(1) *The amount of chemical action in any one cell of a circuit is equivalent to that in any other cell.* This law applies to all cells in the circuit, whether battery cells or electrolytic cells.

Thus, if a current from, say, six Daniell's cells be passed through a circuit of three voltameters arranged for the electrolysis of copper sulphate, silver nitrate, and water respectively, then the amount of copper deposited in *each* battery cell is the same as that deposited on the cathode and dissolved from the anode of the copper sulphate voltameter. Also, the amounts of zinc, copper, silver, oxygen, hydrogen, etc., liberated in the several cells are all chemically equivalent. For example, for every 108 parts of silver deposited in the silver nitrate voltameter, 65 parts of zinc are dissolved in *each* battery cell. By arranging a circuit of this nature, and determining quantitatively the amount of chemical action in each cell, this law may be put to the test of experiment, and will be found to be strictly true.

(2) *The amount of chemical action in any cell, in a given time, is directly proportional to the quantity of electricity which has passed through the cell in that time;* that is, if m denote the mass of a particular ion liberated in a given cell by a current C in a time t, then—

$$m \propto \mathrm{C}\,t,$$

for $\mathrm{C}\,t$ denotes the quantity of electricity which has passed through the cell in the time t.

The mass of an ion liberated in one second by a current of one ampere is called the *ampere electrochemical equivalent*

of the ion. If this equivalent for any element be denoted by w, then it is evident from this second law that the amount of the element deposited in a time t by a current C is given by —

$$m = w \cdot C t,$$

that is,

$$m = C w t.$$

The electrochemical equivalent of silver has been very carefully determined, and is found to be equal to 0·001118 grams per ampere; that is, a current of one ampere passing through a solution of silver nitrate deposits silver on the cathode at the rate of 0·001118 grams per second. From this determination for silver the electrochemical equivalent of any other element may be determined by calculation, for the *electrochemical* equivalents of any two elements are directly proportional to their *chemical* equivalents. Thus, if w denote the electrochemical equivalent of hydrogen, then—

$$108 : 1 :: 0{\cdot}001118 : w,$$

or

$$w = \frac{0{\cdot}001118}{108} = 0{\cdot}00001035,$$

and the electrochemical equivalent of any element is readily found by multiplying this value for hydrogen by the chemical equivalent of the element. For example, the electrochemical equivalent of oxygen is—

$$\frac{16}{2} \times {\cdot}00001035 = 0{\cdot}00008281 \text{ grams per ampere,}$$

and that of zinc is—

$$\frac{65}{2} \times {\cdot}00001035 = 0{\cdot}0003364 \text{ grams per ampere,}$$

and so on.

It is evident that this law forms a basis for a method of measuring current. Thus, the ampere, the practical unit

of current, may be taken as that constant current which, when passed through a solution of silver nitrate in water, deposits silver at the rate of 0·001118 grams per second. Hence, if an unknown current C be passed through a solution of silver nitrate in water, and deposit m grams of silver in t seconds, then from the relation—

$$m = C w t,$$

we have—

$$C = \frac{m}{0{\cdot}001118 \times t} \text{ amperes.}$$

This definition of the ampere is that adopted in the Report to the Board of Trade by the Electrical Standards Committee, and appended to their Report is a specification giving detailed instructions for the use of a silver voltameter in the measurement of current. We quote a portion of this specification which explains in a very clear way the different operations involved in the process.

"In employing the silver voltameter to measure currents of about one ampere, the following arrangements should be adopted. The kathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10 cm. in diameter, and from 4 to 5 cm. in depth.

"The anode should be a plate of pure silver some 30 square cm. in area, and 2 or 3 mm. in thickness.

"This is supported horizontally in the liquid near the top of the solution by a platinum wire passed through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling on to the kathode, the anode should be wrapped round with pure filter paper, secured at the back with sealing wax.

"The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

"The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The

total metallic resistance of the circuit should not be less than 10 ohms.

"*Method of making a measurement.* The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a dessicator. When thoroughly dry, it is weighed carefully.

"It is nearly filled with the solution, and connected to the rest of the circuit by being placed on a clean copper support, to which a binding screw is attached. This copper support must be insulated.

"The anode is then immersed in the solution so as to be well covered by it, and supported in that position; the connections to the rest of the circuit are made.

"Contact is made at the key, noting the time of contact. The current is allowed to pass for not less than half an hour, and the time at which contact is broken is observed. Care must be taken that the clock used is keeping correct time during this interval.

"The solution is now removed from the bowl, and the deposit is washed with distilled water and left to soak for at least six hours. It is then rinsed successively with distilled water and absolute alcohol, and dried in a hot air bath at a temperature of about 160° C. After cooling in a dessicator, it is weighed again. The gain in weight gives the silver deposited.

"To find the current in amperes this weight, expressed in grammes, must be divided by the number of seconds during which the current has been passed and by ·001118.

"The result will be the time-average of the current, if during the interval the current has varied."

In laboratory practice it is often more convenient to use a small copper voltameter, made up with two copper plates immersed in a solution of pure copper sulphate, instead of a silver voltameter; and the method is seldom used for the direct measurement of current, but usually to determine the constant of a tangent galvanometer or other current-measuring instrument.

For this purpose the galvanometer and voltameter are

arranged in series in the circuit, and precautions taken to secure a constant current. The current is then allowed to pass for a given time, and since the same current passes through the voltameter and the tangent galvanometer, say, then—

$$C = \frac{m}{w\,t} = k \tan \delta,$$

where δ denotes the deflection of the galvanometer. Hence we have—

$$k = \frac{m}{w\,t \;.\; \tan \delta},$$

and the constant k is thus determined in terms of m, w, t, and δ.

CHAPTER XV.

HEATING EFFECTS OF CURRENTS.

85. Conservation of Energy in a Voltaic Circuit. In dealing with the conservation of energy in a circuit there are three essential points that require consideration. These are, first, the work done in the voltaic cell or battery; second, the work done in the external circuit; and, third, the energy of the current. Of these the first—the chemical work done in the cell—is the source of energy in the circuit and gives rise to the other two, which must therefore be together equivalent to it.

The chemical work done in a voltaic cell in a given time is the mechanical equivalent of the heat which would be generated by the total chemical action going on in the cell in that time. Thus, in a Daniell's cell, if 32·5 milligrams of zinc are dissolved in one hour, then the chemical work done in the cell in the hour is the mechanical equivalent of the heat generated by the solution of 32·5 milligrams of zinc in sulphuric acid, and the solution in water of the zinc sulphate so formed, *less* the heat absorbed by the deposition of 31·5 milligrams of copper from copper sulphate, and the abstraction of the necessary copper sulphate from its solution in water.

The work done in the external circuit may be either of a chemical or magnetic nature; for example, the circuit may include a voltameter in which electrolytic chemical work may be done, or an electromagnet whose power of attracting iron may be employed in doing mechanical work. In either case the energy expended in doing work in the

external circuit takes no part in maintaining the current in the circuit: only that portion of the cell's energy which is not spent in doing other work in the circuit is available for the production of current, and the energy of the current so produced is entirely dissipated as heat in the circuit.

From what has been said it is evident that Ohm's law requires some modification, for, in accordance with this law, a battery of fixed electromotive force E, working in a circuit of resistance R, should give a constant current $C = \frac{E}{R}$. From what we have just said, however, it appears that the current in the circuit may be varied by varying the work to be done in the external circuit, for the less the work to be done, the greater the proportion of the battery's energy available for the production of current, and therefore the strength of the current should increase as the work done in the external circuit diminishes.

Now it is found that wherever work is done in the external circuit, a "back" E. M. F. is set up in opposition to the main E. M. F. of the battery, and in applying Ohm's law this back E. M. F. has to be considered. In the relation $C = \frac{E}{R}$, E denotes the *effective* or *resultant* E. M. F. in the circuit, and is therefore equal to the E. M. F. of the cell, *less* all back electromotive forces set up at points where work is done in the external circuit. When the external circuit consists only of simple metallic conductors, then there is no back E. M. F. to be considered; but if a water voltameter, for example, be included in the circuit, then a back E. M. F. due to polarisation in the voltameter is set up; and if e denote this back E. M. F., and E the E. M. F. of the cell, then the current in the circuit, according to Ohm's law, is given by—

$$C = \frac{E - e}{R},$$

where R denotes the resistance of the circuit.

It is important to notice that, in the case of a silver or copper voltameter, *no* back E. M. F. is set up, for in each of these cells the same metal is dissolved from the anode and

deposited on the cathode, so that on the whole *no* chemical work is done and no polarisation takes place.

It is evident from this that the work done in maintaining the current for a given time may be determined by considering the work done in raising the quantity of electricity which flows through the circuit in that time, against a difference of potential equal to the effective E. M. F. in the circuit.

Hence, if a current C flow for a time t round a circuit in which the *effective* E. M. F. is E, then a quantity of electricity, Ct, is raised through a difference of potential E, and the work done in maintaining the current for the time t is given by—

$$W = C t E.$$

If C and E be expressed in electromagnetic units, and t in seconds, then W is given in ergs, but if C is in amperes and E in volts, then W is given in units of work called *Joules*, one joule being equal to 10^7 ergs.* Thus, C amperes $= \frac{C}{10}$ electromagnetic units, and E volts $= E \times 10^8$ electromagnetic units, and therefore—

$$C t E \text{ joules} = \frac{C}{10} t \times E \times 10^8 \text{ ergs},$$

that is,

$$C t E \text{ joules} = C t E \times 10^7 \text{ ergs};$$

and therefore

$$1 \text{ joule} = 10^7 \text{ ergs}.$$

The work done on the current *per second*, that is, the *activity* of the cell or battery in supplying the current, is evidently given by the product CE, and if C and E are expressed in practical units—amperes and volts respectively—then the activity is given in units known as *Watts*, a watt being one joule per second, therefore equal to 10^7 ergs per second.†

* In *ergs* because the electromagnetic system of units is based on the C.G.S. system, of which the *erg* is the unit of work.

† One horse-power = 746 watts (nearly).

86. Laws of the Development of Heat in the Circuit. We have in this way found an expression for the work done on the current, that is, the work done by the cell in maintaining the current, and we have now to consider what becomes of the energy thus supplied to the current. As already stated, *all* the energy of the current is spent in developing heat in the circuit, and, if R denote the resistance of the circuit, then since

$$W = C E t, \text{ and } E = C R,$$

we have

$$W = C \cdot CR \cdot t = C^2 R t.$$

That is, the heat produced in a circuit of resistance R by a current C in a time t is proportional to $C^2 R t$. Hence, the laws of the development of heat in a circuit carrying a current may be formulated thus:—

(1) The heat developed in a given time in a circuit of given resistance is directly proportional to the *square* of the current strength.

(2) The heat developed by a given current in a given time is directly proportional to the resistance of the circuit.

(3) The heat developed by a given current in a circuit of given resistance is directly proportional to the time the current passes.

These laws are found to be true, *whatever* be the nature of the circuit considered. Thus, if a current, C, flow through a circuit made up of a battery of resistance B, a wire of resistance W, a galvanometer of resistance G, and two voltameters of resistance V_1 and V_2, then the heat developed in the circuit in a time t is proportional to $C^2 R t$, where R denotes the *total* resistance of the circuit, and is therefore equal to $(B + W + G + V_1 + V_2)$. Further, the laws may be applied not only to the circuit considered as a whole, but also to *any portion of the circuit.* Thus, in the example given above, the heat developed in the wire is proportional to $C^2 W t$, in the battery to $C^2 B t$, and so on. This application of the laws to a portion of the circuit gives the useful rule that the amounts of heat developed in different portions of the same circuit are directly proportional to the resist-

ances of the portions considered. For example, in the case considered above, the heat developed in the battery is to the heat developed in the wire, as B, the resistance of the battery, is to W, the resistance of the wire. Hence, if it is desired to confine the development of heat as much as possible to one part of the circuit, the resistance of every other part should be as small as possible; for example, in incandescent lamps.

In applying these rules to voltameter cells in which chemical work is done, students beginning the subject may find it hard to understand how any chemical work can be done in the cell if the whole energy of the current is spent in producing heat. It has already been explained that in general, when electrolysis takes place in a voltameter cell, the products of electrolysis accumulate on the plates of the cell and set up polarisation. The back E. M. F. due to this polarisation acts in opposition to the E. M. F. of the battery, and the current is thereby reduced below the value it would have if a simple conductor of the same resistance as the voltameter cell were substituted for it in the current. The energy supplied by the battery has now two things to do: first, to overcome the back E. M. F. due to polarisation; second, to maintain a current in the circuit—the energy expended in the first case appears as chemical work in the voltameter cell, and that expended in the maintenance of the current appears as heat in the circuit. Thus, if E denote the E. M. F. of a battery and e the back E. M. F. due to polarisation of an electrolytic cell in the circuit, then the current produced in a circuit of resistance R is given by—

$$C = \frac{E - e}{R}.$$

The total work done by the battery per second is expressed by CE, and of this a portion Ce is spent in doing chemical work in the voltameter cell, while the remainder $C(E-e)$, that is, $C^2 R$, is spent in maintaining the current C in the circuit, and appears as heat in the circuit. Thus all the energy of the current is dissipated as heat in the circuit, though all the work done in the battery may not

be spent in producing current—in fact, the energy of the current represents the *difference* between the work done in the battery and that done at other points in the circuit, and really represents the waste of energy in the circuit. Hence, if the chemical work to be done in a voltameter cell is equal to or greater than that which can be done in the battery, then no current will flow. This explains why a single Daniell's cell cannot decompose water—the energy necessary to decompose water into oxygen and hydrogen is greater than that supplied by the chemical action going on in the Daniell, or, in other words, the back E. M. F. due to polarisation in the voltameter cell is greater than the E. M. F. of a single Daniell.

87. Experimental Verification of the Laws of Development of Heat in a Current. The truth of the laws given above may be verified in a general way by the following experiment. A divided circuit of two branches is prepared as follows: one branch, A *a* B (Fig. 151), is made up of two pieces of thick copper wire united by a thin piece of brass or platinum wire — say two feet long — coiled into a spiral; the other branch, A *b c* B, is similarly made up of the thick copper wire and *two* spirals of the same thin wire—the length of wire in each spiral is to be exactly one-quarter that of the single spiral in the branch A *a* B, and the total length of thick copper wire should be one-half that used in A *a* B. Prepared in this way, the resistance of the branch A *b c* B is exactly one-half that of A *a* B; and if the points A and B be connected to the terminals of a battery, the current flowing through A *b c* B will be twice that in A *a* B. Now let us consider the relative amounts of heat developed in the spirals at *a*, *b*, and *c*. The same current flows through *b* and *c*, and these spirals are of equal resistance;

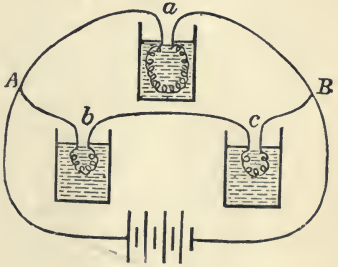


Fig. 151.

hence the rate of development of heat will be the same in each. Again, comparing either *b* or *c* with *a*, the current in *a* is one-half that in *b* or *c*, and therefore, if its resistance were the same, the heat developed in it would be only one-quarter that developed in *b* or *c*; but the resistance of the spiral being four times greater than that of *b* or *c*, the heat actually developed in it per second must be exactly the same as for *b* and *c*. Hence, if the laws of heating are correct, heat should be produced at the same rate in each of the spirals, *a*, *b*, and *c*. To test if this is so, let the spirals be immersed in exactly equal amounts of water in three exactly similar calorimeters. Under these conditions the temperature of the water in each calorimeter should rise at precisely the same rate, and if the temperatures be noted and compared from time to time, this will be found to be the case.

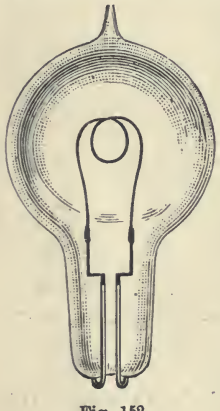


Fig. 152.

One of the most important applications of the heating effects of current is the *incandescent lamp*. This consists of a glass globe (Fig. 152), completely exhausted of air, and containing a fine filament of *carbon* of high resistance attached to terminals of platinum wire fused through the glass. When a strong current is sent through the filament, the carbon becomes white hot, and, being in a vacuum, does not burn, but glows with a steady bright light.

In *arc lamps* a strong current passes across a small gap between the ends of two carbon rods, and the incandescent matter in the gap, together with the incandescent ends of the rods, is the source of light. The difference of potential between the ends of the carbon rods (about 50 or 60 volts) is not nearly high enough to spark across an air gap between them; but if the carbons are put in contact for an instant, the heat developed at the point of contact is great enough to heat the ends of the rods to incandescence; and

if they are then separated a little, the incandescent matter in the gap conducts the current across. The ends of the rods are thus kept glowing, and the bright *arc* between them persists as long as the current passes.

CHAPTER XVI.

MAGNETO-ELECTRIC INDUCTION.

88. Fundamental Experiments. In the preceding chapter it has been stated that when *work is done* in the external circuit of a battery a back E. M. F. is set up in opposition to the main E. M. F. of the battery. From this it follows that if by any means we can *do work* in the external circuit, then an E. M. F. in the same direction as the E. M. F. of the battery will be established in the circuit. Now if we are dealing with chemical work, we can do work in the external circuit by replacing the voltameter, which absorbs energy in the circuit, by a voltaic cell, which supplies energy, and consequently establishes an E. M. F. in the circuit in the same direction as the E. M. F. of the battery. If we are dealing with the mechanical work done by an electromagnet in the circuit, then we may do work in the external circuit by doing mechanical work *against* the attracting or repelling force of the magnet. In doing work in this way an E. M. F. is established in the circuit, acting in the same direction as the E. M. F. of the battery, and the current is increased.

These effects with an electromagnet are neatly shown by the apparatus of Fig. 153. The circuit contains an electromagnet, M, and a small incandescent lamp, L. When the magnet is allowed to do work in attracting the keeper K, the back E. M. F. produced diminishes the current, and the lamp momentarily ceases to glow, or glows less brightly. If, however, work be done against the attracting force of the magnet by quickly withdrawing K, then the E. M. F. produced increases the current, and the lamp glows out

more brightly for an instant. In the same way, if another magnet be employed instead of the bar K, the forces of attraction and repulsion will be stronger, and the effects are

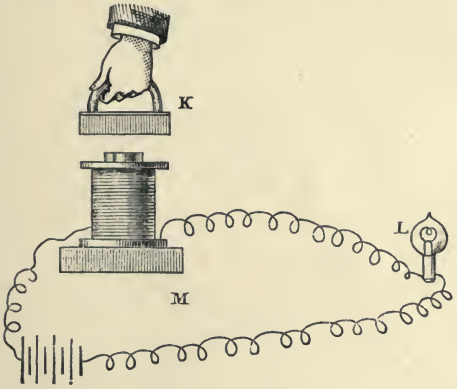


Fig. 153.

more marked. If the results of this experiment be carefully noted, it will be seen that an E. M. F. is produced in the circuit only while work is being done on or by the magnet; hence, with an electromagnet in the circuit there is no *continuous* back E. M. F. as there is with a voltameter, for in a voltameter chemical work is always going on, and therefore the back E. M. F. is continuous, but with an electromagnet the back E. M. F. appears only when work is done, and its magnitude depends upon the rate at which work is done.

It is not, however, necessary to work with an electromagnet to get effects similar to these. Let an insulated wire be coiled round a permanent bar magnet, and its ends joined to the terminals of a reflecting galvanometer. Then if the magnet be made to do work by attracting a piece of iron up to it, an E. M. F. is set up in the circuit, and the current produced in the circuit causes a deflection of the galvanometer needle. Similarly, if work is done by

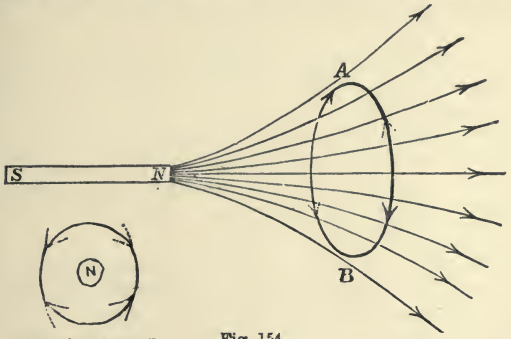
withdrawing the iron from the magnet, then an E. M. F. in the opposite direction is produced, and the resulting current deflects the galvanometer needle in the opposite direction. Further, from what has been said above in connection with the electromagnet, it is evident that the current produced in the first case flows round the circuit in a direction opposite to that of a current which would magnetise the magnet with its existing polarity—that is, in a direction which would be opposed to that of the main current in the circuit if the magnet were an electromagnet. Similarly, in the second case, the current produced would be such as to magnetise the magnet with its existing polarity.

It will be understood that the energy of currents produced in this way is the equivalent of the work done by or against the magnetic force due to the magnet. Such currents are said to be *induced*, and the phenomena connected with them are dealt with under the head of magneto-electric induction. When a magnet does work, for example, in attracting a piece of iron towards it, we know that the work is done by means of the intervening medium. The strained medium possesses energy, and if the state of strain be altered in any way, then some energy may be freed to do work, or additional energy may be absorbed from work done.

Hence, in dealing with induction it is simplest to deal with the state of the medium surrounding the circuit in which the induced currents are produced. Considered in this way, it is found that induced currents may be produced in any circuit by altering the flow of magnetic force through that circuit, that is, a current is produced in any circuit whenever a change takes place in the number of lines of force running through the area bounded by the circuit.

For example, in Fig. 154, the number of lines of force which run through the coil of wire AB from the north pole of the magnet NS may be changed by *any* relative motion of the magnet and the coil. Thus, if AB be moved to the right away from NS, or if NS be moved to

the left away from AB, then the number of lines of force passing through the coil will be decreased, and a current



End view from E. Fig. 154.

will be induced in it in a direction indicated by the full arrowheads in the figure. Similarly, if the coil and magnet be made to approach each other by the motion of either, then the number of lines of force running through the coil is increased, and an induced current traverses the coil in the direction indicated by the dotted arrowheads, that is, in a direction opposed to that of the current induced by the decrease in the number of lines of force. Further, if the magnet NS be suddenly reversed,

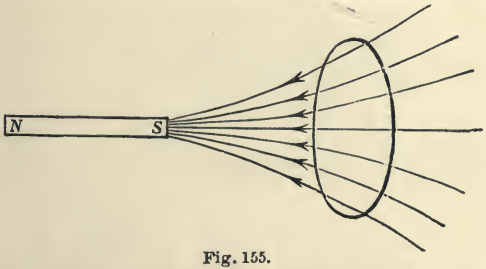


Fig. 155.

as in Fig. 155, so as to bring its south pole next the coil, the flow of force through the coil will also be reversed,

and a strong induction current will be produced in the coil. The number of lines of force passing through the coil are now numerically the same as before, *but their sign is opposite*, so that by the reversal of the magnet the number of lines of force running through the coil have been *reduced* from a *positive* value down to a numerically equal *negative* value.

If the magnet and coil be moved relative to one another as before reversal, the same effects will be produced, but the directions of the induced current will be opposite to those indicated in Fig. 154, when the north pole of the magnet was next to the coil.

The direction of the current in the coil under given conditions may be determined by the following simple rule. If a person look through the circuit along the positive direction of the lines of force, that is, in the direction in which a north pole would be urged, then, for a *decrease* in the number of lines running through the circuit, the induced current flows round the circuit in the same direction as the hands of a clock move. It is of the greatest importance to understand that these induced currents are momentary, and last only while the flow of force through the circuit is changing.

The experiment illustrated in Fig. 154 usually takes a more elaborate form, suitable for the experimental verifica-

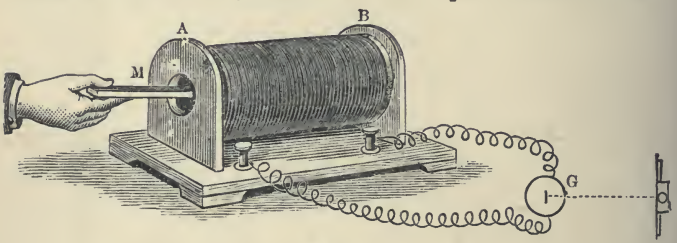


Fig. 156.

tion of the phenomena of induction. The single coil of wire is replaced by a long coil of wire, wound on a large bobbin, AB (Fig. 156), the ends of the wire being attached

to binding screws on the baseboard to which the bobbin is fixed. The terminals of a reflecting galvanometer, G, are connected to these screws, and the coil of the galvanometer thus completes the circuit of the coil on AB. If a strong magnet M be now moved relatively to AB—for example, by thrusting it into the interior of the coil, and then withdrawing it—well marked induction currents are produced, and are indicated by the deflection of the galvanometer in the circuit. Thus, if the north pole of the magnet be thrust into the coil, the spot on the scale is momentarily deflected in one direction, and then comes to rest; on withdrawing the magnet a deflection in the opposite direction is produced; and if the south end of the magnet be inserted and withdrawn, reverse effects are obtained.

Since, in order to produce induction currents in a circuit, it is only necessary to change in any way the flow of magnetic force through the circuit, it is evident that any relative displacement of the magnetic system and the circuit, or the introduction of any magnetic body into the system, may induce a current in the circuit. For example, in Fig. 154, if a piece of soft iron be brought up to AB from the side remote from the magnet, then an increased number of lines of force will be drawn through the coil up to the iron, and a current will therefore be induced in the coil; or, if the coil AB be made to rotate round a diameter at right angles to the plane of the paper, then it is evident that, as the coil rotates, the flow of force through it will be continually changed, and induction currents will be set up in it; and the student will find it a useful exercise to determine, by aid of the rule given above, the direction of the induced current at any stage in the revolution.

From this it will be understood how the magnetic field of the earth may be made to yield induction effects. Let a coil of wire wound on a large circular frame be constructed so as to rotate freely round one diameter as an axis. If this coil be placed with its axis vertical, and caused to rotate, the number of lines of force passing through the coil will vary as the coil spins, and currents will be induced in the coil. Since the axis of the coil is vertical, the plane

of the coil will always be vertical, and therefore the vertical component of the earth's magnetic field need not be considered, for no vertical lines of force can in this case cross the circuit of the coil. Hence, under the conditions here considered, we shall have to do only with H, the horizontal component of the earth's field. Imagine the coil to start rotating from a position at right angles to the magnetic meridian, and the east side of the coil to rotate towards the south. When at right angles to the magnetic meridian, the greatest possible number of lines of force cross the circuit of the coil, for the plane of the coil is at right angles to the horizontal component of the earth's field, and H lines of force cross each unit of area enclosed by the coil. As the coil rotates the number of lines passing through the coil decreases until, when parallel to the magnetic meridian, the number is diminished to zero. The total flow of force across the coil when at right angles to the meridian is given by nAH, where n denotes the number of turns of the coil and A the area enclosed by the coil; for in the case of any one turn the number of lines of force passing across it is AH, if A denote the area of the single coil; and as this is true for every one of the n turns, it is evident that the total flow of force across the coil is nAH. Hence, during the first 90° of a revolution, starting from a position at right angles to the magnetic meridian, the flow of force across the coil *decreases* from nAH to zero, and the direction of the induced current will be from west to east (for the starting position) along the upper half of the coil. The direction of the induced current is readily determined by the application of the rule given above. Suppose a person to stand on the south side of the coil, so as to look, as specified in the rule, through the coil along the lines of force—that is, towards the north; then, as we are here dealing with a *decrease* of the flow of force through the coil, the induced current will flow round the coil in the same direction as the hands of a clock move—that is, from west to east, along the upper half of the coil. During the next 90° of rotation the flow of force will evidently *increase* from zero to nAH, and the direction of the induced current will

be from east to west (for the final position), along the upper half of the coil—that is, the current will still be in the same direction *in the coil*; for although its direction *in space* is reversed, the position of the coil is similarly reversed by revolution, so that the direction of the current in the coil is the same during the first 180° of revolution, starting from the position in which the flow of force through the coil is a maximum. During the next 180° of revolution the same changes in flow of force will take place as described above, and a current will be induced *in the coil* in the *opposite* direction to that induced during the first half-revolution, owing to the lateral reversal of the coil by that half-revolution. Hence during each complete revolution the direction of the induced current changes as the coil passes through the position at right angles to the direction of the lines of force in the field. If the terminals of the coil be connected, one to each half of a ring cut into two halves across a diameter, a continuous current in one direction may be obtained by fixing the split ring on the axis of the coil and drawing off the current by springs resting on the rotating ring, and so placed that each spring changes from one half of the ring to the other at the same time as the reversal of current in the coil takes place. Such an arrangement would be a simple form of dynamo.

If the coil be fixed with its axis horizontal and parallel to the magnetic meridian, then the horizontal component of the earth's magnetic field will not enter into the varying flow of force across the coil, for the position of the plane of the coil is such that it is always parallel to the horizontal lines of force. Hence, under these conditions, the vertical component of the earth's field only need be considered.

If the terminals of the coil are connected with a ballistic galvanometer, and the throw of the needle observed, (1) when the axis of rotation is vertical, and the coil is quickly rotated through 180°, starting from a position at right angles to the magnetic meridian; (2) when the axis is horizontal and parallel to the magnetic meridian, and the coil is quickly rotated through 180°, starting from a horizontal position—then the angle of dip can be ascertained with some

accuracy from the data obtained. For in the first case the change in the flow of force given by $2n\text{AH}$ is proportional to H, and in the second case the change given by $2n\text{AV}$ is proportional to V. Hence (see next article) the quantity of electricity flowing through the galvanometer during the half-revolution is in the first case proportional to H, and in the second case to V; so that if d_1 and d_2 denote the throws of the galvanometer needle in the two cases, we have—

$$\frac{\text{V}}{\text{H}} = \frac{d_2}{d_1}.$$

But from Art. 47—

$$\frac{\text{V}}{\text{H}} = \tan\delta, \text{ and therefore } \tan\delta = \frac{d_2}{d_1}.$$

The arrangement described above is sometimes called an *Earth Inductor* or *Delezenne's Circle.*

Another simple arrangement for obtaining induced currents by motion of a conductor in a magnetic field is known as *Faraday's Disk* or *Barlow's Wheel.*

A circular disk or wheel of copper is fixed so as to rotate freely round a central axis at right angles to its plane. If made to rotate with its plane at right angles to the direction of the lines of force in a magnetic field, and a metal spring be made to press lightly on its edge, a continuous current will be found to flow from the spring to the centre of the wheel, and can be drawn off by making connection with the spring and the metal axis of the wheel. In some forms of the arrangement the edge of the wheel or disk (which is sometimes serrated like a circular saw) passes through a small trough of mercury in one part of the revolution, and connection is thus made with less friction than with a spring pressing on the edge.

89. Simple Laws of Magneto-Electric Induction. In the preceding article several general laws have been given which may, perhaps, for convenience, be again referred to here. It has been found:—

(1) That a current is induced in a circuit if the number of lines of magnetic force running through the circuit is changed.

(2) That the induced current is transient, and lasts only

while the change in the flow of magnetic force through the circuit is taking place.

(3) That the direction of the induced current may be determined by the empirical rule given in Art. 88.

In addition to these general laws, there are two more precise laws which we must notice. The intensity of an induced current is found to depend on the *rate* at which the number of lines of force passing through the circuit is changed. Thus, in the experiment illustrated by Fig. 156, the induced currents will be very feeble if the magnet is moved slowly, but strong if it is moved very quickly. From investigation of this question it is found—

(4) That the electromotive force of the induced current in a circuit is measured by the change per second in the number of lines of force passing through the circuit. For example, if the lines of force initially running through the circuit number N, and in t seconds they are reduced, say, to N′, then the average E. M. F. of the current induced during the change is given by—

$$E = \frac{N - N'}{t},$$

and if R denote the resistance of the circuit, the intensity of the current is given by—

$$C = \frac{E}{R} = \frac{N - N'}{R t}; \text{ or, } C t = \frac{N - N'}{R},$$

where Ct denotes the *quantity* of electricity induced in the circuit in time t.

The application of this law will be more fully realised by considering a simple example. Take the case of the *Earth Inductor* described in the preceding article: the change in the flow of force through the coil during each *half* revolution is $2 n A H$; hence if the coil make m revolutions per second, the total change in the flow of force per second is $4 m n A H$, and this is the difference of potential between the ends of the coil so long as it revolves uniformly at the rate of m revolutions per second. Also if R denote the resistance of the coil and the external circuit between its terminals, then $4 m n A H / R$ gives the current in the circuit. If A is in square centimetres and H in dynes per unit pole, the quantity $4 m n A H$ is in

absolute C. G. S. electro-magnetic units of potential, and can be expressed in volts by dividing it by 10^8; then if R is in ohms, the current in the circuit can be calculated in amperes.

Again, in the case of Faraday's disk, consider a radial strip of the disk passing from the centre to the point of contact with the spring. As this strip rotates through a small angle, θ, the area it cuts out is $\frac{1}{2}\theta r^2$ where r is the radius of the disk and θ is taken in circular measure, and the number of lines of force added to the circuit is $\frac{1}{2}\theta r^2 \mathrm{H}$ where H denotes the intensity of the magnetic field in which the disk rotates. If the disk make n revolutions per second, then the time it takes to describe an angle θ is $\theta / 2\pi n$ seconds, and the rate of change of the number of lines of force passing through the circuit is $(\frac{1}{2}\theta r^2 \mathrm{H}) / (\theta / 2\pi n) = \pi n r^2 \mathrm{H}$, and this measures the E. M. F. or difference of potential between the centre and the circumference of the rotating disk.

These laws of induction are frequently expressed somewhat differently, in the following form. If any conductor (not forming a continuous circuit) be moved in a magnetic field so as to cut across lines of force, then an electromotive force, measured by the rate at which the lines are cut, is induced in the conductor. Hence, if a continuous circuit be moved in any way in a magnetic field, different portions of it will, in general, cut across the lines of force at different rates, and the electromotive force induced in the circuit will be the algebraic sum of the electromotive forces induced in each element of the circuit considered as a separate conductor. It is evident that if the algebraic sum of the electromotive forces induced in the circuit is zero—that is, if the algebraic sum * of the lines of forces cut in unit time by each element of the circuit is zero—then the total number of lines of force passing through the circuit is constant. Hence it follows that no current is induced in the circuit so long as the number of lines of force crossing the circuit is constant, and also that the induced electromotive force is measured by the rate of change in the number of lines of force passing through the circuit.

* Cut lines passing into the circuit and lines passing out of the circuit must be taken with opposite signs.

Another law of importance is *Lenz's law*. It states—

(5) That when a current is induced in a circuit by the relative motion of the circuit and a magnet (or its equivalent), the reaction of the induced current is always such as to tend to stop the motion which produces it.

For example, in Fig. 156, if the magnet be held as in Fig. 154 with its north pole next the coil, then, when the magnet is moved *away* from the coil, the direction of the induced current is such that the end A of the coil is equivalent to the *south* pole of a magnet, and therefore attracts the receding north pole of the magnet. The reaction of the induced current thus tends to stop the motion which gives rise to it.

Another illustration of the action of Lenz's law is found in Arago's well-known experiment. A disk of copper is made to rotate in a horizontal plane immediately below a delicately balanced magnetic needle, the axis of rotation of the disk being vertically below the pivot of the needle. As the disk rotates it is found that the needle is gradually deflected in the same direction as the rotation, and if the rate of rotation is sufficiently high, finally takes up a motion of rotation in the same sense as the disk, but at a slower rate. This result is explained by the fact that currents are induced in the copper disk by its rotation relative to the magnet, and the reaction between the disk and the needle is in accordance with Lenz's law such as to tend to stop the motion of the disk, but the needle being movable and not fixed the result of this is that the needle is itself set in motion. The direction of the induced current in the disk is such that a current always flows along the diameter of the disk vertically below the needle in such a direction as to deflect the needle in the same direction as the rotation of the disk. The current flowing along this diameter spreads out at one end, and curving round the sides of the disk unites again at the other end; hence if the disk has a number of radial slits cut in it, the inductive effect on the needle is much less marked.

In the same way, if the copper disk is fixed, and the needle made to oscillate, the oscillations quickly die away and the needle comes to rest, as the result of the interaction of the induced currents and the oscillating needle. This

damping effect of a copper plate under a magnetic needle is sufficiently well marked to be of practical use.

Similarly, if a disk of copper be made to spin rapidly between the poles of an electromagnet in a plane at right angles to the lines of force, it is much more difficult to spin when the current is on than when it is off; and if the disk be put in rapid motion with the current off. it is quickly brought to rest when the current is put on. The interaction of the induced currents and the poles of the magnet is such as to stop the motion of the disk.

It has already been explained that a solenoid, or coil of wire carrying a current, is equivalent to a magnet; hence if, in the experiment illustrated by Fig. 156, the magnet M be replaced by a coil of wire wound on a core or bobbin small enough to be thrust into the interior of the coil AB, currents may be induced in the outer coil by moving the inner coil while it carries a *constant* current or by varying the strength of the current while the coil remains at rest in the interior of AB. For example, if the current be stopped by breaking the circuit of the inner coil, then a current is induced in the outer coil in the same direction as that induced by the complete withdrawal of the coil when it carries a constant current. Similarly the current induced when the current is started is in the same direction as that obtained by thrusting the coil carrying the steady current into the interior of AB. If, while the coil is inside AB, the current be reversed, then the flow of magnetic force through AB is also reversed; and the induced current is the same as that which would be obtained if the coil carrying the steady current were suddenly withdrawn from AB and immediately reinserted with its ends reversed.

To determine the direction of the induced current in any given case, it must be remembered that the "north pole" of the coil is that end round which the current appears, when the coil is viewed from that end, to circulate in a direction opposite to that of the hands of a watch. The application of this fact to what has been said above gives the following simple rule for the direction of the induced current: When the current in the inner coil is stopped or decreased,

or when the coil carrying a steady current is withdrawn, *the direction of the induced current in the outer coil is the same as that of the current in the inner coil*, and consequently when the current in the inner coil is first made or increased, or when the coil carrying a steady current is inserted into the outer coil, the direction of the induced current in the outer coil is opposite to that of the current in the inner coil.

It is of the utmost importance for the student to realise that these induced currents are in *all* cases only momentary and exist only while the variations of the magnetic field to which they are due are in progress.

90. The Induction Coil. Ruhmkorff's *Induction Coil* is an arrangement of two coils for the production of induced currents of high electromotive force. The coils are arranged one inside the other, the inner, or *primary*, coil consisting of a small number of turns of thick wire, and the outer, or *secondary*, coil of a very large number of turns of thin wire. The coils are fixed in position and the induced currents in the secondary coil are obtained by arranging that a strong current in the primary coil shall be automatically made and broken a large number of times per second by an automatic "make" and "break" contact in the primary circuit. The induced currents thus obtained in the secondary coil are of very high potential, and the terminals of the coil may be made to yield sparks and "shocks" in the same way as the prime conductor of a frictional electrical machine.

* Figure 157 shows a common form of induction coil: the primary and secondary coils, one inside the other, form the cylinder A B, but to strengthen the inductive action of the primary on the secondary, an iron core made up of a bundle of soft iron wires is fitted as the axis of the primary coil. This core increases the inductive effect by increasing along the axis of the coil the magnetic field due to the current in the coil, so that when the current is made or broken the change in the flow of force through the coil is greater than it otherwise would be. The automatic 'make' and 'break' is shown at C; the terminals of the

primary coil are seen at *a*, *b*; the current entering, say at *a*, flows beneath the baseboard to the base of the pillar *c*,* and from thence to the spring hammer *d* across the screw contact *s* between *c* and *d*. The end of the screw *s* is tipped with platinum, and rests on a small disc of platinum attached to the spring *d*, so that, as these platinum surfaces always remain clean, the contact is always good. The current then passes round the coil, and returns to the terminal *b*. The action of the 'make' and 'break' will now be evident: when the current first passes, the iron

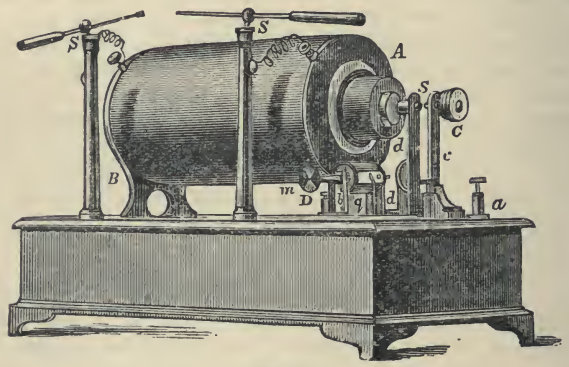


Fig 157.

core of the primary coil becomes an electromagnet and attracts to it the mass of soft iron which forms the head of the spring hammer *d*; this results in breaking the contact with *s*, and the current being now interrupted the iron core ceases to be an electromagnet, the hammer *d* now springs back into contact with *s*, and again makes the current. In this way the current is continuously made and broken at a very rapid rate. The arrangement shown at D is a commutator by which the current in the primary circuit may be reversed at will. The essential parts are

* For simplicity the connections to the commutator D (described below) are not here referred to.

the cylinder *l*, which can be rotated by the screw *m*, and the two upright springs *p* and *q*; the cylinder is of ebonite, and has two strips of brass parallel to its length let into its surface, and 180° apart. When it is desired to start the current these strips are brought into contact with the springs, *p* and *q*, and the direction of the current may be reversed by turning the handle *m* through 180° and thus interchanging the strips in contact with the springs. From the section shown in Fig. 158 it will be seen how the commutator acts—the terminals, *a*, *b*, (Fig. 157) are connected to the pillars *a'*, *b'* (Figs. 157, 158) which support the brass axis of the cylinder. This axis, as the figure shows, is divided, and one end is connected to one of the brass strips on the cylinder and the other end to the other strip. The springs *p* and *q* are directly connected to the ends of the primary coil through the make and break C. From this

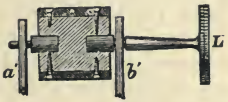


Fig. 158.

description of the connections it will at once be evident how the current in the primary coil is reversed by rotating the cylinder *l* through 180°, so as to interchange the springs in contact with the brass strips.

The terminals of the secondary coil are shown at S S carrying discharging rods, between the ends of which sparks can be made to pass when their distance apart is not too great.

* **91. Self-Induction.** It has already been explained that a change in the flow of magnetic force through a coil of wire produces an induced current in the coil. Now, when a current is first started in a coil of wire a magnetic field due to this current is suddenly established in the medium around the coil, and consequently there is a sudden change in the flow of magnetic force through the circuit. The result of this is that a current is induced in the coil, and

from the law given above the direction of this induced current will be opposed to that of the primary current to which it is due. Similarly, when the current in the circuit is broken, the flow of force due to it through the coil is suddenly destroyed, and an induced current in the same direction as the primary current is produced. As these induced currents are the result of very rapid changes in the flow of magnetic force through the circuit in which they are induced, they are of very high electromotive force. This is readily seen when the current in a coil of several turns is broken: a spark will be seen to leap across the small gap made at the point of interruption of the current, showing that the E. M. F. of the induced current is great enough to spark across the gap in the circuit.

These induction currents are said to be due to the *self-induction* of the circuit because they are the result of the magnetic effects attendant on the making and breaking of the current in a circuit on that circuit itself, and not on a secondary circuit, as in the experiments of Art. 88.

These induced currents due to self-induction are a source of trouble in the action of induction coils. When the current is made in the primary coil, the induced current opposed to it retards the rise of the primary current,* and consequently diminishes the inductive effect on the secondary. Similarly, when the current is broken, the induced current in the same direction as the primary, sparks across the interruptor, thus making the break less sudden and definite than it otherwise would be. The sparking would also spoil the surfaces of contact of the interruptor. To prevent these effects, a condenser, made of sheets of tin-foil and paraffined paper, is connected with the primary circuit; one pole of it is connected to the pillar of the screw *s* (Fig. 157), and the other to the spring hammer *d*. The action of this condenser may be briefly explained thus. When the current is made,

* When a current is made or broken in a circuit, it does not attain its full intensity nor become reduced to zero instantaneously, but a finite time is required for the change to be effected, and it is during this small interval of rapid change that the induction currents are produced.

the condenser is at the same time charged, and thus the current takes a comparatively long time to attain its maximum intensity, and the inductive effect attendant on the make (both on its own circuit and on the secondary circuit) is very greatly reduced. Again, when the current is broken the self-induced current, having to charge the condenser, is not able to spark across the break, and thus the current is very suddenly broken. Further, the electricity so stored in the condenser being unable to leap across the break in the circuit, at once discharges round the primary coil, but in the opposite direction to that of the primary current—that is, in the opposite direction to that in which it would have passed had it been able to spark across the break. In this way the extra-current at break is made a help instead of a hindrance. The secondary induced current at make, we see, is thus practically suppressed, while that at break is intensified, so that in an ordinary induction coil the secondary induced currents are those due to the breaks of the primary current.

*** 92. A simple form of Dynamo.** Clark's machine, shown in Fig. 159, although now practically obsolete, is a simple form of *Dynamo*, or machine in which the current is produced by magneto-electric induction. The coils *c*, *c*, are rotated by means of the wheel W in front of the compound horse-shoe magnet M. The coils *c*, *c*, are wound on cores of soft iron, and the wire with which they are wound is continuous, and so wound on the coils that the current induced in one, say when leaving the north pole of the magnet, is in the same direction along the wire as that induced in the other coil when leaving the south pole. If the student follow out the directions of the induced currents in the coils he will find that, as in the case of the Earth Inductor, the direction of the current in the wire is reversed at each *half revolution*, and hence a commutator C has to be connected to the circuit of the coils. The ends of the wire wound on the coils are connected to a split ring fitted on the axis of revolution of the coils. This axis is of ebonite or other insulating material, and the split ring is a broad ring of brass split into two halves by two slits parallel to its length. Two upright springs, *s*, *s*, press on this ring, and

are connected at their lower ends to the terminals of the machine. As the coils revolve the current in the wire is, as we have said, reversed at each half-revolution; but at each half-revolution there is also an interchange of the halves of the split ring in contact with the contact springs,

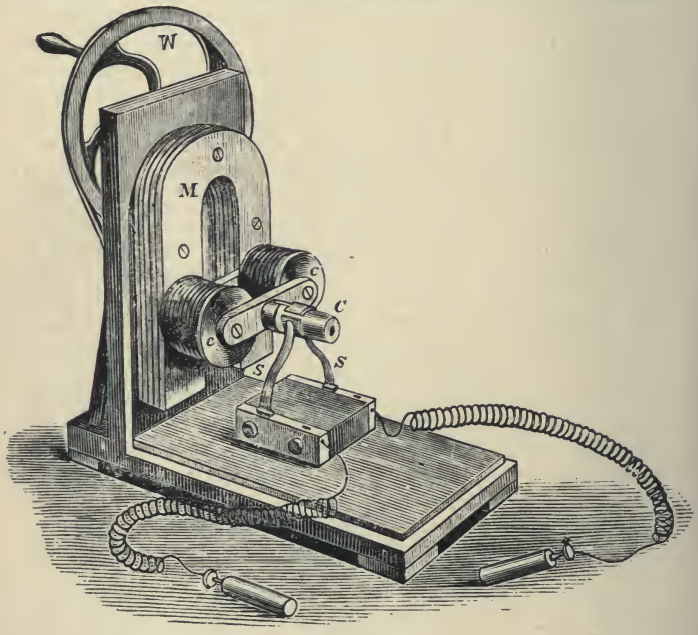


Fig. 159.

and thus the current derived from the terminals of the machine is always in the same direction.

Rapid rotation of the coils in front of the poles of the magnet results in a continual change in the flow of magnetic force though the coils, and if the magnet is strong and the number of turns of wire on the coils very great, the E. M. F. of the induced currents may be very great.

The well-known "medical coils" are made in this way, but the magnet is usually provided with a keeper of soft iron whose position may be adjusted so as to vary the strength of the magnetic field in which the coils revolve, and thus regulate the strength of the induced currents. For example, when the keeper is placed across the poles of the magnet most of the lines of force pass through it, and the external field in which the coils revolve is very small; but as the keeper is withdrawn from the poles the strength of the external field gradually increases, as the distance between the keeper and the magnet poles increases.

In all modern forms of dynamos electro-magnets are used as *field-magnets* instead of permanent magnets, and the current by which they are excited may be provided by a separate battery (*separately excited*), or more usually by the current induced in the coils. In this latter case (*self-excited*) the current is started in the coils by the action of the iron cores of the electro-magnet as feeble permanent magnets, for if these cores have been once strongly excited they always retain more or less residual magnetism, according to the quality of the iron.

In self-excited dynamos the current induced in the coils may all pass round the coils of the field-magnets, or a portion of it may be shunted round them. In the former case the dynamo is said to be *series wound*, and in the latter case *shunt wound*. When both methods of winding are combined the machine is said to be *compound* wound.

CALCULATIONS.

93. THE electromagnetic unit of current is that current which, flowing in an arc of unit length and unit radius, produces a magnetic field of unit intensity at the centre of the arc. (Art. 71.)

The intensity of the magnetic field at the centre of a thin coil of n turns carrying a current C is given by—

$$I = \frac{2\pi nC}{r}, \qquad \text{(Arts. 70, 71.)}$$

where r denotes the mean radius of the coil.

In a tangent galvanometer with a thin coil of n turns and mean radius r the relation between the current, C, and the deflection, α, which it produces, is given by—

$$C = \frac{rH}{2\pi n} \tan \alpha,$$

where H denotes the horizontal intensity of the magnetic field in which the needle lies—usually this is the horizontal component of the earth's magnetic field. The factor $\frac{rH}{2\pi n}$ is known as the *working constant* or *reduction factor* of the galvanometer and is frequently denoted by k, the above relation being written $C = k \tan \alpha$. (Art. 73.)

Similarly in a sine galvanometer under the same conditions we have—

$$C = \frac{rH}{2\pi n} \sin \alpha,^*$$

or

$$C = k \sin \alpha, \qquad \text{(Art. 74.)}$$

where k denotes the reduction factor of the galvanometer.

* The needle of the galvanometer is here supposed to be short like that of a tangent galvanometer.

By Ohm's Law we have—

$$C \text{ (amperes)} = \frac{E \text{ (volts)}}{R \text{ (ohms)}}.$$

1 volt = 10^8 C.G.S. units.
1 ohm = 10^9 ,, ,,
1 ampere = $\frac{1}{10}$,, unit
or 10 amperes = 1 ,, ,,

(Arts. 76, 79.)

A *coulomb* is the electromagnetic unit of quantity of electricity, and is the quantity conveyed by a current of one ampere in one second.

If R denote the resistance of a conductor of length l and area of cross section a made of material of specific resistance S, then—

$$R = S \frac{l}{a}. \qquad \text{(Art. 78.)}$$

Hence the specific resistance of any substance is the resistance of a conductor of that substance of unit length and unit area of cross section.

The resistance of a conductor, in general, increases with temperature. If R_t denote the resistance of a given conductor at $t°$ C., R_0 its resistance at 0° C., and a the co-efficient of increase of resistance with temperature, then as an approximate relation we have—

$$R_t = R_0 (1 + at). \qquad \text{(Art. 78.)}$$

[For most pure metals the value of a is about ·0038.]

The joint resistance of two conductors of respective resistances r_1 and r_2 joined in parallel arc is given by—

$$R = \frac{r_1 r_2}{r_1 + r_2}, \qquad \text{(Art. 78.)}$$

and the ratio of the currents in the two conductors is the inverse of that of their resistances; that is,

$$\frac{C_1}{C_2} = \frac{r_2}{r_1}. \qquad \text{(Art. 78.)}$$

The current given by a battery of n cells *in series*, each of electromotive force E and internal resistance r, through an external resistance R, is given by—

$$C = \frac{n E}{nr + R}.$$

With the same cells *in parallel* the current is given by—

$$C = \frac{E}{\frac{r}{n} + R} = \frac{nE}{r + nR} \qquad \text{(Art. 79.)}$$

If we have m n cells, each of electromotive force E and resistance r, arranged in n rows in series each row containing m cells grouped in parallel, then the current given by this grouping through an external resistance R is given by—

$$C = \frac{nE}{\frac{nr}{m} + R}. \qquad \text{(Art. 79.)}$$

To obtain the strongest current from this arrangement we must have—

$$\frac{nr}{m} = R.$$

That is,

$$nr = mR. \qquad \text{(Art. 79.)}$$

If the resistance between two points on a simple conductor be denoted by R and the current in the conductor by C, then the difference of potential between those two points is given by—

$$e = CR. \qquad \text{(Arts. 76, 79.)}$$

[This relation is the algebraic expression of Kirchhoff's second law.]

The *chemical equivalent* of an element is the number obtained by dividing the atomic weight of the element by its valency.

The *electrochemical equivalent* (per *ampere*) of an element is the mass of that element liberated in one second by a current of one ampere. The electrochemical equivalent of any element is obtained by multiplying the electrochemical equivalent of hydrogen (0·00001035 gram) by the chemical equivalent of the given element.

The relative masses of different elements liberated by the electrolytic action of the *same* current are directly proportional to their chemical equivalents. (Arts. 83, 84.)

The mass of an ion liberated in a time t by a current C is given by—

$$m = Cwt,$$

where w denotes the *electrochemical equivalent* of the ion. (Art. 84.)

If a current C flow for a time t round a circuit in which the effective E. M. F. is E, then the work done in maintaining the current for the time t is given by—

$$W = Ct.E. \qquad \text{(Art. 85.)}$$

If C and E be expressed in electromagnetic units, and t in seconds, then W is given in ergs.

From this we have that the *activity* of the battery supplying the current is given by CE. (Art. 85.)

Here, if C be expressed in *amperes* and E in volts, then the activity is given in *watts*.

The watt is a unit of power or activity equal to 10^7 ergs per second, or such that 746 watts equal one horse-power. (Art. 85.)

The heat produced in a conductor of resistance R by a current C in a time t is given by—

$$H = \frac{C^2Rt}{J}, \qquad \text{(Art. 86.)}$$

where J (Joule's equivalent) denotes the quantity of work equivalent to unit quantity of heat.

[In C.G.S. units $J = 4{\cdot}2 \times 10^7$ ergs.]

If C be expressed in *amperes*, R in *ohms*, and t in *seconds*, then this relation reduces to—

$$H = \frac{C^2Rt}{4{\cdot}2} = C^2Rt \times 0{\cdot}24 \text{ gram-degree.}$$

[The process of reduction is as follows—

$$\begin{aligned} C \text{ (amperes)} &= C \times 10^{-1} \text{ C.G.S. units.} \\ R \text{ (ohms)} &= R \times 10^9 \quad ,, \quad ,, \end{aligned}$$

$$H = \frac{C^2Rt}{J} = \frac{C^2 \times 10^{-2} \times R \times 10^9 \times t}{4{\cdot}2 \times 10^7} = \frac{C^2Rt \times 10^7}{4{\cdot}2 \times 10^7} = \frac{C^2Rt}{4{\cdot}2}.]$$

Examples IV.

1. The coil of a tangent galvanometer consists of 10 turns of fine wire on a narrow ring of 22 cm. radius. Find the intensity of the magnetic field at the centre of the coil when a current of one ampere passes through it.

The intensity of the magnetic field at the centre of the coil is given by—

$$I = \frac{2\pi nC}{r}.$$

Here $n = 10$; C = 1 ampere $= \frac{1}{10}$ C.G.S. unit; and $r = 22$ cm.

Therefore—

$$I = \frac{2 \times 22 \times 10 \times 1}{7 \times 22 \times 10} = \frac{2}{7} \text{ dyne.}$$

2. Calculate the reduction factor of the tangent galvanometer referred to in the preceding question, and find the deflection which a current of 0·21 ampere would produce when passed through the instrument.

The reduction factor of the galvanometer is given by—

$$k = \frac{r\,\mathrm{H}}{2\,\pi\,n}.$$

Hence, taking H = 0·18 dyne, we have—

$$k = \frac{22 \times \cdot 18 \times 7}{2 \times 22 \times 10} = \cdot 063, \text{ for C.G.S. units.}$$

Also if α denote the deflection produced by a current of 0·21 ampere, that is, 0·021 C.G.S. units, then from—

$$\mathrm{C} = k \tan \alpha,$$

we have—

$$\cdot 021 = \cdot 063 \tan \alpha,$$

or

$$\tan \alpha = \frac{\cdot 021}{\cdot 063} = \frac{1}{3}.$$

That is, α is an angle whose tangent is $\frac{1}{3}$, and is therefore an angle of about 18° 26′.

3. A sine galvanometer with a short needle is used as a tangent galvanometer, and when a given current is passed through it a deflection of 30° is produced. Find the deflection which the same current should produce if the instrument were used as a sine galvanometer.

Here, when the galvanometer is used as a tangent galvanometer, we have—

$$\mathrm{C} = k \tan \alpha_1,$$

and when used as a sine galvanometer we have—

$$\mathrm{C} = k \sin \alpha_2.$$

And from the conditions of the question we have—

$$k \tan \alpha_1 = k \sin \alpha_2,$$

or

$$\tan \alpha_1 = \sin \alpha_2.$$

But

$$\alpha_1 = 30°;$$

therefore,

$$\tan 30 = \sin \alpha_2,$$

or

$$\frac{1}{\sqrt{3}} = \sin \alpha_2.$$

That is, α_2 is an angle whose sine is $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ (= ·577), and is therefore an angle of about 35° 15′.

4. Find the specific resistance of copper at 0° C., given that the resistance at 15° C. of a copper wire 1 metre long and 0·0376 cm. in diameter is 0·157 ohm.

Here, applying the relation—

$$R = S\frac{l}{a},$$

we get—

$$S = R\frac{a}{l}.$$

And from the data of the question—

$$R = 0{\cdot}157 \text{ ohm.}$$

$$a = (\pi r^2) = 3{\cdot}1416 \times (0{\cdot}0188)^2 = 0{\cdot}00111 \text{ sq. cm.}$$

$$l = 1 \text{ metre} = 100 \text{ cm.}$$

Therefore,

$$S = \frac{0{\cdot}157 \times 0{\cdot}00111}{100} = {\cdot}0000017427 \text{ ohm.}$$

This is the specific resistance of copper at 15° C.

To determine the specific resistance at 0° C. we must know the co-efficient of increase of resistance with temperature. For copper the value of this co-efficient is 0·00387.

Hence, in the relation—

$$R_t = R_0 (1 + at),$$

we have—

$${\cdot}0000017427 = R_0 (1 + {\cdot}00387 \times 15),$$

$$= R_0 (1{\cdot}058).$$

$$\therefore R_0 = \frac{{\cdot}0000017427}{1{\cdot}058} = {\cdot}0000016472 \text{ ohm.}$$

That is, the specific resistance of copper at 0° C. is ·0000016472 ohm or 1647·2 C.G.S. units.

5. A battery made up of 5 Bunsen's cells, each of E. M. F. 1·8 volts and internal resistance ·2 ohm, has its terminals joined by two wires, A and B, arranged in parallel. The resistance of A is 6 ohms and that of B is 3 ohms; find the strength of the current in each wire.

Here, the total E. M. F. in the circuit is—

$$5 \times 1{\cdot}8 = 9 \text{ volts.}$$

The internal resistance of the battery is—

$$5 \times \cdot 2 = 1 \text{ ohm.}$$

The external resistance of the circuit is given by—

$$\frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2 \text{ ohms.}$$

Therefore the *total* resistance of the circuit is 3 ohms, and by Ohm's Law the total current in the circuit is given by—

$$C = \frac{E}{R} = \frac{9}{3} = 3 \text{ amperes.}$$

In the external circuit this current of 3 amperes is divided between A and B in the inverse ratio of their resistance.

That is,

$$\text{Current in A} = \frac{3}{9} \times 3 = \frac{1}{3} \times 3 = 1 \text{ ampere,}$$

and

$$\text{Current in B} = \frac{6}{9} \times 3 = \frac{2}{3} \times 3 = 2 \text{ amperes.}$$

6. A battery is made up of 10 Daniell's cells joined in series. A very high resistance galvanometer is connected to the terminals of the battery and shows a deflection of 156 divisions. Without disconnecting the galvanometer, the terminals of the battery are now joined by a wire of 10 ohms resistance, and the galvanometer deflection falls to 52 divisions. Find the average resistance of the battery cells.

Here, as in Art. 79, if E denote the E. M. F. of the battery and e the potential difference of the terminals, then—

$$E = C\ (10 + B),$$

and

$$e = C \times 10,$$

where C denotes the current in the circuit when the terminals are joined by the wire, and B the internal resistance of the battery. Hence we have—

$$\frac{E}{e} = \frac{10 + B}{10}.$$

But

$$\frac{E}{e} = \frac{156}{52} = \frac{3}{1}; \qquad \text{(Art. 79.)}$$

therefore,

$$\frac{10 + B}{10} = \frac{3}{1},$$

or

$$B = 20 \text{ ohms.}$$

Hence the average resistance of a battery cell is—

$$\frac{20}{10} = 2 \text{ ohms.}$$

7. The same current passes through a copper and a silver voltameter arranged in series in the circuit of a battery of 10 Daniell's cells. If 10·8 grams of silver are deposited in the silver voltameter in a given time, find the amount of copper deposited in the copper voltameter, and the amounts of copper and zinc respectively deposited and dissolved in the battery in the same time.

The amount of the elements deposited in *each cell in the circuit* will be proportional to the chemical equivalents of the elements. The chemical equivalents of silver, copper, and zinc are (approximately)—

$$\begin{aligned} Ag &= 108. \\ Cu &= 31{\cdot}5. \\ Zn &= 32{\cdot}5. \end{aligned}$$

Hence we have for each cell—

$$Ag : Cu : Zn :: 108 : 31{\cdot}5 : 32{\cdot}5$$

That is,

$$10{\cdot}8 : w_1 : w_2 :: 108 : 31{\cdot}5 : 32{\cdot}5$$

Therefore

$$w_1 = 3{\cdot}15$$

and

$$w_2 = 3{\cdot}25$$

That is, in each cell in the circuit 3·15 grams of copper are deposited and 3·25 grams of zinc are dissolved during the time that 10·8 grams of silver are deposited.

Therefore,

Copper deposited in voltameter cell	=	3·15 grams.
„ „ „ battery (10 cells)	=	31·5 „
Zinc dissolved „ „ „	=	32·5 „

8. In an experiment to determine the reduction factor of a tangent galvanometer it was found that, in 30 minutes, 1·1081 grams of copper were deposited on the negative plate of a copper sulphate voltameter. The deflection of the galvanometer was constant at 35° during the experiment.

Determine from these data the required constant, taking the electrochemical equivalent of copper to be ·0003275 gram per ampere. From the relation—

$$m \quad C w t$$

we have—

$$C \quad \frac{m}{w t}.$$

Also

$$C = k \tan \delta,$$

where k denotes the reduction factor of the galvanometer.

Hence

$$\frac{m}{w t} \quad k \tan \delta.$$

That is,

$$k = \frac{m}{w t \,.\, \tan \delta}. \qquad \text{(Art. 84.)}$$

Substituting the data of the question in this relation we get—

$$k = \frac{1{\cdot}1081}{{\cdot}0003275 \times 30 \times 60 \times \tan 35^\circ}.$$

From tables we get $\tan 35^\circ = {\cdot}7$.

Therefore,

$$k = \frac{1{\cdot}1081}{{\cdot}0003275 \times 30 \times 60 \times {\cdot}7} = 2{\cdot}6845.$$

This is the value of the reduction factor which, when used in the relation $C = k \tan \delta$, gives C in *amperes*.

9. The same current is passed through a silver voltameter and a platinum wire of 10 ohms resistance. In twenty minutes ·6708 gram of silver are deposited in the voltameter; find the rate at which heat is developed in the wire.

Here from—

$$m = C w t \text{ or } C = \frac{m}{w t},$$

we have—

$$C = \frac{{\cdot}6708}{{\cdot}001118 \times 20 \times 60} = {\cdot}5 \text{ ampere.}$$

Hence the heat developed in the wire *per second* is—

$$H = C^2R \times {\cdot}24 = ({\cdot}5)^2 \times 10 \times {\cdot}24 = {\cdot}6 \text{ calorie.}$$

10. Water is heated by a spiral of platinum wire immersed in it.

The resistance of the wire is 74·6 ohms and it carries a current of 10 amperes; find, in horse-power, the rate at which energy is spent in heating the water.

Here the rate at which energy is expended in heating the platinum wire (and therefore the water) is given by—

$$\text{Activity} = C^2R = (10)^2 \times 74{\cdot}6 \text{ watts} = \frac{100 \times 74{\cdot}6}{746} = 10 \text{ horse-power.}$$

11. Describe the action of an electric current on a magnet, and explain how it is made use of in a galvanometer.

There are 25 turns of wire in a galvanometer coil, the mean radius of which is 150 cm. Assuming the value of H to be 0·18, find the current which will deflect a magnet placed at the centre of the coil 45°. If the resistance of the circuit including the battery be 3 ohms, find the electromotive force required to produce the current.

12. Show that the strength of a current passing round a tangent galvanometer is proportional to the tangent of the angle of deflection.

A circuit is formed containing galvanometer, battery, and connecting wires, the total resistance of the circuit being 4·85 ohms. The galvanometer shows a deflection of 48½°. When a piece of platinum wire is introduced into the circuit, the deflection falls to 29°. From these data determine the resistance of the platinum wire, given tan 48½° = 1·130, and tan 29° = 0·554.

13. Explain fully the meaning of Ohm's Law. What is the resistance of a conductor?

A Menotti cell of 40 ohms internal resistance is connected by thick wires with the terminals of a tangent galvanometer, formed by a single ring of stout copper wire. The deflection is 45°. Three similar cells of the same resistance are then connected in series with the first. What is the deflection of the galvanometer needle? Would any other arrangement of the four cells give a stronger current, and why?

14. A length of uniform wire, of resistance 12 ohms, is bent into a circle, and two points at a quarter of the circumference apart are connected with a battery whose resistance is 1 ohm and electromotive force 3 volts; find the currents in the different parts of the circuit.

15. A battery of 4 ohms internal resistance is sending a current through an external resistance of 6 ohms. The terminals of the battery are connected to the electrodes of a reflecting electrometer, and the deflection on the scale is 100 divisions. What will the deflection be when, everything else remaining the same, the external circuit is broken?

16. Three voltaic cells *A*, *B*, *C*, whose respective electromotive forces and resistances are as follows, namely—

	A	*B*	*C*	
Electromotive force . . .	1·07	1·54	1·9	volts,
Resistance	·72	2·3	·1	ohms,

are connected in series, and the circuit is completed by a wire of resistance 5·9 ohms. Determine the strength of the current produced.

If the cell *B* were removed and replaced in the circuit with its terminals inverted, what would be the strength of the current?

17. State the law of the division of an electric current in two branches of a circuit, when part of the current is shunted.

A tangent galvanometer of 4 ohms resistance forms part of a circuit whose total resistance is 80 ohms. The galvanometer is then shunted by a wire whose resistance is 4 ohms. Find the ratio of the currents in the galvanometer before and after it is shunted.

18. Define carefully *strength of electric current*, *electromotive force*, and *resistance*. What resistance should a wire have which, when connected across the terminals of a galvanometer whose resistance is 3663 ohms, would let one-hundredth part of the whole current pass through the galvanometer, and $\frac{99}{100}$ths through itself?

19. What is the current in a circuit which consists of a battery of 10 similar cells arranged in series and a wire of 24 ohms resistance? (Each of the cells has an electromotive force of 1·8 volts, and an internal resistance of ·3 of an ohm.)

20. Eight cells, each with half an ohm internal resistance, and 1·1 volts E. M. F, are connected, (*a*) all in series, (*b*) all in parallel, (*c*) in two parallel sets of four cells each: calculate the current sent in each case through a wire of resistance ·8 ohm.

21. An electrical current may pass from A to B by either of two wires ACB, ADB, the resistances of which are 3 and 7 respectively. What will be the resistance of a single wire which replaces ACB and ADB in such a way as not to produce any alteration in the current in the rest of the circuit?

22. A wire is formed into a circle, 1 foot in diameter, and two points, *A* and *B*, a quarter of the circumference apart, are connected to the poles of a battery of E. M. F. 3 volts and resistance 5 ohms. If 1 foot of the wire have a resistance of 6 ohms, find the current in the battery and in the two parts of the wire.

23. Explain the effects on the current from a battery of the size of the plates, their closeness together, the number of cells in series, and the number in parallel (*a*) when the external resistance is small, (*b*) when it is large.

24. A current from three Daniell's cells in series is passed through solutions of nitrate of silver, of sulphuric acid, and of sugar of lead, all in series; platinum electrodes being used. How much hydrogen is liberated, how much sulphuric acid decomposed, and how much lead, copper, and silver deposited, by the time half a gram of zinc has been fairly dissolved in each cell? If all this occurs in 30 minutes, what is the strength of the current? being given that unit current (one ampere) deposits 4 grams of silver in an hour.

N.B.—Atomic weights: H = 1, O = 16, S = 32, Cu = 63·5, Zn = 65, Ag = 108, Pb = 207.

25. A current produces 35 c.c. of mixed hydrogen and oxygen per

minute in an electrolytic cell, and at the same time produces 155 units of heat per second in a wire; if the current be increased until it produces 105 c.c. of mixed gases per minute, calculate how much heat will now be produced per second in the wire.

26. The same current produces heat in a wire, and decomposes a solution of copper sulphate. What is the relation between the heat produced and the copper deposited, and how may it be verified?

27. If the current of a battery of 10 Grove's cells connected in series is sent simultaneously through two voltameters, containing respectively a solution of cupric sulphate and a solution of silver nitrate, and placed one after another in the circuit, show how much copper and how much silver will be deposited while 3·25 grams of zinc are dissolved in the entire battery. Also show how much copper and silver would be deposited during the solution of the same quantity of zinc, if the battery were arranged in two parallel series, each of 5 cells, instead of in a single series of 10 cells.

[Zn = 65·0; Cu = 63·4; Ag = 108.]

28. The internal resistance of a voltaic battery is equal to the resistance of three metres of a particular wire. Compare the quantities of heat produced, both inside and outside the battery, when the poles are connected by one metre of this wire, with the quantities produced in the same time when they are connected by 27 metres of the same wire.

29. Ten voltaic cells, each of internal resistance 2 and electromotive force 1·5, are connected (*a*) in a single series, (*b*) in two series of five each, the like ends of the two series being joined together. If the terminals are in each case connected by a wire of resistance 10, show what is the strength of the current in the wire in each case, and compare the rates of consumption of zinc.

30. A battery of electromotive force e and resistance R is employed in sending a current through a wire of resistance r, and there are no other resistances in the circuit. How much work will be done per second by the battery, and how much heat will be generated in the wire?

31. What is meant by the electrochemical equivalent of a substance? If a current of 1 ampere decompose per minute 0·0056 gram of water, what quantity of silver will a current of 0·5 ampere separate from a solution of silver nitrate in an hour?

[108 grams silver are chemically equivalent to 1 gram hydrogen.]

32. State the laws of electrolysis. A current deposits 5 grams of copper per hour in a voltameter and also produces 5 units of heat per minute in a wire; if it be increased so as to deposit 10 grams of copper per hour, how much heat will it produce in a minute in the wire?

CHAPTER XVII.

THERMO-ELECTRICITY: ELEMENTARY FACTS.

*** 94. Introductory.** It is found that if a circuit be made up of two wires of different metals, and one of the junctions be heated, a current is set up in the circuit. Currents so produced are called *thermo-electric currents*, and the E. M. F. to which they are due is called *thermo-electromotive force*. For example, if a piece of iron wire and a piece of copper wire of about equal length have their loose ends soldered or twisted together, a thermo-electric current may be established in the circuit by heating either of the junctions, or rather by establishing a difference of temperature between the two junctions, for a current may also be produced by cooling either of the junctions. The existence of this current may be shown by including a galvanometer in the circuit; this is readily done by cutting either of the wires and joining the terminals of the galvanometer to the loose ends of the cut wire, or by taking about equal lengths of copper and iron wire, twisting one pair of ends together and joining the other pair to the terminals of the galvanometer,—the ends twisted together form one junction which can be heated or cooled, and the coil of the galvanometer itself forms the other junction which always remains at the temperature of the room and is therefore practically constant. It should here be noted that the junction between two wires may be effected in any way without influence on the result, so long as the temperature of the extremities of whatever forms the junction is the same: for example, in the case of the galvanometer just mentioned, the wire of the coil forms one

junction; and if the extremities of the coil, that is, the terminals of the galvanometer, are of the same temperature, then the result will be the same as if the ends of the wires were directly twisted together, and the junction so formed maintained at the temperature of the terminals of the galvanometer. The point here mentioned may be more generally put in this way:—in a compound circuit made up of several conductors of different materials, any difference of temperature of the several junctions produces a current in the circuit, *but the thermo-electric effect of any element of the circuit is nil if the extremities of that element have the same temperature*—that is, such an element may be considered as a mere junction between the two adjacent elements, and the temperature of the junction is the common temperature of the extremities of the element. Further, in such a compound circuit the total electromotive force in the circuit is the sum of all the thermo-electromotive forces at the several junctions in the circuit.

The production of thermo-electric currents by heating junctions of different metals was first discovered by **Seebeck** in 1822.

* **95. A Simple Thermo-Electric Couple.** Let the extremities of two pieces of wire of different metals, say copper and iron,† be twisted together and a low-resistance reflecting galvanometer included in the circuit. Let one junction be slowly heated by a Bunsen flame while the other remains at the temperature of the room. It will be noted that as the temperature of the junction rises the deflection of the galvanometer at first increases, indicating an increase of the thermo-electric current in the circuit. This increase, however, becomes less and less rapid as the heating goes on, before long it ceases altogether, and the current attains a maximum. If the heating of the junction is still continued the current now begins to *decrease*, and the decrease goes

† Copper and iron are chosen simply because the couple they form conveniently exhibits the phenomena to be studied; any other pair of metals could be made to give the same results, but in most cases the necessary range of temperature would not be easily obtained.

on continuously until the current is first reduced to zero and then *reversed* in the circuit. The important features of this experiment are : first, the current first increases, attains a maximum and then decreases ; and second, the reversal of the current in the circuit after it has been reduced to zero.

If the experiment be so conducted that the temperature of the heated junction can be noted during the experiment—for example, if the junction be inserted in a long glass tube closed at one end and dipping into an oil bath—the following facts will be noticed. First, the temperature at which the current attains its maximum value is always the same, no matter what the temperature of the other junction may be. Second, the temperature at which reversal of the current takes place is as much above the temperature of maximum current as that temperature is above the temperature of the cool junction, that is the temperature of maximum current, is the mean of the temperature of reversal, and the temperature of the cool junction. For example, in the case of the copper and iron couple here considered, supposing the cold junction to be at 20° C. the temperature of maximum current will be about 260° C. † and the temperature of reversal about 500° C. Or, if the cold junction be kept at 0° C. then the temperature of maximum current will be the same as before—about 260° C.—but the temperature of reversal will be about 520° C.

The temperature of maximum current for any couple is known as the **neutral point** for that couple, and it is always the mean of the junction temperatures when reversal takes place. It will be evident from what has been said that the neutral point is a constant for a given couple, but that the temperature of reversal depends upon the temperature of the cold junction.

* **96. The Peltier Effect.** In 1834 it was discovered by Peltier that if a current was sent round the circuit of a thermo-electric couple, heat was evolved at one junction and absorbed at the other—that is, one junction was heated

† The temperature varies slightly with different specimens of iron and copper.

and the other cooled. Further, it was found that the junction which was cooled was the one which, if heated, would give a thermo-electric current in the same direction round the circuit as the current which causes the cooling effect. For example, in the copper-iron couple considered above, for temperatures below the neutral point, the *thermo-electric* current flows round the circuit, so as to pass from the copper to the iron at the hot junction; hence, if a current be sent round the circuit of this couple, the junction at which it passes from copper to iron will be cooled, and the other will be heated. From this it will at once be seen that, when the thermo-electric current passes round the circuit of the couple, it absorbs heat at the hot junction and evolves heat at the cold junction. This result indicates the source of energy in a thermo-electric couple,—the heat absorbed at the hot junction is greater than that evolved at the cold junction, and the difference between the two quantities of heat is the source of energy to which the current in the circuit is due. In fact, a thermo-electric couple acts as a small heat-engine, taking in heat at the hot junction as its source, and giving out heat at the cold junction as its condenser.

* **97. Thermo-Electric Power.** If the temperature of one of the junctions of a thermo-electric couple be changed, there will be a corresponding change in the thermo-electromotive force of the couple; and if these changes be supposed to be very small, the thermo-electric power of the junction may be defined as the *ratio* of the change of thermo-electromotive force to the change of temperature to which it is due. This ratio varies with the temperature at which the changes are supposed to take place. The meaning of the term *power* used in this relation is rather ambiguous; the probable reason of its use will be understood by considering the following case. Suppose a couple to have both its junctions at the *same* temperature—say that of the room; then, if the temperature of one of them is slightly changed, a thermo-electromotive force is set up in the circuit, and the ratio of this E. M. F. to the small change of temperature to which it is due gives an idea of the thermo-electric

power of the couple at the temperature of the room. The thermo-electric power of a couple is evidently least in the neighbourhood of the neutral point, and is zero at that temperature, for as the current approaches its maximum value its rate of change with temperature gradually diminishes, and finally changes sign (that is, changes from a rate of *increase* to a rate of *decrease*) after passing through the value zero.

***98. The Thomson Effect.** From theoretical considerations of the action of thermo-electric couples, Sir William Thomson was led to assume that the Peltier effect is zero when the thermo-electric power is zero: that is, if the temperature of a junction is at the neutral point for the couple, then at that junction there is no Peltier effect. If however this junction is the hot junction, then we meet with a difficulty in explaining where the energy of the current is derived from; for if the Peltier effect at the hot junction is zero no heat is absorbed there, and at the cold junction heat is *given out*, hence heat must be absorbed in a thermo-electric circuit at other points than at the junctions. This result was first pointed out by Sir William Thomson, and he showed that heat must be absorbed in one or both of the wires in virtue of the difference of temperature between the ends, or that heat may be absorbed in one wire and generated in the other, but that the quantity of heat absorbed must, in the case considered, be greater than that given out. This effect is known as the *Thomson effect*. Just as the Peltier effect may be considered as the reverse of the Seebeck effect, so also there is a reverse to the Thomson effect. The Thomson effect states that the passage of a current along a conductor whose ends are at different temperatures is attended with an absorption or evolution of heat according to the material of the conductor; and the reverse of this evidently is, that if the ends of a conductor are maintained at different temperatures, then a current will be produced in the conductor. The Thomson effect will of course vary with the direction of the current in the conductor—that is, if when the current flows from the hot to the cold end, and heat is absorbed, then heat will be evolved when it flows

from the cold to the hot end. Thus, when a current flows in a copper conductor from the cold to the hot end heat is absorbed, and when it flows from hot to cold heat is evolved. In iron the effects are opposite. Lead is the only metal in which there is no appreciable Thomson effect.

The experimental verification of the points referred to above are somewhat difficult. The assumption that the Peltier effect vanishes when the thermo-electric power is zero has not been directly verified, but there is some indirect experimental confirmation of the fact. The Peltier and Thomson effects have both been experimentally confirmed; but on account of the masking effect of the heat evolved in the circuit, in accordance with Joule's law, special precautions have to be taken in order to satisfactorily exhibit these effects.

*** 99. Thermo-electric Piles.** The thermo-electric current produced by a single couple is usually extremely small—for

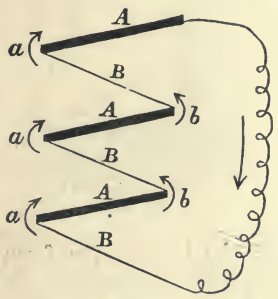


Fig. 159.

example, in our iron-copper couple, with its junctions at 0° C. and 100° C. respectively, its E. M. F. is only about 1143 micro-volts †—and the current produced will of course depend, in accordance with Ohm's law, on the resistance of the circuit. Other pairs of metals may, however, be chosen so as to make more effective couples. A couple of bismuth

† A micro-volt is the millionth part of a volt—that is, about the millionth part of the E. M. F. of a Daniell's cell.

and antimony gives perhaps the greatest E. M. F. for a given difference of temperature between its junctions: for example, at ordinary temperatures for 1° C. difference of temperature such a couple gives an E. M. F. of about 117 micro-volts. The E. M. F. in a thermo-electric circuit may, however, be increased by using, in the same circuit, a large number of couples of small resistance, all arranged so as to give a current in the same direction, just as the cells of a battery are arranged in series. Thus, in a single bismuth-antimony couple the current flows in the

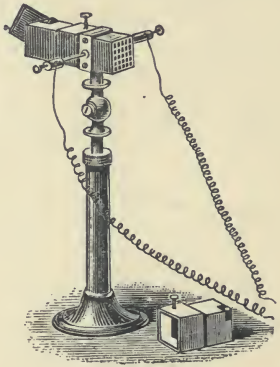


Fig. 160.

circuit so as to cross the hot junction from bismuth to antimony; if, then, a number of couples be arranged as in Fig. 159, with *alternate* junctions a, a, a, and b, b, b, brought near together, then a current may be set up in the circuit by keeping one set (a, a, a), say, hot, and the other set (b, b, b) cold. For example, if the figure represent an arrangement of bismuth-antimony couples of which the junctions a, a, a, are hot and b, b, cold, then the direction of the current in the circuit is shown by the arrows. The intensity of this current will depend upon the difference of temperature between the two sets of junctions; and when this difference is small, the intensity of the current may be said to be directly

proportional to it. This fact suggested the use of an arrangement of thermo couples for the measurement of difference of temperature. If a large number of couples be arranged as in Fig. 159, and small slips of mica or other insulating material be placed between the bars of each couple, then the whole arrangement may be folded up into a bundle and mounted as shown in Fig. 160. The instrument here shown is known as a thermo-pile, and is used with a low-resistance reflecting galvanometer as a very delicate differential galvanometer.*

* See Text-Book of Heat, Art. 56

CHAPTER XVIII.

PRACTICAL APPLICATIONS.

100. Electric Bells. These are simple applications of electro-magnetism; they are of two typical kinds, viz., *Tremblers*, which ring continuously as long as the current passes, and *Strikers*, which give only a single sound each time contact is made; the former are commonly used in houses and offices, the latter for signalling purposes. Fig. 162 shows diagrammatically the construction of a trembler. On a wooden base (not drawn) is mounted a small horse-shoe electro-magnet LMNR. The end L of the wire which is wound on it is permanently attached to one of the terminals T of the instrument, and the other to a nut K. Fixed to K is a steel spring S carrying a soft-iron armature I, to the other end of which is fitted an arm terminating in the clapper C. The spring S is so adjusted that when no current passes, the armature rests lightly against the point of a brass screw E, which for purposes of adjustment is made to work in a bearing H, and from this bearing, which is also of brass, a wire proceeds and is permanently attached to the other terminal T'. In the external circuit are a battery B, and bell-push or contact-maker P.

When contact is made a current passes via the route BPT'HISKRNMLT. This magnetises the core of the electro-magnet, draws I towards its poles, and makes the clapper C strike the gong G. Contact being thus broken

between the point of the screw E and the armature, the core becomes demagnetised, and the armature, by reason of the elasticity of the spring S, falls back against the said point. This re-establishes contact, the core is again magnetised and the armature attracted again immediately

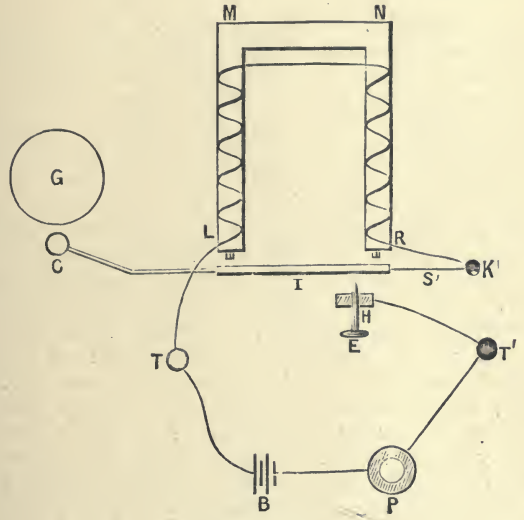


Fig. 162.

to fall back, and so on, the result being a continuous titinnabulation. Two small brass projections are attached to the poles of the electro-magnet in order to prevent actual contact between them and the armature and consequent magnetic retention, which would prevent the armature falling back when the current stopped. It is usual also to tip the screw E with platino-iridium and to attach a minute plate of the same alloy to the armature for this tip to play against; in this way on account of the hardness of the alloy wear and tear are diminished.

In *strikers* the screw E and bearing H are absent, the terminal T′ being connected directly to the nut K. Thus as soon as contact is made by the push P the armature is attracted and the clapper gives one stroke, but as there is now nothing to break the circuit automatically it does not fall back until the finger is taken off the push.

101. Telegraphy. To deal anything like thoroughly with this subject would require a book of several hundred pages; all, therefore, we shall attempt is to give an outline of its general principles together with a brief account of one of the most commonly employed systems, viz. the Morse.

The object of the telegraph is to enable two persons at distant places to communicate with one another. The place from which the message is sent is called the *sending* station and that to which it is sent the *receiving* station. Now call the places A and B, and first suppose that it was only required to send messages from A to B and not from B to A. This could be effected by placing A and B in a a circuit containing a battery, providing A with a contact-maker or "key" and B with some form of receiving instrument or "recorder" capable of responding to the current, the message of course being spelled out by a system of preconcerted signals. But as it is always necessary to be able to send messages *both* ways there must also be in the circuit a key at B and a recorder at A, and this calls for a special device, partly to prevent the sender transmitting the messages through his own recorder (which, however, would not much matter), and partly to prevent the key at the receiving station breaking the circuit when the person at the sending station was endeavouring to transmit. Fig. 163 shows the arrangement commonly adopted; it should be noted that in this as in all telegraphic systems only *one* wire (called the *line*) is used to connect the stations, the circuit being completed through the earth:—

Each of the stations A and B is provided with a battery (*zc*, *z′c′*) as well as with a key (K, K′) and recorder (R, R′). The zinc plates of both batteries are in good

electrical communication with the earth by means of earth-plates E E′ while their copper plates are capable of contact with the front ends of the keys. The recorders are placed in branch circuits as shown, and when a key is *not* depressed by the sender (as at K′) its back end is kept, by means of a spring, in contact with the recorder while it is lifted from

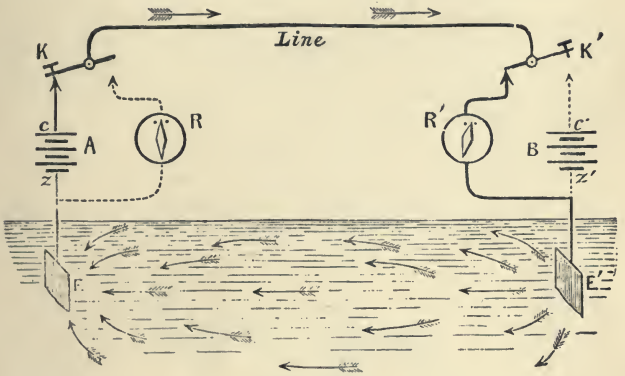


Fig. 163.

the battery. On the other hand when it *is* so depressed (as at K) its fuse-end makes contact with the battery while it is lifted from the recorder. The result is that each station is always in readiness to receive messages from the other and that the sender does not transmit through his own recorder: the figure shows A transmitting to B. Obviously, however, the two stations cannot signal to each other *simultaneously*; this can be done by the "duplex" system.*

In the Morse system above referred to, the recorder consists essentially of an electro-magnet and armature similarly arranged to those of the "striker" form of electric bell (see preceding section), the armature thus

* For an account of this and many other important matters the student is referred to Preece and Sievewright's *Telegraphy*.

remains drawn towards the magnet just so long as the key is pressed down. In the most modern form of recorder, known as the *ink-writer*, the armature is attached to a small wheel covered with printers' ink and under this is drawn by means of clockwork a long narrow strip of paper; when the armature is not attracted the wheel remains clear of the paper, but when attraction occurs it comes in contact therewith and prints a dot thus . or a dash thus — according as the duration of the current is "short" or "long." The letters of the alphabet, stops, etc., are represented by combinations of dots and dashes according to the annexed code:—

A	· —	P	· — — ·
B	— · · ·	Q	— — · —
C	— · — ·	R	· — ·
D	— · ·	S	· · ·
E	·	T	—
F	· · — ·	U	· · —
G	— — ·	V	· · · —
H	· · · ·	W	· — —
I	· ·	X	— · · —
J	· — — —	Y	— · — —
K	— · —	Z	— — · ·
L	· — · ·	Full stop	· · · · · ·
M	— —	Comma	· — — — — ·
N	— ·	Interrogation	· · — — · ·
O	— — —	Hyphen	— · · · · —

Another form of Morse recorder is the *sounder*. In this no printing is done but the person at the receiving station listen to the clicks of the instrument and notices whether the intervals between them are short or long. A short interval corresponds to a dot and a long one to a dash, the code of course being the same as above. The sounder is much more difficult to work with than the ink-writer, as the ear can interpret the signals only after careful training.

It frequently happens, especially over long distances, that the current from the sending station is too weak to

work the recorder. To remedy this a delicate auxiliary electro-magnet, called a *relay*, is made use of, it and the aforesaid current being made to pass through this *and not through the recorder at all*. Each time the current passes the relay armature is attracted and matters are so arranged that this completes a local circuit through the recorder and a battery at the receiving station. It is thus this latter battery which is the *direct* producer of the signals. Fig. 165 still shows the arrangement of the line, etc., *only that R, R' now represent the relays*, the recorders and local batteries not being depicted.

The key (K or K', Fig. 163) employed in the Morse system is of special construction and is shown in detail in Fig. 164. Its centre A works on a pivot connected with the "line" when not pressed down its back-end *a* is kept by the spring *f* in contact with the terminal *b*, which runs to the relay at

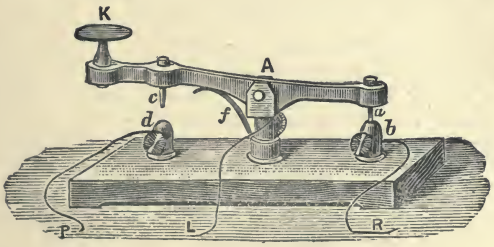


Fig. 164.

its own station (K', Fig. 163), while when pressed down its fuse-end *e* makes contact with the terminal *d* running to the battery (K, Fig. 163).

102. The Telephone. This instrument, which was invented by Prof. Graham Bell in 1876, depends for its sending power upon the fundamental fact of "magneto-electricity," and for its receiving power upon that of "electro-magnetism"; the former is that if a coil of wire be placed in a magnetic field and if anything be done to alter the strength of the part of the field where the coil is

situate, then so long as that alteration is in progress a current is induced in the coil, which goes one or the other way round according as the said part of the field is being strengthened or weakened; and the latter is that if a current circulates round a magnet it strengthens or weakens the latter according to whether it goes one way round or the other.

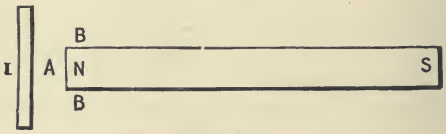


Fig. 165.

Before describing the instrument, let us consider the simple device of Fig. 165. NS is a bar magnet and I a piece of soft iron in front of one of its poles N. When I was absent there was a certain system of lines of force surrounding the magnet. When I is brought near, by virtue of its high permeability, the lines in the region A between the pole and the iron become more concentrated, while those in the lateral region BB become less so. Moreover, these effects are more marked the nearer the iron be to the pole. Confining our attention then to the region BB, we see that *any approach of I weakens while any recession strengthens it.* Now suppose N surrounded by a bobbin coiled with insulated iron, and let I be made to vibrate to and fro; then during approach a current will flow one way and during recession the opposite way through this wire. Moreover, if the ends of the latter be connected with those of another on a distant bobbin similarly surrounding one of the poles of another bar-magnet, in front of which is another piece of iron, these varying currents will alternately strengthen and weaken that pole and cause the iron (if suitably mounted) to vibrate in unison with the first piece,* though with much

* Approach of the first piece may of course produce either approach or recession of the second according to the polarity of the magnets and the way the bobbins are wound.

less vigour because by no means all the energy of the first is conveyed to the second, a great deal being dissipated as heat, etc., in the wires.

The above is the essence of the telephone; Fig. 166 shows the actuality in section. AA is a cylindrical bar-magnet fixed in a wooden case, and CC a bobbin surrounding one of its poles, and wound with fine insulated copper wire, the ends of which are connected to the terminals F, F. In front of the pole which the bobbin encircles, and supported at its edges by the wooden case, is a very thin diaphragm of soft iron D, and in front of this a mouthpiece. At the distant station is a precisely similar contrivance, and the two are in communication by "line," and "earth" connections to their terminals. The sender speaks into the mouthpiece of his instrument while the receiver holds his to his ear; the vibrations of the voice are communicated to the diaphragm of the former and these, as explained in the preamble, to that of the latter. Hence the receiver hears the words of the speaker, though with diminished strength and a somewhat "tinny" clang.

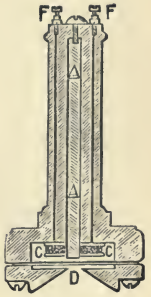


Fig. 166.

It is now usual for both sender and receiver to be supplied with *three* instruments, viz., one for each ear and a larger one for the mouth. The inconvenience of shifting is thus avoided and better effects obtained.

103. Dynamos. From what has been said above it is known that if a closed circuit be moved in a magnetic field in such a way as to vary the magnetic flux through the circuit then a current is induced in the circuit. In the case of the earth inductor, described in Article 88, it is shown how by rotation of a coil of wire round an axis at right angles to the direction of the magnetic field a continuous variation in the magnetic flux through the coil may be obtained. It has also been explained that during

each complete revolution of the coil the direction of the induced current changes as the coil passes through the position at right angles to the direction of the lines of force, but that by the action of a split ring commutator and spring brushes resting on the ring a continuous current in a fixed direction may be obtained. Such an arrangement constitutes a simple form of dynamo. A dynamo is thus a machine for the production of a current by means of magneto-electric induction. Theoretically, the essentials of such a machine are a magnetic field and the coil in which the induced current is produced. The magnetic field usually employed is the field between the poles of a suitably constructed magnet. In a very few cases a permanent steel magnet is employed, but in all modern dynamos an electro-magnet is used. Fig. 167

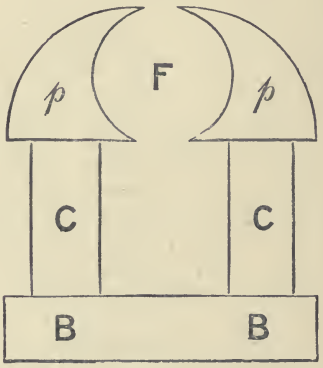


Fig. 167.

shows diagrammatically the arrangement of the iron circuit of *field magnets* of a dynamo. The iron cores C, C, are fixed in the bed-plate B, and carry the pole picees *p*, *p*, which are so shaped as to give a fairly uniform field in the suitably shaped space F between the poles. The

field magnet coils are wound round the cores C C, the number of turns being adjusted to suit the strength of current they will carry and the magnetic quality of the iron. The iron circuit, or as it is sometimes called the *magnetic circuit*, of the dynamo may be cast all in one piece or in separate pieces, but when separate the pieces must be closely and accurately fitted together in order to ensure the efficient action of the electro-magnet. The current through the field coils may be obtained from a suitable battery in which case the dynamo is said to be *separately excited*. It is, evident, however, that if the field magnets have once been excited there will, even when no current passes through the field coils, be a weak magnetic field in the space F. If a coil is rotated in this field a weak current is induced in the coil, and if the field coils be connected either in series or in parallel with this coil then this weak current increases the magnetism of the field magnets and therefore strengthens the field at F. This increase in the intensity of the field causes an increase in the induced current, and this again results in a further increase in the field intensity. This action may go on until the field magnets approach saturation, when an increase in the current gives little or no increase in the intensity of the field at F. By properly adjusting windings of the coil rotating between the poles and the field coils it is thus possible for a dynamo to self-excite its field magnets. Dynamos which work in this way are said to be *self-excited*—most modern dynamos are of this type. When the rotating coil and the field coils are in series the machine is said to be *series wound*, when they are in parallel or when the field coils are connected as a shunt on the rotating coil the machine is *shunt wound*. When the winding combines the series and shunt methods the machine is said to be *compound wound*.

Figs. 168, 169, 170, show diagrammatically the connections of the coils in a series wound, shunt wound and compound wound dynamo.

The coil which rotates in the field F is usually wound on a properly constructed frame containing soft iron, the

object of the soft iron being to increase by its permeability the magnetic flux through the coil. This frame with the coil constitutes the *armature* of the dynamo. The armature is mounted by the spindle, fitted through its frame on bearings, and may be rotated from an engine by a belt

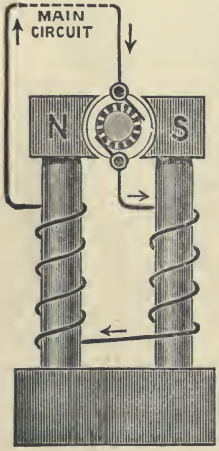


Fig. 168.

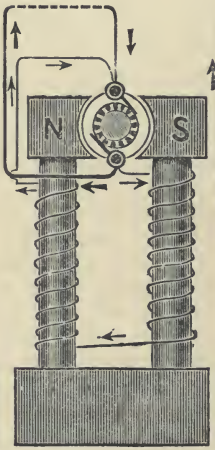


Fig. 169.

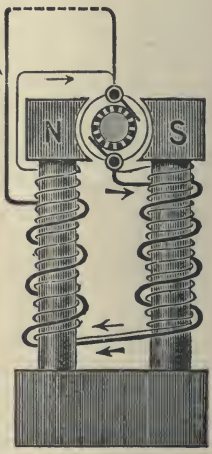


Fig. 170.

fitted round the fly wheel of the engine and on a pulley on one end of the armature spindle, or, as in some cases, the engine may be directly coupled to the armature without the intervention of a belt. The mechanical work done in driving the armature against the resistance due to the field at F is spent in producing the electrical energy developed in the circuit of the dynamo.

The coils of the armature are not closed circuits. In the simplest case the armature coil may be one continuous wire, one end of which is connected to one half of the split ring commutator and the other end to the other half. Brushes or springs rest upon the commutator ring and

they are connected to the terminals of the external circuit so that the closed circuit in which the current is induced consists of the armature coil as internal circuit and the external circuit connected to the brushes. In the case of most dynamos the armature coil is wound in a number of separate sections and the commutator ring is split into a corresponding number of sections or bars.

The commutator ring, with each section insulated, is fitted on the armature spindle and the brushes are usually carried by a brush holder, which allows them to be raised off or pressed on the ring at the proper points.

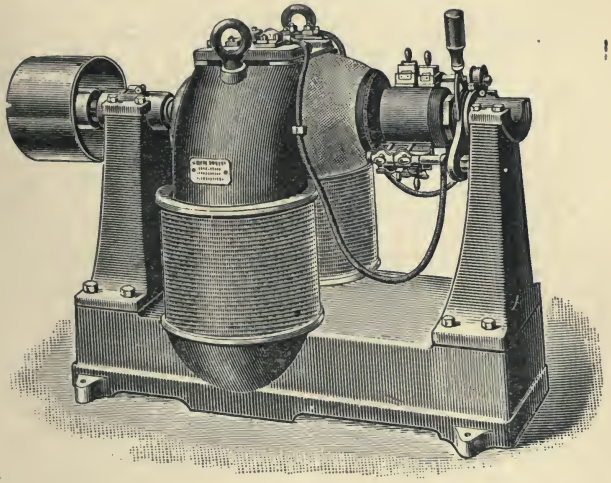


Fig. 171.

Fig. 171 shows a simple form of the modern dynamo. The student should be able to make out the various parts for himself.

A dynamo, such as has been described above, gives, by the action of the commutator, a *continuous* or one-direction current in the external circuit, and is called a *continuous current* dynamo. If, instead of the commutator, we have

two insulated rings or collars fitted on the spindle, one ring connected to one end of the armature coil and the other to the other end, then by means of brushes, resting one on each of these collars, we can obtain a current which changes direction in the circuit twice during each revolution of the armature. Such a current is an instance of what is called an alternating current, and a dynamo supplying an alternating current is called an *alternator*. The number of times a current *alternates* during a complete revolution of the armature depends upon the arrangement of the field magnets and the coils of the armature.

104. Electromotors. In a dynamo mechanical energy is expended in causing the rotation of the armature in the magnetic field due to field-magnets in the space between the pole pieces—and by its action mechanical energy is expended in producing electrical energy. In an electromotor the reverse action takes place. A current from a battery or another dynamo, is passed through the coils of the armature, and the field magnets (either in shunt or in series), and the couple acting on the armature, due to the force exerted on its coils carrying current in a magnetic field, causes its rotation. If the armature is properly connected to suitable mechanism the torque causing rotation may be made to do work. In construction a motor is essentially the same as a dynamo—in fact any well constructed dynamo will run as a motor, but there are numerous differences in design necessitated by the very different uses to which dynamos and motors are put.

When a motor is running, and the rotating armature is made to do work, then the motor is a sink of energy in the circuit, and is therefore a seat of back electromotive force. This is also evident from the fact that as the armature rotates the machine acts as a dynamo, and tends to produce a current in a direction opposed to that which causes it to run as a motor, and thus a back E. M. F. is set up. The more rapidly the motor runs the higher the back E. M. F., and the smaller the current in the circuit. Thus if R denote the resistance of the circuit, E the E. M. F. of the

battery or dynamo, and e the back E. M. F. of the motor, then the current in the circuit is given by

$$C = \frac{E - e}{R}.$$

Also, under the same conditions, EC denotes the *activity* of the dynamo or battery, eC the activity of the motor, and EC $- e$C or C(E $- e$) the loss of power due to the heating of the circuit by the current.

The electrical efficiency of this dynamo-motor or battery-motor arrangement is evidently given by the ratio eC/EC or e/E for, of the power EC given out by the dynamo or battery, an amount eC is absorbed by the motor. This ratio approximates to unity when e is nearly equal to E. Hence the more rapidly a motor runs, that is the lighter the load at which it works, the greater the efficiency of the arrangement and the less the loss of power in the circuit.

The activity, eC, of the motor refers to the power it absorbs in the circuit as an equivalent of the mechanical work done by the motor. If we include the energy dissipated as heat in the coils of the motor then the total power absorbed from the circuit is eC + C^2r, where r denotes the resistance of the coils of the motor. Or, if e' denote the difference of potential between the terminals of the motor, then e'C gives the total power absorbed by the motor, that is, e'C = eC + C^2r. The ratio of the power absorbed as the equivalent of mechanical work done by the motor to the total power absorbed by the motor, that is, the ratio eC/e'C or e/e' is known as the *electrical efficiency* of the motor. The mechanical work done by the motor as the equivalent of eC is not all useful external work; part of it is spent in the motor doing internal work against the friction of the bearings, air resistance, and the inertia of the moving parts of the machine. If P denote the power of the motor to do external work, as measured say by a suitable brake, then the ratio P/eC gives what is called the *mechanical efficiency* of the motor.

When the connection is first made between a motor and the battery or dynamo which is to run it there is no back

E. M. F. in the circuit, and an extra resistance has to be put in to prevent the production of an excessively high current, then when the motor has got up speed the resistance may be taken out. For this purpose all simple motors are provided with a starting switch.

Motors may be series wound, shunt wound or compound wound just as dynamos are. Most small toy motors are series wound owing to the simplicity of the winding, but series wound motors of a large size are not much used. They are much more variable in speed than shunt wound machines, which, within certain small limits, automatically regulate their speed according to the load. Motors with compound winding may be adjusted to have practically a constant speed at all loads.

105. Electroplating and Electrotyping. The term electro-*plating*, which was originally applied to the process of coating a baser metal with silver, is now extended to include the coating of any object with a coherent layer of any metal; in practice, however, the metals employed as the coat are usually either silver, gold, nickel, or copper, while the object may be either another metal, *e.g.* iron, or some non-conducting material such as wood or plaster-of-Paris.

The process is an application of the principle of electrolysis. The object to be coated is attached to the negative pole of a battery or dynamo, and immersed in a suitable solution or "bath" of some salt of the metal it is desired to deposit thereon; a plate of the last-named metal is also immersed in the solution and is attached to the positive pole. If the object be a non-conductor its surface must first be rendered conducting by covering it with a thin layer of graphite or sulphide of silver. When the current passes, the object (the *kathode*) receives the desired deposit from the "bath," while the other plate (the *anode*) becomes dissolved, thus maintaining the solution of approximately constant strength.

When copper is the metal to be deposited it is not unusual to abolish the source of external excitation and simply to make the object the positive or high-potential plate of a Daniell cell.

Great care has to be taken to employ solutions of appropriate composition and strength, and also properly to regulate the "density," as it is called, of the current, *i.e.* its strength per unit area of the object; otherwise the the deposit is not adherent.

The most important electroplating industry is electro-*silvering*. In this case the salt is a double cyanide of silver and potassium, the bath being prepared by dissolving one part (by weight) of silver cyanide and two of potassium cyanide in 40 of distilled water, and the things to be silvered have to be first freed from grease by means of a solution of potassic hydrate, then washed in weak sulphuric acid, then "scoured," *i.e.* washed in a mixture of sulphuric and nitric acid and salt, and finally amalgamated by immersion in a dilute solution of mercuric sulphate. The most appropriate current "density" is from four to five ampères per square foot, and this is maintained until the deposit amounts to about one ounce per square foot, which corresponds to a thickness of about $\frac{1}{800}$ inch. On removal from the bath the silvered articles are immersed in a solution of potassium cyanide, washed in boiling water and dried in sawdust.

Electro*gilding* is the art of coating objects with gold. The bath is a solution of the double cyanide of gold and potassium. The details are similar to those for electro-silvering except that the current employed is weaker (about one ampère per sq. ft.) and the deposit thinner.

Electro*nickeling* is the art of coating objects (usually of steel, as the handle-bars of bicycles) with nickel. The bath is a slightly acid solution of the double sulphate of nickel and ammonium. Prior to its immersion in the bath the object is cleaned and polished; the current is made fairly strong to begin with and then slackened: a good coat consists of about $\frac{2}{3}$ ounces per square foot, which represents about $\frac{1}{1000}$ inch in thickness.

Electro*coppering* is the easiest of all the plating process. The bath is simply a strong and very slightly acid solution of copper sulphate; it is best to have the current at first

very weak and subsequently to increase it to a density of from 2 to 20 ampères per square foot. When *iron* is the metal to be coppered the simple copper sulphate bath is unsuitable because it would be rapidly decomposed by the iron; in this case the bath consists of an alkaline solution of the double tartrate of copper and sodium obtained by adding excess of sodic hydrate to a solution of copper sulphate and tartaric acid.

Electrotyping is a particular application of electrocoppering, and is used for the production of pictures and diagrams in books. The author first makes a drawing on paper, and a left-handed copy of this is made on wood by a draughtsman, when the wood being cut away by the engraver the lines of the picture show up in relief. The wood, as thus prepared, constitutes the "block" and could be employed directly for printing off *a few* copies, but not many, because on account of its softness it would soon wear away and the lines become indistinct. Accordingly, instead of so using it a mould of it is made in gutta percha, and this is electrocoppered, as above explained, when we get a very thin relief impression (left-handed) of the original drawing, which is strengthened by filling up the hollows with melted type-metal. This is called the "cliché" or "electro," and is capable of yielding some 80,000 impressions. When more are required the electros are sometimes hardened by "steeling"; this consists in plating them with iron, the requisite bath being prepared by passing a current between two plates of iron immersed in an aqueous solution of ammonium sulphate. A double salt (ferrous-ammonium sulphate) is thus formed which does not readily deteriorate by absorbing oxygen from the air.

APPENDIX FOR SCIENCE AND ART STUDENTS.

(A) Spark and Brush Discharge.

[*To be read after Article* 35.]

When two conductors at different potentials are made to approach one another a passage of electricity takes place between them across the intervening dielectric accompanied by heat, light, and a crackling sound. The phenomenon is in general momentary and the potentials of the conductors become more or less equalised, but by connecting the latter to an electrical generator, as in the case of the knobs of a Wimshurt machine, it is possible to restore the original potential difference as fast as it runs down and the discharge then becomes practically continuous.

The appearance presented may be either that of an *unbroken* line of light (straight, curved, or zigzag), when it is called a "spark," or that of a *broken* line with branches diverging in an irregular way like the veins of a leaf when it is called a "brush."

The distance which separates the two conductors when the discharge takes place is called their *striking distance*, and for a given pair of conductors similarly placed is found to be very nearly proportional to their potential difference, but for differently shaped pairs (or even for the same pair with differently shaped portions of their surfaces facing one another), it depends also upon the shape. Thus for a pair of spheres the striking distance is about one inch per 100,000 volts (300 es. units), while for a point facing a flat plate it is much greater, viz., about one inch per 20,000 volts, and for a pair of points greater still. There appears to be no practical difference so far as striking distance is concerned between the "spark" and the "brush," that is taking a given pair of conductors in a given relative

position and at a given difference of potential and causing them gradually to approach, we do *not* get a brush at one distance and a spark at another but we get *either a spark or a brush* at a definite distance. Which of the two forms the discharge assumes depends partly on the capacity of the conductors, but mainly on the shape of their opposing surfaces; rounded surfaces tend to produce sparks, and pointed ones brushes.

It should be carefully noted that the term "striking distance" refers to the *starting* of the discharge and not to its *maintenance*. For example, suppose we have two spheres which by means of a machine are kept at a potential difference of 10,000 volts, then in order to start a spark between them they must be brought to within $\frac{1}{10}$ inch (the striking distance), but after the spark has once started they may be gradually separated to a much greater distance and it will still continue. This circumstance finds application in electric arc lamps where a mechanism is employed to bring the carbons into close proximity in order to start the light and afterwards to separate them.

The figures given above suppose the discharge to take place in air at the ordinary pressure. For solid (and liquid) dielectrics the striking distance is *ceteris paribus* much less owing to their greater mechanical rigidity. For the same pressure the striking distance is under similar conditions much the same in all gases, but when the pressure is diminished the striking distance increases up to a certain point, and afterwards diminishes until when the exhaustion is very complete no discharge occurs however great the difference of potential. For high but not extreme exhaustion the appearance is neither that of the spark or brush, but of delicate striae, and is known as the electric "glow."

All these phenomena appear to indicate that the discharge (in whatever form it occurs) is not a simple passage of "electricity" from one of the conductors to the other, but that this passage is effected by means of the material molecules of the dielectric. According to the

kinetic theory of matter, the molecules of every substance are in a state of rapid vibration and mutual collision, the length of the journey performed by each molecule (called its "free path") being extremely small, and less the greater the mechanical rigidity, and if we regard the discharge as due to the handing on of electricity from one molecule to another during the vibratory movements an explanation is suggested of the connection between rigidity and striking distance. Moreover, in the case of gases, the kinetic theory renders it certain that as the pressure diminishes the free path increases, which explains the corresponding increase of striking distance.

A remarkable fact with regard to the discharge is its *oscillatory* character; although the spark be practically instantaneous, in reality it occupies a finite though very small time, and when it is examined by a revolving mirror it is found to consist of a series of discharges alternately in opposite directions. If a piece of paper or cardboard be held between the knobs of a Wimshurst so as to allow the spark to pierce it, it makes a hole, which, instead of being indented on one side and protuberant on the other like a pin-prick, is *protuberant on both sides*, the molecules having apparently oscillated to and fro, and on the whole moved outwards in both directions.

(B) Force exerted on a Conductor carrying a Current in a Magnetic Field.

[*To be read after Article* 75.]

It has been shown in Articles 70 and 71 that the intensity of the magnetic field at c due to the element ab is given by

$$i = \frac{C \times ab'}{oc^2}$$

the direction of the field at c being at right angles to the plane abc.

If we denote ab by l, oc by r, and the angle aoc by a, then

$$i = \frac{C\, l \sin a}{r^2}$$

This expression for the intensity of the magnetic field at c gives the force which a unit north pole would experience if placed at that point. It therefore gives the force which a unit north pole at c would exert on the element ab. But the strength of the magnetic field at o, due to a unit north pole at c, is $1/r^2$. Hence the force exerted on the element ab in a magnetic field of intensity $1/r^2$ is

$$C \,.\, l \sin a \,.\, 1/r^2$$

That is, in a field of intensity H the force exerted on the element ab of the conductor AB carrying a current C is

$$C\, l \sin a \,.\, H \quad \text{or} \quad CH\, l \sin a$$

and the direction of this force is at right angles to the plane determined by ab and the direction of the magnetic field at o.

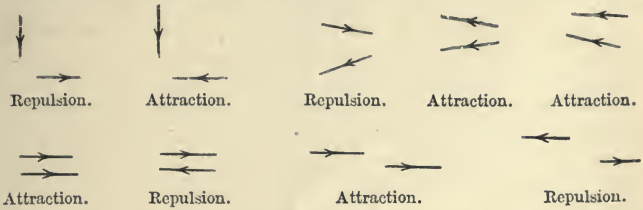
The result given above may evidently be put in the following simple form. The product $H\, l \sin a$ is the area of a parallelogram whose adjacent sides, inclined at an angle a, represent H and l respectively in magnitude and direction. Hence, if we construct a parallelogram whose adjacent sides represent in magnitude and direction the intensity of the magnetic field and the length of an element of the conductor carrying a current C, and if A denote the area of this parallelogram, then the force exerted on this element is given by CA, and is at right angles to the plane of the parallelogram. If the conductor is straight and the field uniform then this result may be applied to a conductor of finite length.

(C) Mutual Action of Currents.

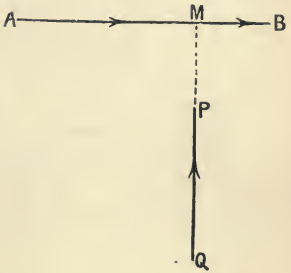
[*To be read after Article* 75.]

If we place two separate spirals of wire a short distance apart with their axes in the same straight line and pass currents round them, then (provided they have suitable freedom of movement) they are found to approach or recede

from one another, thus indicating respectively attraction or repulsion, according as the currents circulate round them



in the same or in opposite directions. Bearing in mind that the current renders each spiral practically a magnet, and also the fundamental law of electromagnetic polarity, it is easy to see that these actions are in accordance with the laws of attraction and respulion between two magnets.



But they may also be referred directly to the currents, in fact, seeing that a circuit of whatever shape carrying a current is equivalent to a magnet, we should expect a pair of such in general to attract or repel each other according to circumstances. That this is so was first shown experimentally by Ampère, who demonstrated in the case of straight circuits the following laws :—

(1) *Two* PARALLEL *currents attract or repel one another according as they flow in the same or in opposite directions.*

(2) *Two* NON-PARALLEL *currents attract one another if both approach or both recede from the point of meeting of their directions while they repel one another if one approach and the other recede from that point.**

These laws are illustrated in the figures above, which the

* If the currents be not in the same plane the words "point of meeting of" should be replaced by "common perpendicular to."

student should carefully study. In the latter the vertical current attracts the portion AM of the horizontal one and repels the portion MB, but since AM is longer than MB there is, on the whole, attraction; if M were the middle point of AB the total force would be *nil*.

The experiments whereby Ampère's laws are established merely consist in having two circuits, one fixed and the other moveable, placing them in various relative positions and passing currents in sundry directions, when the moveable circuit is observed to behave in accordance with the said laws.

A striking illustration of Ampère's first law is afforded by *Roget's vibrating spiral*; this is simply a fine copper spiral suspended vertically by its upper end from a fixed support, and with its lower end just dipping into mercury; one pole of a battery is connected to the support and one to the mercury, and when the current is passed the spiral vibrates up and down. The reason is that the coils carrying the current first attract one another; this shortens the spiral, causing its lower end to leave the mercury, the current then stops, the coils cease to attract, and the lower end again falls into the mercury, thus renewing the current and causing the movement to repeat.

(D) Resistance of Liquids.

[*To be read after Article* 81.]

Liquids like solids offer resistance to the passage of the current. The amount varies greatly with the nature of the liquid; thus turpentine, paraffin, and pure water are practically insulators, mercury is a good conductor, while electrolytes, *e.g.*, a solution of cupric sulphate, conduct fairly well. Moreover, the resistance of such solutions is found in general to depend on their strength, diminishing as the latter increases, up to a certain point, and then increasing until saturation is attained; thus

in the case of sulphate of copper the least resistance is found to correspond to $CuSO_4 + 45\ H_2O$ and in that of sulphate of zinc to $ZnSO_4 + 24\ H_2O$.

The *specific resistance* of a liquid, like that of a solid, is defined as the resistance of a "centimetre cube" thereof, that is a cube each of whose edges is one cm. long, it being understood that the current flows through it in a direction perpendicular to a pair of opposite faces. Moreover, as in the case of solids, if S denote the specific resistance and R that of a column of uniform cross section a (of any shape) and length l, we have

$$R = Sl/a$$

The most important case of liquid resistance is that of an electrolyte, and here its practical measurement is attended with difficulties arising from the back E. M. F. which generally accompanies electrolysis and which renders most of the methods employed for metals, *e.g.*, the Wheatstone Bridge, inapplicable. Moreover, owing to polarization, this back E. M. F. does not remain constant for any length of time. In the following method both these difficulties are obviated :—

The solution is contained in a tall cylindrical jar of known internal cross-section a square centimetres; at the bottom rests a platinum electrode just fitting the jar and connected by a gutta-percha-covered wire to one of the poles of an external battery; the other electrode similarly connected to the other pole is also of platinum and of the same size *but is arranged so as to be capable of sliding up and down the jar*. A tangent galvanometer and contact-breaker are placed in the circuit between one of the electrodes and the corresponding battery-pole.

Now let E denote the E. M. F. of the battery, E′ the back E. M. F. of the solution, and X the joint resistance of the battery, galvanometer, and connecting wires, all these being unknown. Also let S be the specific resistance of the solution which it is our object to determine. To do this we first place the electrodes a distance l_1 centimetres (ascertained by measurement) apart, make contact and

note the galvanometer-deflection d_1; if k denote the reduction factor of the instrument we have, by Ohm's law, etc.,

$$\frac{E - E'}{X + \frac{Sl_1}{a}} = k \tan d_1 \ldots . (1).$$

We next change the distance between the electrodes to a new (measured) value l_2 and note the deflection d_2, *the operation being performed quickly so as not to allow time for the back E. M. F. appreciably to alter*. We then have in like manner

$$\frac{E - E'}{X + \frac{Sl_2}{a}} = k \tan d_2 \ldots . (2).$$

Lastly, we change the distance to a third value l_3, again operating quickly, when we have

$$\frac{E - E'}{X + \frac{Sl_3}{a}} = k \tan d_3 \ldots . (3).$$

Dividing (2) and (3) respectively by (1) we obtain

$$\frac{X + \frac{Sl_1}{a}}{X + \frac{Sl_2}{a}} = \frac{\tan d_2}{\tan d_1} \quad \text{and} \quad \frac{X + \frac{Sl_1}{a}}{X + \frac{Sl_3}{a}} = \frac{\tan d_3}{\tan d_1}$$

from which two equations we can eliminate X and find S.

Another good method devised by Kohlrausch is a Wheatstone's bridge method. The liquid may be placed in a jar or tube fitted with electrodes as described above. The usual connections are made, but the secondary of an induction coil is used instead of a battery, and a telephone instead of a galvanometer. The adjustment is correct when the buzzing sound heard in telephone, due to the action of the make and break of the induction

coil, is reduced to zero or to a minimum. The resistance of the column of liquid is then calculated in the usual manner.

(E) Secondary Batteries.

[*To be read after Article* 84.]

Viewed from the standpoint of energy, the principle of any voltaic cell is that we take certain substances which react chemically upon one another; during this process the chemical potential energy of the system diminishes, and the equiavlent of the loss is obtained as electrical energy, which ultimately becomes all heat or partly heat and partly mechanical work according as the external circuit consists of a simple resistance or contains some kind of electromotor. In a *voltameter* precisely the reverse transformations occur, the salt employed is made to undergo decomposition, whereby the chemical potential energy of the system is augmented, and the increase is furnished at the expense of the electrical energy of the current employed to effect the electrolysis. If now, the latter having been performed, the external current be taken away and the plates of the voltameter connected by a wire, re-union occurs between the materials deposited on the two electrodes, the chemical potential energy of the system diminishes and the equivalent of the loss is again obtained as electrical energy; in short *when the plates of a voltameter after electrolysis are connected it behaves as a battery.* This is the principle of "secondary batteries" or "accumulators" which are simply commercial forms of voltameter used mainly in connexion with electric lighting; the process of performing electrolysis in them is known as "charging," while when subsequently used to furnish the current they are said to be "discharged."*

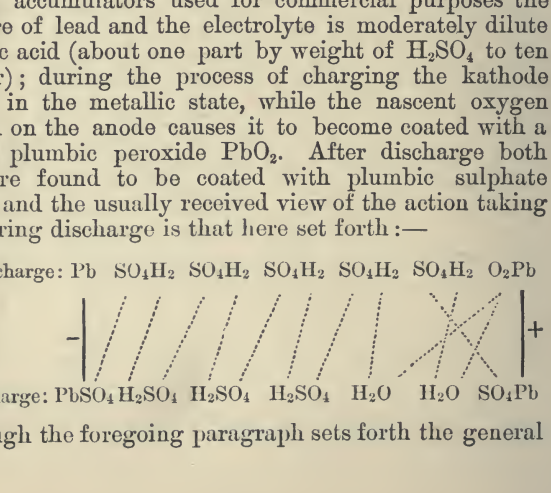
The pioneer of modern accumulators was *Grove's gas-battery.* It consists of two long glass tubes open at the

* It is a common popular notion that they are charged with electricity. Of course this is not the case; what they are really charged with is chemical potential energy.

bottom and closed at the top with brass caps having binding screws attached; inside each tube and extending nearly the entire length thereof is a platinum plate attached by a wire to the brass cap. The tubes are filled with very dilute sulphuric acid and immersed mouth downwards in a vessel of the same liquid, when the "battery" is complete. To "charge" it the binding screws are connected to the poles of an external battery when electrolysis occurs, hydrogen and oxygen accumulating and being partly absorbed or "occluded" by the platinum plates. The battery is then removed when the occluded gases will remain for a considerable time without escape, but as soon as the plates are connected the gases reunite and the battery is discharged. This instrument is, however, too feeble and the current furnished of too short duration for it to be of any practical value.

It should be noted that in this case the action between the materials deposited on the two plates takes place through the string of intervening molecules of liquid in accordance with Grotthüs's theory: the same is true in the accumulators now to be considered.

In all accumulators used for commercial purposes the plates are of lead and the electrolyte is moderately dilute sulphuric acid (about one part by weight of H_2SO_4 to ten of water); during the process of charging the kathode remains in the metallic state, while the nascent oxygen liberated on the anode causes it to become coated with a layer of plumbic peroxide PbO_2. After discharge both plates are found to be coated with plumbic sulphate ($PbSO_4$) and the usually received view of the action taking place during discharge is that here set forth:—



Although the foregoing paragraph sets forth the general

action of accumulators it omits several details. The earliest accumulators were devised by Gaston Planté in 1860. In these the plates prior to charging were two sheets of ordinary lead coiled together, but prevented from coming into actual contact by a layer of felt. On *once* passing the charging current the + plate became partly peroxidized while the — plate remained unchanged and being in a compact form was during discharge not at all easily attackable by the SO_4 so that the action was very poor. To remedy this the charging was made to consist of passing the current a great number of times in alternate directions; calling now the original anode A and the original kathode B, the first passage peroxidizes A leaving B unchanged, the second peroxidizes B and reduces the portion of A previously peroxidized to metallic lead *in a spongy state*, the third passage similarly reduces B while it peroxidizes at any rate some of the hitherto unattacked portions of A, and so on until eventually one of the plates contains a thick coating of the peroxide while the other consists largely of *spongy* metallic lead, and since this is more easily attacked than the compact sheet the discharging action is much more efficient. The process of charging the Planté cells was, however, very tedious, and to obviate this Fauri in 1881 improved upon them by coating the plates to begin with with a layer of *red-lead* (Pb_3O_4); on charging, the red lead on the kathode became quickly reduced to spongy metal, while that on the anode was peroxidized. In this way much time was saved. In the more recent Sillon-Volckmar cells the plates are made in the form of a grid, the hollows being filled with a paste of red lead; in this way they become more cohesive and less liable to fall to pieces. Both the Fauri and Sillon-Volckmar plates are usually made flat and arranged in "nests," the positive and negative nests locking into one another, while electrically separated by strips of wood or indiarubber.

After any form of accumulator has been discharged it can be charged again by passing a current from an external source; the sulphuric acid thus becomes electrolysed, the

nascent oxygen on the anode converts the $PbSO_4$ into PbO_2 according to the equation

$$PbSO_4 + O + H_2O = PbO_2 + H_2SO_4$$

while the nascent hydrogen on the kathode reduces the $PbSO_4$ to the spongy metallic state thus

$$PbSO_4 + H_2 = H_2SO_4 + Pb.$$

The great advantages of secondary over primary batteries are (i) their low internal resistance, which is only about ·02 ohm persquare foot of plate, and (ii) no appreciable action takes place in them when their terminals are unconnected. Their E. M. F. is about 2 volts per cell.

(F) Calculation of the E.M.F.

[*To be read after Article* 85.]

In a voltaic cell the chemical reactions are essentially of an *exothermic* nature, that is, they are such as to produce under ordinary circumstances, an evolution of energy in the form of heat. In a voltaic cell this energy is not evolved as heat, but the electrical energy evolved is the equivalent of this heat energy. In a voltameter the chemical reactions are of an *endothermic* nature, that is, they are usually accompanied by an absorption of heat energy, and the presence of a voltameter in a circuit therefore means the absorption of an amount of electrical energy from the circuit equivalent to this heat energy. From these considerations the E.M.F. of a given voltaic cell or the back E. M. F. in a voltameter can be calculated if the necessary data are known. Thus if in a voltaic cell h gram-degrees of heat are evolved by the total chemical action which takes place when one gram of the negative element of the cell is dissolved, then Jh denotes the energy evolved in ergs, J being Joule's mechanical equivalent of heat in ergs per gram-degree. But if E denote the

required E. M. F. of the cell in volts, and w the electro-chemical equivalent of the negative element in grams, then E/w denotes in *Joules* the energy evolved per gram of the negative element. Hence

$$J h = E/w \times 10^7$$

or

$$E = \frac{J h w}{10^7}$$

in volts, or as $J = 4{\cdot}2 \times 10^7$.

$$E = 4{\cdot}2\, h\, w.$$

For example, in a Daniell's cell $h = 770$ gram-degrees, roughly, per gram. of zinc dissolved.

Therefore $E = 4{\cdot}2 \times 770 \times {\cdot}000{,}328$ volts;

or $E = 1{\cdot}06$ volts (approximately).

Similarly in the case of a water voltameter h is about 35,000 gram-degrees per gram of hydrogen evolved, and therefore

$$e = 4{\cdot}2 \times 35{,}000 \times {\cdot}00001038 \text{ volts};$$

that is $e = 1{\cdot}5$ volts (approximately).

(G) Coefficient of Self-Induction.

[*To be read after Article* 91.]

When a current flows through a coil it produces a flow of magnetic force through the coil. This flow of force is proportional to the current when the surrounding medium is non-magnetic. In this case if F denotes the flow of force through the coil and C the current in the coil then

$$F = LC$$

where L is a constant. This constant is the co-efficient of self-induction of the coil.

If the surrounding medium is magnetic then L is not a constant, but depends upon the permeability of the medium and therefore varies with C.

When F varies on account of the variation of C then we evidently have the relation

$$\frac{dF}{dt} = L\frac{dC}{dt} \quad \text{that is,} \quad E = L\frac{dC}{dt}$$

This relation supplies another definition of L and shows that the induced electromotive force is proportional to the rate of change of the current in the coil.

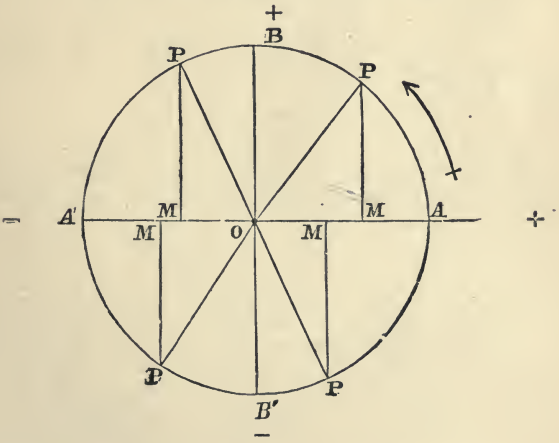
APPENDIX ON TRIGONOMETRY.

1. Let OA be fixed, and let the revolving line OP describe a circle around O. Draw PM always perpendicular to AA'. Then, giving PM, OM their proper signs in accordance with the usual convention, *and taking OP always positive*,

$$\sin AOP = \frac{PM}{OP}, \quad \cos AOP = \frac{OM}{OP},$$

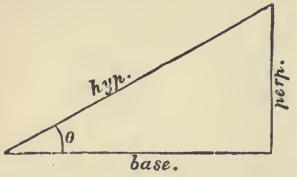
$$\tan AOP = \frac{PM}{OM}, \quad \cot AOP = \frac{OM}{PM},$$

$$\sec AOP = \frac{OP}{OM}, \quad \operatorname{cosec} AOP = \frac{OP}{PM}.$$



A *Definition* must be written as above, but it is useful to remember

that, in a right-angled triangle (see Fig.),



$$\sin\theta = \frac{\text{perp.}}{\text{hyp.}},$$

$$\cos\theta = \frac{\text{base}}{\text{hyp.}},$$

$$\tan\theta = \frac{\text{perp.}}{\text{base}},$$

the perpendicular being the side *opposite* the angle taken.

2. Since $$\frac{PM}{OM} = \frac{PM}{OP} \cdot \frac{OP}{OM} = \frac{PM}{OP} \div \frac{OM}{OP},$$

we get $$\tan AOP = \frac{\sin AOP}{\cos AOP}.$$

3. By Euclid I., 47, $PM^2 + OM^2 = OP^2$;

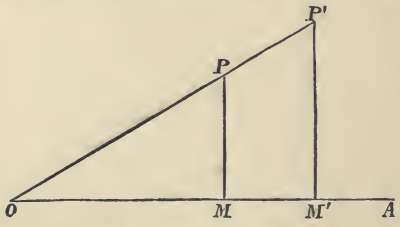
$$\therefore \quad \left(\frac{PM}{OP}\right)^2 + \left(\frac{OM}{OP}\right)^2 = 1;$$

$$\therefore \quad (\sin AOP)^2 + (\cos AOP)^2 = 1;$$

or, as it is generally written,

$$\sin^2 AOP + \cos^2 AOP = 1.$$

4. If OP be taken of any other length, say OP', the lengths of OP', $P'M'$, OM' will be different from those of OP, PM, OM, but the



ratios $\frac{P'M'}{OP'}, \frac{OM'}{OP'}, \frac{P'M'}{OM'}$ will be equal to the ratios $\frac{PM}{OP}, \frac{OM}{OP}, \frac{PM}{OM}$

respectively. For, since the triangles PMO and $P'M'O'$ are equiangular, the sides about the equal angles will be proportional (by Euclid VI., 4).

Hence
$$\frac{P'M'}{OP'} = \frac{PM}{OP} = \sin AOP,$$

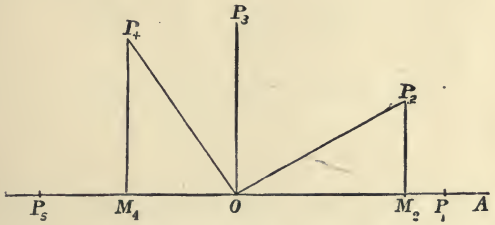
$$\frac{OM'}{OP'} = \frac{OM}{OP} = \cos AOP,$$

$$\frac{P'M'}{OM'} = \frac{PM}{OM} = \tan AOP.$$

Therefore the trigonometrical ratios of an angle only depend on the magnitude of the angle.

5. **Positive and negative trigonometrical ratios.**

Imagine the line OP to start from the position OP_1 along OA, and to revolve about O so as to occupy successively the positions OP_2, OP_3, OP_4, OP_5. Then M, the foot of the perpendicular from P on OA,



starts from P_1, and travels to the left so as to coincide with O when $\angle AOP = 90°$, and to coincide with P_5 when OP has revolved through an angle of 180°. As long as M lies to the right of O, OM is considered positive; but when M lies to the left of O (*i.e.*, when $\angle AOP > 90°$), OM is considered negative. PM is considered positive throughout the whole change of position of OP from OP_1 to OP_5.

Therefore, for **angles between 0° and 90°**, the sine and cosine are **both positive**; while for **angles between 90° and 180°**, the **sine is positive** and the **cosine is negative.**

6. To find the **trigonometrical ratios** for an angle of **30°**.

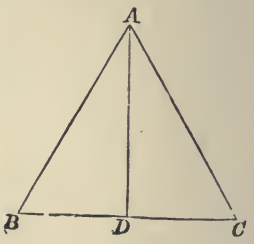
Draw an equilateral triangle ABC.

Join A to D, the middle point of BC.

Then the triangles ABD, ACD are equal in every respect.

Every angle of the equilateral triangle = 60°, because the three angles together = two right angles = 180°.

$\therefore$ $\angle BAD = 30°$, and $BDA = 90°$.



Now let each side of the equilateral triangle $= 2a$;

$$\therefore \quad BD = a, \quad \text{and} \quad AD^2 + BD^2 = AB^2,$$

$$AD^2 = AB^2 - BD^2 = 4a^2 - a^2 = 3a^2;$$

$$\therefore \quad AD = a\sqrt{3};$$

$$\therefore \quad \sin 30° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2},$$

$$\cos 30° = \frac{AD}{BD} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2},$$

$$\tan 30° = \frac{BD}{AD} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

7. To find the **trigonometrical ratios** for an angle of **45°**.

Draw a square $ABCD$, and draw the diagonal AC.

Then $\angle CAB = \frac{1}{2}$ right angle $= 45°$,

$\angle CBA =$ right angle.

And if $AB = BC = a$, $AC^2 = AB^2 + AC^2 = 2a^2$, $\therefore$ $AC = a\sqrt{2}$; therefore

$$\sin 45° = \frac{BC}{AC} = \frac{1}{\sqrt{2}}, \quad \cos 45° = \frac{AB}{AC} = \frac{1}{\sqrt{2}}, \quad \tan 45° = \frac{BC}{AB} = 1.$$

8. To find the **trigonometrical ratios** for an angle of **60°.**

Take the figure of § 6. Then $\angle B = 60°$. Therefore

$$\sin 60° = \frac{AD}{AB} = \frac{\sqrt{3}}{2}, \quad \cos 60° = \frac{BD}{AB} = \frac{1}{2}, \quad \tan 60° = \frac{AD}{BD} = \sqrt{3}.$$

The trigonometrical ratios for these angles are given in the following table, *which should be remembered.*

Angle	0°	30°	45°	60°	90°
sin =	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos =	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan =	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

If there be any difficulty in fixing these in the memory, it may be noticed that, for the common angles 0°, 30°, 45°, 60°, 90°, the sines are the *square roots* of

$$\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4};$$

the cosines are the *square roots* of

$$\frac{4}{4}, \frac{3}{4}, \frac{2}{4}, \frac{1}{4}, \frac{0}{4};$$

and the tangents are the square roots of the first series of numerators divided by the corresponding numerators of the second series.

In any case, note that the series of values of the cosines is the same as the series for sines written backward; and each tangent is the corresponding sine ÷ cosine.

9. **Trigonometrical ratios of angles between 90° and 180°.**

In the fig. of § 5, let

$$\angle P_4OM_4 = \angle M_2OP_2 = A;$$

then $$\angle AOP_4 = 180° - A.$$

$$\therefore \quad \sin(180° - A) = \frac{P_4M_4}{OP_4} = \frac{P_2M_2}{OP_2} = \sin A,$$

$$\cos(180° - A) = \frac{OM_4}{OP_4} = -\frac{OM_2}{OP_2} = -\cos A.$$

Similarly, $$\tan(180° - A) = -\tan A.$$

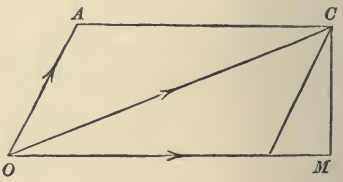
Hence $$\sin 120° = \sin(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2},$$

and $$\cos 120° = \cos(180° - 60°) = -\cos 60° = -\frac{1}{2}.$$

Similarly for the trigonometrical ratios of angles of 135° and 150°.

10. Magnitude of the resultant of two forces.

Let OA, OB represent the two forces P and Q in magnitude and direction.

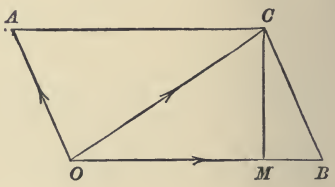


Complete the parallelogram $OACB$; then OC will represent the resultant R in magnitude and direction.

Draw CM perpendicular to OB.

Then $$OM = OB + BM$$

in all all cases, whether M falls within or without OB. In the case when M falls outside OB this is obvious. When M lies between O and B, we have



$$OM = OB - MB;$$

and, remembering that lines measured in opposite directions have opposite signs, we see that $MB = -BM$.

Hence $$OM = OB + BM$$

in this case also.

By Euclid I., 47,

$$OC^2 = OM^2 + CM^2 = (OB + BM)^2 + CB^2 - BM^2$$
$$= OB^2 + BC^2 + 2OB \,.\, BM.$$

But
$$BC = OA = P,$$
$$OB = Q,$$
$$BM = BC \,.\, \frac{BM}{BC} = BC \cos CBM = BC \cos AOB.$$

Hence, if α is the angle between the two given forces,

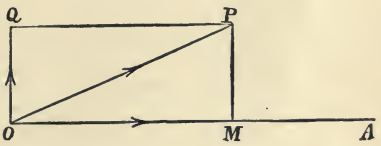
$$R^2 = P^2 + Q^2 + 2P \,.\, Q \cos \alpha.$$

Putting α equal to 0°, 30°, &c., successively, we get the results of examples.

11. To resolve a force into two components respectively parallel and perpendicular to a given line.

Let OP represent the given force, OA the given straight line; and let

$$\angle OAP = \alpha.$$



Draw PM perpendicular to OA, and complete the rectangle QM. Then OQ and OM represent the two components.

$$OM = OP \,.\, \frac{OM}{OP} = OP \cos \alpha,$$

and
$$OQ = PM = OP \,.\, \frac{PM}{OP} = OP \sin \alpha.$$

Hence, if P be a given force, its component parallel to a line making an angle α with it is $P \cos \alpha$, and its component perpendicular to that line is $P \sin \alpha$.

By the help of the results given in § 8, we may thus obtain the results of Examples

12. The sides of a triangle are proportional to the sines of the opposite angles.

One of the angles of triangle, say B, will be acute. C may then be acute, obtuse, or right.

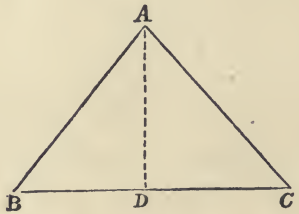


Fig. 1.

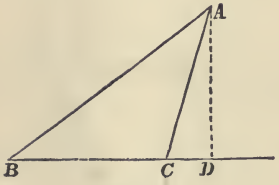


Fig. 2.

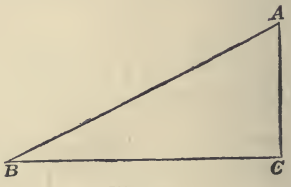


Fig. 3.

Draw AD perpendicular to the base, or base produced.

Then, in all cases, if B be acute, $AD = AB \sin B = c \sin B$.

If C be acute (Fig. 1), $AD = AC \sin C = b \sin C$.

If C be obtuse (Fig. 2),

$$AD = AC \sin ACD = b \sin (180° - C) = b \sin C.$$

If C be right (Fig. 3),

$AD = AC = AC \sin C$ (since $\sin C = \sin 90° = 1$) $= b \sin C$.

Hence, in each case, $c \sin B = b \sin C$

$$\therefore \quad \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{\sin A}{a}, \text{ by symmetry.}$$

RESULTS IN MENSURATION.

It is necessary for the student of Magnetism and Electricity to be acquainted with the following facts in Mensuration. For proofs of them, we must refer to any treatise on Elementary Solid Geometry, such as Wilson's, or to the Appendix at the end of Casey's Euclid.

(1) The **length of the whole circumference of a circle** is $2\pi r$, where r = radius of a circle, and π is an "incommensurable" number—that is, a number which cannot be exactly expressed, either as a vulgar fraction or as a decimal, with a finite number of significant figures. The value of π lies between 3·141592 and 3·141593. The value 3·1416 is sufficiently accurate for all practical purposes, and for most purposes the value 22/7 may be used

(2) The area enclosed by a circle is πr^2.

(3) The **volume of a sphere** is $\frac{4}{3}\pi r^3$.

(4) The **area of the surface of a sphere** is $4\pi r^2$.

(5) The **volume of a cylinder** is $\pi r^2 h$, r being the radius of the base, and h the height of the cylinder.

(6) The **area of the curved surface of a cylinder** is $2\pi rh$.

ANSWERS.

EXAMPLES I. (pages 27, 28).

6. 1 dyne.

7. 1 dyne parallel to AB.

8. $\frac{\sqrt{3}}{1000}$ dyne.

9. $\frac{\sqrt{2}}{5}$ dyne.

10. $2\sqrt{2} : 3\sqrt{3}$.

11. 14·4°.

12. $3\sqrt{3}\,\frac{q}{4a^2}$ dynes, where q denotes the magnitude of each of the equal charges, and a the side of the hexagonal base in cms.

EXAMPLES II. (pages 98—100).

8. $8\sqrt{2}$; $8+\frac{4\sqrt{5}}{5}$.

9. 5.

10. $\frac{2a}{a+b}$, $\frac{2b}{a+b}$.

11. $\frac{1}{4}\frac{Q^2}{C}$.

12. 10 cm. and 12 cm.

13. 25 : 189.

14. 25 : 24.

18. $\frac{1}{16}$, $\frac{1}{4}$, $\frac{1}{16}$.

20. The jar retains $\frac{1}{3}$ of its charge, and the final potential of both jars is $\frac{1}{3}$ of the initial potential of the first charged jar.

22. 1 dyne and $1\frac{9}{16}$ dyne for charges of the same sign, 1 dyne and $\frac{9}{16}$ dyne for charges of opposite sign.

23. 108 : 36.

Examples III. (page 184).

7. $2500\sqrt{2} : 3249$. **8.** 1080 C.G.S. units.

9. $(7{\cdot}5)^2 : (8{\cdot}3)^2 : (10{\cdot}4)^2$. **10.** 9·31.

Examples IV. (pages 319—321).

11. 1·72 ampere; 5·16 volts. **12.** 6·337 ohms.

13. Deflection is practically unchanged. See Art. 79.

14. $1\frac{2}{3}$ ampere, $\frac{9}{13}$ ampere, $\frac{3}{13}$ ampere. **15.** $166\frac{2}{3}$ divisions.

16. ·5 ampere, ·1585 ampere. **17.** 39 : 20.

18. 37 ohms. **19.** $\frac{2}{3}$ ampere.

20. (*a*) $1\frac{5}{6}$ amperes, (*b*) $1\frac{1}{6}\frac{9}{0}$ amperes, (*c*) $\frac{4}{5}$ ampere.

21. 2·1. **22.** $\frac{84}{250}$ ampere, $\frac{21}{250}$ ampere, $\frac{63}{250}$ ampere.

24. H. = ·0154 gram; H_2SO_4 = ·7546 gram; Pb = 1·5923 grams; Cu = ·1884 gram; Ag = 1·6615 grams. Current = ·83075 ampere.

25. 1395 units of heat.

27. ·317 gram Cu, 1·08 grams Ag; ·634 gram Cu, 2·16 grams Ag.

28. 225 : 4 (inside); 25 : 12 (outside).

29. The current in each case is ·5 ampere. One-half as much zinc is consumed in case (*b*) as in (*a*) in the same time.

30. $\frac{e^2}{R+r}$ units of work; $\frac{e^2 r}{J(R+r)^2}$ units of heat.

31. 2·016 grams. **32.** 20 units of heat.

INDEX.

	PAGE
ACLINIC line	163, 165
Action of magnet in uniform magnetic field	139
Action of one magnet on another ...	142
,, ,, points	57
,, ,, voltaic cell	185
Activity of electromotor	343
Alternator	342
Ampère	248
Ampère's rule	209
Annual variations of dip	170
Anode	214
Arc lamp	288
Armature	339
Arrangement of cells	259
,, ,, in series	197, 259
,, ,, in parallel	198, 259
Artificial magnets	125
Astatic galvanometer, Nobili's ...	221
,, needle	220
Attracted-disc electrometers ...	101
Axis, Magnetic	118, 157
BALLISTIC galvanometer ...	273
Barlow's wheel	298
Batteries of cells	197
Batteries, secondary	355
Battery, Planté's	357
Biot's experiment	53
Bridge, Wheatstone's	268
Brush discharge	347
Bunsen's cell	191
CALIBRATION of galvanometer	222
Capacity, Electrical	73
Capacity, Specific inductive ...	33, 85, 106
Cell, Bunsen's	191
,, Daniell's	192
,, Grove's	192
,, Latimer Clark's	247
,, Leclanché	193
,, Poggendorff's bichromate ...	190
,, Smee's	189
Cells, Two-fluid	190
,, Various forms of	189
,, Voltaic	185
Chemical effects of currents ...	213, 276
,, equivalence	276
,, equivalent	277
Circuit, Divided	248
Clark's machine	307
Coefficient of self-induction ...	359
Coil, Induction	303
Comparison of E. M. F.'s	264
,, intensities of magnetic fields	175
,, magnetic moments of magnets ...	178
,, potentials	92
,, resistances	268
Compass, Mariner's	131
Compensating resistance	256
Compound magnets	130
,, wound dynamos	339
Condensers, Electrical	82
Condensing electroscope	89
Conductivity	234
Conductors	6, 7
Conductor carrying current	349
Consequent poles	129
Conservation of energy in a voltaic circuit	282
Constant working of galvanometer	231
Contact theory	193
Convection discharge	58
Coulomb	248
Crown of cups	199
Current	248
,, Extra	306
,, from cells in series and in parallel	259
Currents, Mutual action of	350
DANIELL'S cell	192
Declination	155
Delezenne's circle	298
Demagnetising force	149
Development of heat in circuit ...	285
Diamagnetic substances	119
Diamagnetism	150
Dielectric	85
Dip	155, 158
,, circle	158
Dipping needle	158
Discharging tongs	88
Distribution of electricity on conductors	52
Distribution of magnetism	128
Diurnal variations of dip	170
Divided circuit	248

PAGE

Duplex system... 333
Dynamos 337
,, compound wound ... 339
,, self-excited 339
,, series wound 339
,, shunt wound 339
,, simple form of 307

EARTH inductor 298
Earth's magnetic field 154
Effects of currents 204
,, ,, Chemical... 213, 276
,, ,, Heating ... 204, 282
,, ,, Magnetic 207, 223
Electric bells 330
,, force exerted by charged sphere 100
,, force exerted by plane surface of indefinite extent 101
,, spark 51
Electrical absorption 107
,, attraction and repulsion 15
,, capacity 73
,, condensers 82
,, efficiency of electromotor 343
,, machines 61, 108
,, potential 66
,, units, Table of 275
Electrics 1
Electrification by friction 4
Electrochemical equivalent... ... 278
Electrodes 214
Electrolysis 213, 214
,, Laws of... 277
Electrolytes 203, 204
Electromagnetic current of unit strength 227
Electromagnets 210
Electrometer, Thomson's Quadrant 76
Electrometers 76, 101
Electromotors 342
Electromotor, Activity of 343
,, Electrical efficiency of 343
,, Mechanical efficiency of... 343
Electromotive force 193, 246
,, forces, Comparison of 264
,, ,, Poggendorff's comparison of 266
,, ,, Wiedemann's comparison of 265
Electrophorus 46
Electroplating 344
Electroscope 3, 33, 41
,, Condensing 89
Electrotyping 344, 346
E. M. F., Calculation of 358
E M. F.'s of cells in series and parallel 247
Energy of electrification 24
Equipotential surface 70

PAGE

Equivalence, Chemical 276
Equivalent, Chemical 277
,, Electrochemical ... 278
Essentials of good cell 199
Extra current ... ... , ... 306

FARADAY'S disc 298
,, ice-pail experiment 42
,, laws... 277
,, net 54
Fauri 357
Field magnets 338
Field of electrical force 19
,, magnetic force 132
Floating battery 213
Frictional machines 61

GALVANOMETER, Astatic ... 221
,, Ballistic ... 273
,, Reduction factor of... 231
,, Reflecting ... 234
,, Sine ... 232
,, Tangent ... 229
Galvanometers... 228
Galvanoscopes 211
Gas battery, Grove's 355
Glow, Electric 348
Gold-leaf electroscope 33
Grove's cell 192
Grove's Gas battery 355
Guard-plate 101
Guard-ring 101

HEATING effect of currents 204, 282
Henley's Quadrant Indicator 65
Holtz' Influence Machine 113
Horizontal component of the earth's magnetic field 155, 165
Horse-shoe magnet 130
Hysteresis 242

ICE-PAIL experiment 42
Incandescent lamp 288
Inclination 155, 158
Induction coil 303
,, electrical machines ... 108
,, ,, Holtz 113
,, ,, Voss 113
,, ,, Wimshurst ... 115
,, Electrostatic 29
,, Magnetic 121
,, Magneto-electric ... 290
Insulators 6, 7
Intensities of magnetic field, Comparison of 175
Intensity of electrical field 19
,, ,, magnetic field 133
,, ,, magnetisation ... 146, 150

PAGE
Ions 215
Iron, magnetisation of ... 236
Isoclinic lines 165

JOULE 284
„ effect... 285

KEEPERS 130
Kirchhoff's Laws ... 248, 258

LAMP, Incandescent ... 288
„ Arc 288
Latimer Clark's cell 247
Leclanché cell 193
Lenz's Law 301
Leyden jar 87
Lines of force 20, 133
Liquids, Resistance of ... 352
Local action 198

MAGNETIC attraction and repulsion, Laws of ... 132
„ axis 118, 157
„ effects of currents 207, 223
„ elements 155
„ „ Variations of 170
„ field 133
„ „ of a current ... 223
„ induction ... 121, 150
„ meridian 155
„ moment 140
„ moments of magnets, Comparison of ... 178
„ needle 131
„ permeability 152
„ saturation 239
„ susceptibility... ... 152
Magnetisation by inductive action of earth's magnetic field... 170
„ Intensity of 146
„ Methods of 125
„ of iron 236
Magnetite (Fe_3O_4) 117
Magneto-electric induction, laws of 290, 298
Magnetometer... 166
Magnets 117, 125
„ Compound 130
Mance's method 272
Mariner's compass 131
Mechanical efficiency of electro-motor 343
Molecular theory of magnetisation 127
Moment, Magnetic 140
Morse system 333
Mirror galvanometer... 234

NATURAL magnets 117
Needle, Magnetic 131
Neutral condition 5
Neutral point 324
Nobili's astatic galvanometer ... 221
Non-conductors 8

ŒRSTEDT'S experiment... ... 208
Ohm 249
Ohm's law 246, 256

PARALLEL arrangement of cells 198, 259
Paramagnetic substances 119
Peltier effect 324
Permeability 138, 152, 240
Pile, Thermo-electric... 327
„ Voltaic 199
Planté 357
Plate electrical machine 62
Poggendorff's bichromate cell ... 190
„ method of comparison of E.M.F.'s ... 266
Point charge 19
Points, Action of 57
Polarisation 188
Polarity... 118
Poles, Consequent 129
„ of a battery 197
„ „ cell... 187
„ „ magnet 117, 128
„ of the earth 163
Potential, Electrical 66
„ energy of a charge ... 69
„ of a condenser 91
Potentiometer 267
Power, Thermo-electric 325
Proof plane 10

QUADRANT Electrometer, Thomson's 76
Quadrant Indicator, Henley's ... 65

REDUCTION factor of galvanometer 231
Reflecting galvanometer, Thomson's 234
Relay 335
Replenisher, Thomson's 111
Residual charge 89, 107
Resistance 195, 249
„ box 250
„ coils 249
„ of conductors in series and parallel 252
„ of liquids... 352
„ Specific 251, 353
„ Variation of, with temperature 256
Resistances, Comparison of... ... 268
„ Comparison of, Mance's method 272
„ Comparison of, Wheatstone's bridge method 268

PAGE

Roget's vibrating spiral 352
Ruhmkorff's induction coil ... 303

SATURATION, Magnetic ... 239
Secondary batteries 355
Secular variations 170
Seebeck's discovery 323
Self-excited dynamos 339
Self-induction 305
,, Coefficient of ... 359
Series arrangement of cells 197, 259
Series wound dynamos 339
Shunts 254
Shunt wound dynamos 339
Sine galvanometer 232
Smee's voltaic cell 189
Solenoid... 212
Spark and brush discharge 347
Specific conductivity 252
,, inductive capacity ... 33, 85, 106
,, resistance 251, 353
Surface density 57
Susceptibility, Magnetic 152

TABLE of electrical units ... 275
Tangent galvanometer... ... 229
Telegraphy 332
Telephone 335
Terrestrial magnetism 148 154
Thermo-electric couple, Simple ... 323
,, couple, Neutral point of 324
,, currents 322

PAGE

Thermo-electric piles... 327
,, power 325
Thermo-electricity 322
Thermo-electromotive force ... 322
Thomson effect 326
Thomson's method of measuring galvanometer resistance 271
Thomson's Replenisher 111
Tongs, discharging 88
Torsion Balance 21, 173
Two-fluid cells 190

UNIT of current strength... ... 227
,, quantity of electricity ... 18

VARIATIONS of magnetic elements 170
Vibrating spiral 352
Volt 246
Voltaic cells 185
,, pile 199
Voltameter 215
Volta's experiment 200
Voss's Influence Machine 113

WATT... 284
Wheatstone's bridge 268
Wiedemann's method of comparison of E. M. F.'s... 265
Wimshurst machine 115
Working constant of galvanometer 231

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